Find The Size Of Each Of 9 Payments Made At The End Of Each Year Into A 7% Rate Sinking Fund Which Produces

Find The Size Of Each Of 9 Payments Made At The End Of Each Year Into A 7% Rate Sinking Fund Which Produces a specified future amount is a common problem in financial mathematics, particularly in the context of savings plans, loan amortizations, and corporate sinking funds. This article provides a comprehensive guide to calculating the annual payment required to accumulate a desired future sum in a sinking fund that earns interest at a 7% rate, with payments made at the end of each year over a period of nine years.

Understanding the Concept of a Sinking Fund

What Is a Sinking Fund?

A sinking fund is a means by which a company or individual sets aside money over time to repay a debt or replace assets in the future. It involves making regular payments into an interest-earning account, which grows over time due to compound interest.

Importance of Sinking Funds

- Ensures funds are available for future liabilities
- Reduces the burden of a lump sum payment at maturity
- Helps manage cash flows efficiently

- Provides a disciplined savings approach

Basic Assumptions in Calculations

When calculating payments for a sinking fund, the following assumptions are typically made:

- Payments are made at the end of each period (ordinary annuity)
- The interest rate is compounded annually
- The future value (FV) or the amount desired at the end of the period is known
- Payments are of equal size

Formulating the Problem: Key Variables

To solve for the size of each payment in a sinking fund, we need to understand the variables involved:

- **FV**: The future value or the amount the fund should reach after 9 years
- **PMT**: The annual payment amount (what we are solving for)
- **i**: The annual interest rate (7% in this case)
- **n**: Number of payments (9 payments)
- **Type of payment timing**: Payments are made at the end of each year (ordinary annuity)

Mathematical Foundations: The Future Value of an Ordinary Annuity

Future Value Formula for End-of-Year Payments

The future value (FV) of an ordinary annuity (series of equal payments made at the end of each period) can be calculated using the formula:

$$
FV = PMT \times \frac{(1 + i)^n - 1}{i}
$$

Where:
- FV is the future value of the sinking fund after n payments
- PMT is the annual payment
- i is the interest rate per period
- n is the total number of payments

This formula assumes payments are made at the end of each period, which aligns with the scenario described.

Calculating the Payment Amount (PMT)

Rearranged Formula

To find the size of each payment, we rearrange the formula:

$$
PMT = \frac{FV \times i}{(1 + i)^n - 1}
$$

This formula allows us to compute the annual payment required to reach a specific future value after n years at a given interest rate.

Step-by-Step Calculation

Suppose the future value FV is known, say, $100,000 (or any other target amount). The calculation proceeds as follows:

1. Convert interest rate to decimal: $i = 7\% = 0.07$
2. Determine the number of periods: $n = 9$
3. Calculate $(1 + i)^n$:

$$
(1 + 0.07)^9 = 1.07^9
$$

4. Calculate numerator and denominator:

$$
PMT = \frac{FV \times 0.07}{1.07^9 - 1}
$$

5. Plug in the target FV value and compute.

Practical Examples: Computing Payments for Different Future Values

Example 1: Future Value of $100,000

Assuming you want to accumulate $100,000 in 9 years:

$$
PMT = \frac{100,000 \times 0.07}{1.07^9 - 1}
$$

Calculations:

- $1.07^9 \approx 1.8385$
- $1.8385 - 1 = 0.8385$
- Numerator: $100,000 \times 0.07 = 7,000$

Therefore:

$$
PMT = \frac{7,000}{0.8385} \approx 8,349.68
$$

Result: Each annual payment should be approximately $\$8,349.68$.

Example 2: Future Value of $50,000

Similarly, for a target future value of $50,000:

$$
PMT = \frac{50,000 \times 0.07}{1.07^9 - 1} \approx \frac{3,500}{0.8385} \approx 4,174.84
$$

Result: Each annual payment should be approximately $4,174.84.

Factors Affecting the Calculation

Interest Rate Variations

- A higher interest rate decreases the required annual payment for a given future value.
- Conversely, a lower interest rate increases the annual payment needed.

Number of Payments

- Increasing the number of payments reduces the size of each payment, assuming the same future value.
- Fewer payments mean larger annual contributions.

Target Future Value

- The larger the desired future sum, the larger the annual payments needed, all else being equal.

Additional Considerations in Sinking Fund Calculations

Payment Timing

- The calculations above assume payments are made at the end of each period (ordinary annuity).
- If payments are made at the beginning (annuity due), the formula adjusts slightly.

Inflation and Changes in Interest Rates

- Real-world scenarios may involve changing interest rates or inflation, requiring more complex models.

Tax Implications

- Taxation on interest earned can influence net returns and payment calculations.

Using Financial Calculators and Software

Modern financial calculators and spreadsheet software like Excel simplify these calculations:

- Excel Formula: `=PMT(rate, nper, pv, fv, type)`
- For payments made at the end of the period, `type` = 0.

Example in Excel:

```
=PMT(0.07, 9, 0, -100000)
```

```

This returns approximately \$8,349.68, matching manual calculations.

---

## Summary and Key Takeaways

- To find the size of each of the 9 payments in a sinking fund earning 7% annually, use the future value of an ordinary annuity formula.
- Rearranged, the formula is:

$$
PMT = \frac{FV \times i}{(1 + i)^n - 1}
$$

- Accurate estimation relies on understanding the timing of payments, interest rates, and desired future value.
- Utilizing financial tools can streamline these calculations.

---

## Conclusion

Calculating the required annual payments for a sinking fund involves understanding the relationship between future value, interest rate, payment timing, and payment amount. With the formulas and approach outlined above, individuals and organizations can plan effectively to meet their financial goals, whether saving for a large purchase, repaying debt, or funding future obligations. By carefully considering the variables and employing financial calculators, it becomes straightforward to determine
```

the precise payments needed to achieve the desired future sum in a 7% interest environment over nine years.

Remember: Always verify your calculations and consider consulting a financial advisor for complex or large-scale financial planning.

Frequently Asked Questions

How do I determine the size of each payment in a 7% sinking fund with 9 annual payments?

To find each payment amount, use the sinking fund formula: Payment = (Future Value × Rate) / [(1 + Rate)^Number of Payments - 1]. Here, the future value is typically zero at the start, so adjust accordingly based on the problem specifics.

What is the significance of the 7% rate in calculating sinking fund payments?

The 7% rate represents the annual interest rate earned on the sinking fund, which affects how much needs to be deposited each period to reach the target amount by the end of the term.

If the total amount accumulated at the end of 9 years is known, how can I find the annual payment into the sinking fund?

Use the present value of an annuity formula adjusted for the interest rate: Payment = Future Value × (Rate) / [(1 + Rate)^Number of Payments - 1], or rearranged if the future value is known.

Can I calculate the size of payments if I know the total accumulated amount at the end of 9 years?

Yes, by rearranging the future value of an annuity formula, you can solve for the periodic payment: Payment = Future Value × Rate / [(1 + Rate)^Number of Payments - 1].

What assumptions are made when calculating payments for a sinking fund at 7% interest over 9 years?

The calculations assume that interest is compounded annually at 7%, payments are made at the end of each year, and the interest rate remains constant throughout the period.

How does increasing the interest rate affect the size of each payment in a sinking fund?

An increase in the interest rate decreases the size of each payment needed because the fund earns more interest each year, reducing the amount that needs to be deposited annually.

Additional Resources

Find The Size Of Each Of 9 Payments Made At The End Of Each Year Into A 7% Rate Sinking Fund Which Produces

---

Introduction

In the world of finance and investment planning, sinking funds serve as a vital mechanism for systematically accumulating capital to meet future liabilities, such as bond redemptions or large capital expenditures. A sinking fund is essentially a dedicated savings account into which periodic payments

are made, with the goal of gradually building up a lump sum by a specified date.

This article examines a typical scenario: determining the size of each of nine annual payments made at the end of each year into a sinking fund that earns interest at a 7% annual rate. The objective is to find out how much needs to be deposited each year to reach a target amount, or to understand the future value generated by these payments. We will delve into the financial mathematics involved, explore the relevant formulas, and demonstrate step-by-step calculations for clarity.

Understanding the Scenario

Let's define the parameters of the sinking fund problem:

- Interest rate (i): 7% per annum
- Number of payments (n): 9 payments, made at the end of each year
- Payment amount (PMT): Unknown; to be calculated
- Payment timing: Payments are made at the end of each year
- Future value (FV): Typically, the amount accumulated at the end of the ninth year (or at a specified date)
- Present value (PV): Not specified, but often considered when calculating the necessary annual payment
- Interest compounding: Assumed to be annual and at the same rate as the payment periods

In most cases, the goal is to determine the size of each payment (PMT) necessary to reach a specified future value (FV). Alternatively, if the amount of each payment is known, one might compute the total future value or the present value of the fund.

The Mathematics of Sinking Funds

The Future Value of an Annuity

When making equal payments at regular intervals, the future value (FV) of these payments, invested at a fixed interest rate, can be calculated using the Future Value of an Ordinary Annuity formula:

$$
FV = PMT \times \frac{(1 + i)^n - 1}{i}
$$

Where:
- PMT = the payment made at the end of each period
- i = interest rate per period
- n = number of payments

This formula assumes payments are made at the end of each period, which aligns with our scenario.

The Present Value of an Annuity

Similarly, if the future value is known, we can find the payment amount as:

$$
PMT = \frac{FV \times i}{(1 + i)^n - 1}
$$

Conversely, if the target amount or future value at the end of the ninth year is given, this formula allows us to determine the annual payment required.

Calculating the Payment: Step-by-Step

Suppose we are asked to find the size of each payment required to accumulate a future sum FV after 9 years at 7% interest. Here's how to proceed:

Step 1: Identify the Target Future Value

Let's assume the goal is to accumulate \$100,000 at the end of 9 years.

Step 2: Convert the Interest Rate

Interest rate per annum: $i = 7\%$ or 0.07

Step 3: Apply the Annuity Future Value Formula

Using:

$$
PMT = \frac{FV \times i}{(1 + i)^n - 1}
$$

Plugging in the numbers:

$$
PMT = \frac{100,000 \times 0.07}{(1 + 0.07)^9 - 1}
$$

Calculating:

$$
(1 + 0.07)^9 = 1.07^9
$$

Using a calculator or financial software:

$$
1.07^9 \approx 1.838
$$

Thus:

$$
PMT = \frac{100,000 \times 0.07}{1.838 - 1} = \frac{7,000}{0.838} \approx 8,348.45
$$

Result:
Each annual payment must be approximately \$8,348.45 to accumulate \$100,000 after 9 years at 7% interest, with payments made at the end of each year.

Variations and Additional Considerations

1. Payments Made at the Beginning of Each Year (Annuity Due)

If payments are made at the beginning of each period, the formula adjusts slightly to:

$$
FV = PMT \times \frac{(1 + i)^n - 1}{i} \times (1 + i)
$$

This typically results in a slightly higher accumulated amount for the same payment, since each payment earns interest for an additional period.

2. Targeting a Different Future Value

If the desired future value changes, simply substitute the new FV into the formula and recalculate.

3. Calculating the Present Value of the Fund

Sometimes, the question is reversed: given a series of payments, what is the present value? The Present Value of an Ordinary Annuity is:

$$
PV = PMT \times \frac{1 - (1 + i)^{-n}}{i}
$$

This helps in determining how much the fund is worth today based on future periodic payments.

Practical Application: An Example Scenario

Imagine a corporation planning to redeem bonds in 9 years and aiming to set aside funds annually to ensure they have enough money when the time comes. They decide on a 7% interest rate and want to know how much to deposit each year.

Given:
- Future value needed: \$150,000
- Number of payments: 9
- Rate: 7%

Applying the formula:

$$
```
```

$$
PMT = \frac{150,000 \times 0.07}{(1.07)^9 - 1}
$$

Calculate:

$$
(1.07)^9 \approx 1.838
$$

$$
PMT = \frac{10,500}{0.838} \approx 12,526.20
$$

Conclusion:
The company must deposit approximately \$12,526.20 at the end of each year into the sinking fund to reach \$150,000 in nine years.

Significance of Accurate Calculations

Understanding how to calculate the appropriate payment size ensures that organizations and individuals can plan for future financial obligations effectively. Underestimating payments could leave shortfalls, while overestimating might tie up unnecessary funds. The key is precise application of the formulas, considering the timing of payments and interest rates.

Furthermore, these calculations highlight the importance of interest compounding in wealth accumulation strategies. The 7% rate used here exemplifies a reasonable assumption for investment returns, though actual rates can vary based on market conditions.

Final Thoughts

Determining the size of each payment into a sinking fund is a fundamental aspect of disciplined financial planning. Whether saving for future projects, bond redemptions, or other long-term liabilities, the principles outlined in this article provide a solid foundation for making informed decisions.

By mastering the formulas for the future and present values of annuities, investors, financial managers, and individuals can accurately project required contributions, optimize their savings strategies, and ensure that their financial goals are met within the desired timeframe.

In summary, to find the size of each of the nine payments made at the end of each year into a 7% rate sinking fund that produces a specified future sum, one must:

- Clearly define the target future value
- Convert the annual interest rate to decimal form
- Use the future value of an ordinary annuity formula
- Substitute known values and perform the calculations

This methodical approach guarantees precise planning and effective fund accumulation, underpinning sound financial management.

Find The Size Of Each Of 9 Payments Made At The End Of Each Year Into A 7 Rate Sinking Fund Which Produces

Find The Size Of Each Of 9 Payments Made At The End Of Each Year Into A 7 Rate Sinking Fund Which Produces

Related Articles

- Can Someone Please Help Me, ASAP! Write A Song Poetry By Following The Examples That The Question Explains,
- The Equation Y = 0.7x Represents The Distance Henry Hikes In Miles Over Time In Hours. The Graph Represents
- For Steam At 500C And 10 MPa, Using The Mollier Diagram,a. Compute The Joule-Thomson

Coefficient = (OT/).b.

Back to Home