Find The Particular Solution To Y ' = 2sin(x) Given The General Solution Is Y = C - 2cos(x) And The Initial

Find The Particular Solution To Y ' = 2sin(x) Given The General Solution Is Y = C - 2cos(x) And The Initial problem is a common question in the realm of differential equations, particularly when dealing with first-order linear differential equations. Identifying a particular solution is crucial because it helps us understand the specific behavior of a differential equation under given initial conditions, rather than just its general form. In this article, we will explore the methods to find the particular solution for the differential equation $Y' = 2 \sin(x)$, given that the general solution is $Y = C - 2 \cos(x)$, and discuss how initial conditions influence this solution.

Understanding the Differential Equation and Its General Solution

Before diving into the process of finding the particular solution, it’s essential to understand the structure of the given differential equation and its general solution.

The Differential Equation $Y' = 2 \sin(x)$

This is a first-order differential equation where the derivative of Y with respect to x is expressed as a simple sinusoidal function. It indicates that the rate of change of Y varies sinusoidally with x.

The General Solution $Y = C - 2 \cos(x)$

The general solution includes an arbitrary constant C, which accounts for the family of solutions arising from the indefinite integration process. The presence of $-2 \cos(x)$ suggests that the integral of $2 \sin(x)$ with respect to x results in $-2 \cos(x)$.

Deriving the General Solution

Though the general solution is provided, it’s beneficial to verify how it’s derived.

Method: Integration of the Differential Equation

Given:

$$
Y' = 2 \sin(x)
$$

we integrate both sides with respect to x:

$$
Y = \int 2 \sin(x) \, dx + C
$$

Calculating the Integral

Recall that:

$$
\int \sin(x) \, dx = - \cos(x)
$$

Thus:

$$
Y = 2 \times (- \cos(x)) + C = -2 \cos(x) + C
$$

which confirms the given general solution:

$$
Y = C - 2 \cos(x)
$$

Finding the Particular Solution

The key step now is to find the particular solution Y_p, which satisfies the differential equation for a specific initial condition, often denoted as $Y(x_0) = Y_0$.

What Is a Particular Solution?

A particular solution is a specific function that satisfies the differential equation and the initial condition, as opposed to the general solution which encompasses all such solutions.

Role of Initial Conditions

Initial conditions are specific values of x and Y that allow us to determine the value of the constant C. For example, if given $Y(x_0) = Y_0$, then:

$$
Y_0 = C - 2 \cos(x_0)
$$

which can be rearranged to find C.

Step-by-Step Process to Find the Particular Solution

Let's consider how to find the particular solution given an initial condition.

Step 1: Write Down the General Solution

$$
Y = C - 2 \cos(x)
$$

Step 2: Apply the Initial Condition

Suppose the initial condition is:

$$
Y(x_0) = Y_0
$$

then plugging into the general solution:

$$
Y_0 = C - 2 \cos(x_0)
$$

which yields:

$$
C = Y_0 + 2 \cos(x_0)
$$

Step 3: Find the Particular Solution

Substitute C back into the general solution:

$$
Y_p = (Y_0 + 2 \cos(x_0)) - 2 \cos(x)
$$

This is the particular solution satisfying the initial condition.

Example:

If $Y(0) = 3$, then:

$$
C = 3 + 2 \cos(0) = 3 + 2 \times 1 = 3 + 2 = 5
$$

and the particular solution is:

$$
Y_p = 5 - 2 \cos(x)
$$

General Approach for Any Initial Condition

The process outlined above can be generalized for any initial condition:

1. Start with the general solution $Y = C - 2 \cos(x)$.
2. Plug in the initial values (x_0, Y_0) to determine C:
    - Calculate $C = Y_0 + 2 \cos(x_0)$.
3. Write the particular solution:
$$
Y_p = Y_0 + 2 \cos(x_0) - 2 \cos(x)
$$

Significance of Finding the Particular Solution

Understanding how to find the particular solution is vital in applications because:

- It provides the exact behavior of the system under specific conditions.
- It allows for precise predictions and modeling in physics, engineering, and other sciences.
- It helps in solving real-world problems where initial conditions are known.

Additional Tips and Common Mistakes

Tips for Successfully Finding Particular Solutions

- Always verify the initial conditions before substituting into the general solution.

- Double-check the integral calculations to avoid sign errors, especially with sinusoidal functions.
- Remember that the constant \(C \) can be positive or negative depending on the initial condition.
- Use a calculator or software for complex initial conditions to avoid arithmetic mistakes.

Common Mistakes to Avoid

- Confusing the general solution with a particular solution—remember the general includes an arbitrary constant.
- Incorrectly calculating the integral or algebraic manipulation when solving for \(C \).
- Neglecting to substitute the initial condition correctly, which can lead to an incorrect particular solution.

Conclusion

Finding the particular solution to the differential equation \(Y' = 2 \sin(x) \) when given the general solution \(Y = C - 2 \cos(x) \) is a straightforward process once the initial condition is known. The key steps involve substituting the initial values into the general solution to determine the constant \(C \), thereby obtaining a specific, tailored solution that models the behavior of the system under those initial conditions. This process is fundamental in differential equations, serving as a bridge between the theoretical solutions and practical applications in science and engineering. Mastery of this technique enhances your ability to analyze and interpret real-world phenomena governed by differential equations.

Meta Description: Learn how to find the particular solution to \(Y' = 2 \sin(x) \) given its general solution \(Y = C - 2 \cos(x) \), including detailed steps with initial conditions for precise solutions.

Frequently Asked Questions

How do you determine the particular solution for the differential equation $Y' = 2\sin(x)$ using the given general solution?

Since the general solution is $Y = C - 2\cos(x)$, the particular solution can be found by substituting the initial conditions into this expression to solve for C. Without initial conditions, the particular solution includes the constant C, which can be determined once initial values are known.

What role does the initial condition play in finding the particular solution from the general solution $Y = C - 2\cos(x)$?

The initial condition allows us to solve for the constant C by substituting the given initial values of y and x into the general solution, thus specifying the particular solution that satisfies those initial conditions.

If $y(0) = 3$ is given, what is the particular solution to $Y' = 2\sin(x)$?

Substitute $x=0$ and $y=3$ into $Y = C - 2\cos(x)$: $3 = C - 2\cos(0) \Rightarrow 3 = C - 2(1) \Rightarrow 3 = C - 2$. Therefore, $C = 5$. The particular solution is $Y = 5 - 2\cos(x)$.

Why is the general solution $Y = C - 2\cos(x)$ valid for the differential equation $Y' = 2\sin(x)$?

Because taking the derivative of $Y = C - 2\cos(x)$ yields $Y' = 2\sin(x)$, which matches the original differential equation. The constant C accounts for the family of solutions differentiated by initial conditions.

Can the particular solution be expressed without the constant C? If so, how?

Yes, if initial conditions are provided. For example, given $y(0) = y_0$, you can solve for C and substitute back into the general solution, resulting in a specific particular solution without the constant C explicitly present.

Additional Resources

Find the Particular Solution to \(Y' = 2 \sin(x)\) Given the General Solution \(Y = C - 2 \cos(x)\) and the Initial Conditions

---

Introduction

In the realm of differential equations, understanding how to find particular solutions from a general solution is a fundamental skill, especially when initial conditions are provided. The differential equation $Y' = 2 \sin(x)$ serves as a classic example illustrating the process of deriving a specific solution tailored to particular circumstances. The general solution, expressed as $Y = C - 2 \cos(x)$, encapsulates an entire family of solutions parameterized by the constant C. To pinpoint the exact solution that satisfies given initial conditions, whether it pertains to a specific value of Y at a particular x, or other constraints, is essential for precise modeling and analysis.

This article delves into the systematic approach to identifying the particular solution for the differential equation $Y' = 2 \sin(x)$, exploring the derivation from the general solution, the application of initial conditions, and the significance of these solutions in mathematical and real-world contexts. The discussion emphasizes clarity, depth, and analytical rigor, making it a valuable resource for students, educators, and professionals engaged with differential equations.

Understanding the Differential Equation and Its General Solution

The Differential Equation $Y' = 2 \sin(x)$

At its core, the differential equation $Y' = 2 \sin(x)$ specifies the rate of change of a function $Y(x)$ with respect to x. It indicates that the derivative of Y at any point x is proportional to the sine of x, scaled by a factor of 2.

Key points:

- It is a first-order ordinary differential equation.
- It is linear and separable, which simplifies the process of solving.
- The right side, $2 \sin(x)$, is a known function, making the integration straightforward.

Deriving the General Solution

To find the general solution, we integrate both sides with respect to x:

$$
Y' = 2 \sin(x)
$$

$$
\Rightarrow Y = \int 2 \sin(x) \, dx + C
$$

Calculating the integral:

$$
\int 2 \sin(x) \, dx = -2 \cos(x) + D
$$

where D is an arbitrary constant of integration. However, since the general solution is typically expressed with a single constant C, we set:

$$
Y = C - 2 \cos(x)
$$

This solution encompasses all possible solutions to the differential equation, with the constant C accounting for initial conditions or boundary constraints.

The Role of Initial Conditions in Determining the Particular Solution

What Are Initial Conditions?

Initial conditions specify the value of the solution Y (or its derivatives) at a particular point $x = x_0$. For example, an initial condition might be given as:

$$
Y(x_0) = Y_0
$$

where Y_0 is a known number.

Why Are Initial Conditions Important?

While the general solution contains an arbitrary constant C, the particular solution is the specific function that satisfies both the differential equation and the initial condition. This process involves:

- Substituting known values into the general solution.
- Solving for C.

This ensures that the solution aligns with the real-world or theoretical scenario described by the initial conditions.

Step-by-Step Process to Find the Particular Solution

1. Write Down the General Solution

As established:

$$
Y = C - 2 \cos(x)
$$

2. Apply the Initial Condition

Suppose the initial condition is:

$$
Y(x_0) = Y_0
$$

Substituting the initial point into the general solution:

$$
Y_0 = C - 2 \cos(x_0)
$$

3. Solve for C

Rearranging:

$$
C = Y_0 + 2 \cos(x_0)
$$

This formula allows us to determine the specific value of C given any initial data.

4. Write the Particular Solution

Substituting C back into the general solution:

$$
Y = \left(Y_0 + 2 \cos(x_0) \right) - 2 \cos(x)
$$

This is the particular solution satisfying the initial condition.

Illustrative Example

To make the process concrete, consider a specific initial condition:

$$
Y(0) = 3
$$

Applying this:

$$
3 = C - 2 \cos(0)
$$

Since $\cos(0) = 1$:

$$
3 = C - 2(1) = C - 2
$$

$$
\Rightarrow C = 3 + 2 = 5
$$

Hence, the particular solution is:

$$
Y = 5 - 2 \cos(x)
$$

This function satisfies both the differential equation $Y' = 2 \sin(x)$ and the initial condition $Y(0) = 3$.

Significance and Applications

Theoretical Significance

Understanding how to transition from a general solution to a particular solution is crucial in differential equations because it allows mathematicians and scientists to tailor solutions to specific problems. It demonstrates the power of initial or boundary conditions in refining solutions from an entire family of functions.

Practical Applications

- Physics: Modeling oscillatory systems such as pendulums or electrical circuits, where initial conditions determine the specific amplitude or phase.
- Engineering: Designing control systems with particular starting states.
- Biology: Describing population dynamics with known initial populations.
- Economics: Forecasting financial models based on initial data points.

Limitations and Considerations

While the process seems straightforward, it relies heavily on the availability and accuracy of initial conditions. In real-world scenarios, measurements may contain errors, and the chosen initial conditions might only approximate the true starting point of a system.

Advanced Topics and Related Concepts

Generalizations and Variations

- When the differential equation involves non-constant coefficients, the process to find particular solutions becomes more complex, often requiring methods such as variation of parameters or undetermined coefficients.
- In cases where initial conditions are given for derivatives (e.g., $Y'(x_0) = Y'_0$), similar substitution techniques apply but may involve additional steps.

Numerical Methods

When an explicit initial condition is not available or the equations are more complex, numerical methods such as Euler's method, Runge-Kutta, or finite difference approaches are employed to approximate particular solutions.

Final Thoughts

The ability to find a particular solution to a differential equation like $Y' = 2 \sin(x)$ is foundational in applied mathematics. It bridges the gap between theoretical solutions and real-world problems by incorporating initial data. The process underscores the importance of understanding general solutions, the role of integration constants, and the significance of initial or boundary conditions.

In practice, this methodology empowers scientists and engineers to model phenomena accurately, predict future behavior, and design systems with specific starting conditions. As demonstrated through the example, the steps are systematic, and mastering them enhances problem-solving skills across diverse scientific disciplines.

Summary

- The differential equation $Y' = 2 \sin(x)$ has a general solution $Y = C - 2 \cos(x)$.
- To find the particular solution, apply the initial condition $Y(x_0) = Y_0$.
- Solve for C using the initial data: $C = Y_0 + 2 \cos(x_0)$.
- Write the particular solution: $Y = (Y_0 + 2 \cos(x_0)) - 2 \cos(x)$.
- This particular solution models the specific scenario dictated by the initial data, enabling precise analysis and application.

In conclusion, mastering the process of deriving particular solutions from general solutions is essential for effective problem-solving in differential equations and their myriad applications across science and engineering.

Find The Particular Solution To Y 2sin X Given The General Solution Is Y C 2cos X And The Initial

Find The Particular Solution To Y 2sin X Given The General Solution Is Y C 2cos X And The Initial

Related Articles

- Prompt Think About How The Geologic Time Scale Was Created And How It Is Divided. Then Answer The Following
- Ccording To Roosevelt, What Danger To Society Do Muckrakers Pose? Group Of Answer Choices They Can Convince

- Parallel Lines R And S Are Intersected By A Transversal Line M.Label A Pair Of Corresponding Angle Measures

Back to Home