Find The Exact Value Of The Expression. 6) Cos 20 Cos 40 - Sin 20 Sin 40 A) 1 943 Use The Given Information

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When tackling trigonometric expressions, one of the fundamental skills involves recognizing and applying identities to simplify complex expressions into manageable forms. The problem at hand — Cos 20° Cos 40° - Sin 20° Sin 40° — is a classic example where trigonometric identities can be used to find an exact value efficiently. Understanding how to evaluate such expressions accurately is essential for students, educators, and anyone interested in mathematical problem-solving.

In this article, we will explore how to find the exact value of this expression step-by-step, discuss relevant trigonometric identities, and provide tips for approaching similar problems. By the end, you'll have a comprehensive understanding of the methods involved and how to apply them to various trigonometric expressions.

Understanding the Trigonometric Expression

The expression in question is:

\[\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ \]

At first glance, it might seem complex due to the different angles involved. However, trigonometry offers identities that allow us to simplify such expressions. Recognizing patterns and applying the right identities can turn this into a straightforward calculation.

Key Trigonometric Identities Used

To simplify the expression, we need to recall some fundamental identities:

Product-to-Sum and Sum-to-Product Identities

- Cosine of sum and difference:

\[

$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$

$$
\cos (A - B) = \cos A \cos B + \sin A \sin B
$$

- Complementary identities:

$$
\sin (90^\circ - \theta) = \cos \theta
$$

$$
\cos (90^\circ - \theta) = \sin \theta
$$

Applying Trigonometric Identities to Simplify the Expression

Notice that the numerator of our expression resembles the cosine sum identity:

$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$

Comparing this to our expression:

$$
\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ
$$

we see that:

$$
A = 20^\circ, \quad B = 40^\circ
$$

Therefore, the expression simplifies directly to:

$$
\cos (20^\circ + 40^\circ) = \cos 60^\circ
$$

Calculating the Exact Value

Since the expression simplifies neatly to \(\cos 60^\circ\), we can now determine its exact value:

\[
\boxed{
\cos 60^\circ = \frac{1}{2}
}
\]

This result is exact and well-known:

\[
\boxed{
\cos 60^\circ = 0.5
}
\]

Final Answer and Explanation

Based on the simplification, the exact value of the original expression:

\[
\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ
\]

is:

\[
\boxed{
\frac{1}{2}
}
\]

which corresponds to 0.5 in decimal form.

Additional Insights and Tips for Similar Problems

Understanding how to manipulate and simplify trigonometric expressions is vital for solving more complex problems. Here are some essential tips:

- **Identify familiar identities:** Recognize when expressions match known identities such as sum or difference formulas.
- **Use angle addition formulas:** Know that $\cos (A + B)$ and $\cos (A - B)$ often help simplify products of sines and cosines.
- **Convert products to sums:** Product-to-sum identities can be useful, especially for integrals or more complex algebraic manipulations.
- **Check special angles:** Many common angles like 30°, 45°, 60°, and 90° have known exact values, simplifying calculations.
- **Verify with known values:** Always double-check by plugging in the angles to a calculator or recalling key values to confirm the simplification.

Related Concepts and Further Study

To deepen your understanding of trigonometry, consider exploring the following topics:

1. Sine and Cosine Addition and Subtraction Formulas

- $\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$
- $\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$

2. Double-Angle and Half-Angle Formulas

- $\cos 2A = 2 \cos^2 A - 1$
- $\sin 2A = 2 \sin A \cos A$

3. Practical Applications

These identities are used in fields like physics (wave mechanics), engineering (signal processing), and computer graphics (rotations).

Conclusion

In summary, the key to solving the expression $\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$ lies in recognizing it as a cosine addition formula:

$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$

which simplifies directly to $\cos 60^\circ$. The exact value of $\cos 60^\circ$ is $\frac{1}{2}$, making the solution straightforward once the identity is identified.

Mastering these identities enhances mathematical problem-solving skills and provides a foundation for tackling more complex trigonometric challenges. Always remember to analyze the structure of the expression carefully and recall the most relevant identities to simplify efficiently.

Meta Description: Learn how to find the exact value of $\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$ using fundamental trigonometric identities. Step-by-step explanation with tips for solving similar problems.

Frequently Asked Questions

What is the exact value of the expression cos 20° cos 40° - sin 20° sin 40°?

The exact value is 0.

Which trigonometric identity can be used to simplify cos 20° cos 40° - sin 20° sin 40°?

It can be simplified using the cosine addition formula: cos(A + B) = cos A cos B - sin A sin B.

Using the cosine addition formula, what is cos 20° cos 40° - sin 20° sin 40° equal to?

It simplifies to cos(20° + 40°) = cos 60°.

What is the value of cos 60°?

cos 60° = 0.5.

Is the answer 0.5 or 1 for the expression cos 20° cos 40° - sin 20° sin 40°?

The exact value of the expression is 0.5.

How does the given multiple-choice option '1 943' relate to the problem?

It appears to be a typo or unrelated; the correct exact value of the expression is 0.5, not '1 943'.

What is the significance of recognizing the cosine addition formula in solving this problem?

It allows for direct simplification of the expression into a single cosine function, facilitating an exact value calculation.

Additional Resources

Find The Exact Value Of The Expression. 6) Cos 20° Cos 40° - Sin 20° Sin 40° A) 1 943 Use The Given Information

Understanding and evaluating trigonometric expressions with precision is a fundamental skill in mathematics, particularly in topics related to angles, identities, and functions. The problem at hand involves finding the exact value of the expression:

$$\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$$

This type of problem is common in trigonometry, often designed to test familiarity with core identities and the ability to simplify complex expressions into more manageable forms. In this article, we will thoroughly analyze this expression, explore relevant identities, and demonstrate step-by-step how to arrive at the exact value.

Understanding the Expression

The given expression:

$$\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$$

contains basic trigonometric functions with specific angles. The goal is to evaluate this expression precisely, ideally reducing it to a simple known value or a common trigonometric function.

Key points to consider:
- The angles involved are 20° and 40°, which are multiples of 10°.

- The structure resembles the cosine of a sum or difference, hinting at the use of sum and difference formulas.

Relevant Trigonometric Identities

To evaluate the given expression, it is essential to recall the fundamental identities in trigonometry. The most relevant here are the sum and difference formulas for cosine and sine:

1. Cosine of Sum and Difference
$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$
$$
\cos (A - B) = \cos A \cos B + \sin A \sin B
$$

2. Sine of Sum and Difference
$$
\sin (A + B) = \sin A \cos B + \cos A \sin B
$$
$$
\sin (A - B) = \sin A \cos B - \cos A \sin B
$$

The key identity here is the cosine of sum formula:

$$
\cos (A + B) = \cos A \cos B - \sin A \sin B
$$

This is directly applicable to our expression.

Applying the Identity

Notice that the given expression:

$$ \cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ $$

matches the form of $\cos (A + B)$. Specifically, if we set:

$$
A = 20^\circ,\quad B = 40^\circ
$$

\]

then:

\[
\cos (A + B) = \cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ
\]

which is exactly our original expression.

Therefore:

\[
\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ = \cos (20^\circ + 40^\circ) = \cos 60^\circ
\]

This simplifies our problem to evaluating \(\cos 60^\circ\).

Calculating the Exact Value

The cosine of 60° is a standard value from the unit circle:

\[
\cos 60^\circ = \frac{1}{2}
\]

Thus, the exact value of the original expression is:

\[
\boxed{\frac{1}{2}}
\]

Summary and Final Answer

The original expression:

\[\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ \]

simplifies to \(\cos (20^\circ + 40^\circ) = \cos 60^\circ\), which is a well-known value:

\[
\boxed{\frac{1}{2}}

\]

This method highlights the power of fundamental identities in simplifying seemingly complex trigonometric expressions.

Features and Pros of Using Trigonometric Identities

- Simplification: Identities reduce complex expressions to basic known values.
- Efficiency: Avoids lengthy calculations or approximations.
- Universality: Applicable to a wide range of problems involving angles and functions.
- Educational Value: Reinforces understanding of core mathematical concepts.

Pros

- Quickly simplifies expressions without the need for calculator use.
- Deepens understanding of the relationship between angles and functions.
- Facilitates solving more challenging problems involving multiple angles.

Cons

- Requires familiarity with identities; unfamiliarity can cause confusion.
- Not always straightforward if the expression doesn't match common identities.
- Sometimes multiple steps are needed to recognize the applicable identity.

Additional Insights and Related Problems

While this problem is straightforward owing to the direct application of the cosine sum identity, similar exercises can involve other identities or more complex angles. For instance:

- Evaluating expressions like \(\sin (A + B)\) or \(\tan (A - B)\).
- Using identities to solve equations involving multiple trigonometric functions.
- Simplifying expressions with angles in different units (degrees vs radians).

Related Practice:

1. Find \(\sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ\).
Solution: Recognize as \(\sin (A + B)\) with \(A=30^\circ, B=60^\circ\), which simplifies to \(\sin 90^\circ = 1\).

2. Simplify \(\cos 45^\circ \cos 45^\circ + \sin 45^\circ \sin 45^\circ\).
Solution: Use the cosine of sum identity: \(\cos (A - B)\) or recognize as \(\cos^2 45^\circ + \sin^2

$45^\circ = 1$.

Conclusion

The problem of finding the exact value of $\cos 20^\circ \cos 40^\circ - \sin 20^\circ \sin 40^\circ$ underscores the importance and utility of fundamental trigonometric identities. By recognizing the structure of the expression as a cosine of a sum, the solution becomes straightforward and elegant:

$$
\boxed{\frac{1}{2}}
$$

This approach demonstrates how mastery of identities fosters efficient problem-solving in trigonometry, providing clarity and precision in mathematical calculations. Whether used in academic settings or practical applications like engineering and physics, these identities serve as invaluable tools for simplifying and evaluating complex expressions.

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