Find The Slope Of The Tangent Line To The Polar Curve For The Given Value Of . R=2sin ; =(\pi)/(6) R=1+cos

Find The Slope Of The Tangent Line To The Polar Curve For The Given Value Of . R=2sin ; =(\pi)/(6) R=1+cos

---

Introduction

Understanding the slope of the tangent line to a polar curve at a specific point is a fundamental concept in calculus and analytical geometry. This knowledge not only helps visualize the behavior of curves in the polar coordinate system but also has practical applications in physics, engineering, and computer graphics.

In this article, we will explore how to find the slope of the tangent line to given polar curves at specified points. Specifically, we will analyze the curves defined by the equations:

- $R = 2 \sin \theta$, at $\theta = \frac{\pi}{6}$
- $R = 1 + \cos \theta$, at a given value of θ (though the exact angle isn't specified, we will demonstrate how to compute the slope at any point)

By understanding the derivation process and applying the appropriate calculus techniques, you will be able to determine the slopes for these types of curves with confidence.

Understanding Polar Coordinates and Curves

Before diving into the calculations, it's essential to grasp the basics of polar coordinates and how curves are represented in this system.

What Are Polar Coordinates?

- Polar coordinates represent a point in the plane using a radius R and an angle θ.
- A point P in Cartesian coordinates (x, y) can be expressed as:

$$
x = R \cos \theta
$$
$$
y = R \sin \theta
$$

- Conversely, given R and θ, the Cartesian coordinates can be obtained as above.

Polar Curves and Their Equations

- Polar curves are defined by equations involving R as a function of θ, such as $R = f(\theta)$.

- Examples include circles, cardioids, rose curves, and limacons.
- Understanding how to compute the slope of the tangent line involves differentiating these equations with respect to θ.

Calculating the Slope of the Tangent Line in Polar Coordinates

The key to finding the slope of the tangent line $\frac{dy}{dx}$ at a particular point on a polar curve involves differentiating the parametric equations derived from the polar form.

Parametric Equations from Polar Coordinates

Given:

$$
x = R(\theta) \cos \theta
$$
$$
y = R(\theta) \sin \theta
$$

The derivatives with respect to θ are:

$$
\frac{dx}{d\theta} = R'(\theta) \cos \theta - R(\theta) \sin \theta
$$
$$
\frac{dy}{d\theta} = R'(\theta) \sin \theta + R(\theta) \cos \theta
$$

where $R'(\theta) = \frac{dR}{d\theta}$.

Formula for the Slope $\frac{dy}{dx}$

Using the chain rule:

$$
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{R'(\theta) \sin \theta + R(\theta) \cos \theta}{R'(\theta) \cos \theta - R(\theta) \sin \theta}
$$

This formula allows us to compute the slope at any point θ for a given $R(\theta)$.

Calculating the Slope for $R = 2 \sin \theta$ at $\theta = \frac{\pi}{6}$

Let's analyze the first curve:

$$
R = 2 \sin \theta
$$

and find the slope of the tangent line at $\theta = \frac{\pi}{6}$.

Step 1: Compute $R'(\theta)$

$$
R'(\theta) = \frac{d}{d\theta} (2 \sin \theta) = 2 \cos \theta
$$

Step 2: Evaluate $R(\theta)$ and $R'(\theta)$ at $\theta = \frac{\pi}{6}$

$$
R\left(\frac{\pi}{6}\right) = 2 \sin \left(\frac{\pi}{6}\right) = 2 \times \frac{1}{2} = 1
$$

$$
R'\left(\frac{\pi}{6}\right) = 2 \cos \left(\frac{\pi}{6}\right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}
$$

Step 3: Compute the numerator and denominator for $\frac{dy}{dx}$

$$
\frac{dy}{dx} = \frac{R'(\theta) \sin \theta + R(\theta) \cos \theta}{R'(\theta) \cos \theta - R(\theta) \sin \theta}
$$

Substituting the values:

Numerator:

$$
\sqrt{3} \times \frac{1}{2} + 1 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}
$$

Denominator:

$$
\sqrt{3} \times \frac{\sqrt{3}}{2} - 1 \times \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1
$$

Step 4: Final slope calculation

$$
\boxed{
\frac{dy}{dx} = \frac{\sqrt{3}}{1} = \sqrt{3}

$$
\right)
$$

Conclusion: The slope of the tangent line to the curve $r = 2 \sin \theta$ at $\theta = \frac{\pi}{6}$ is $\sqrt{3}$.

Calculating the Slope for $r = 1 + \cos \theta$ at a Specific θ

Next, consider the second curve:

$$
r = 1 + \cos \theta
$$

Suppose we wish to find the slope at $\theta = \frac{\pi}{3}$ (or any other specified θ). The process is similar.

Step 1: Compute $r'(\theta)$

$$
r'(\theta) = -\sin \theta
$$

Step 2: Evaluate $r(\theta)$ and $r'(\theta)$ at $\theta = \frac{\pi}{3}$

$$
r\left(\frac{\pi}{3}\right) = 1 + \cos \left(\frac{\pi}{3}\right) = 1 + \frac{1}{2} = \frac{3}{2}
$$

$$
r'\left(\frac{\pi}{3}\right) = -\sin \left(\frac{\pi}{3}\right) = -\frac{\sqrt{3}}{2}
$$

Step 3: Compute numerator and denominator for $\frac{dy}{dx}$

$$
\frac{dy}{dx} = \frac{r'(\theta) \sin \theta + r(\theta) \cos \theta}{r'(\theta) \cos \theta - r(\theta) \sin \theta}
$$

Substituting:

Numerator:

\[

$$
- \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{3}{2} \times \frac{1}{2} = - \frac{3}{4} + \frac{3}{4} = 0
$$

Denominator:

$$
- \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{3}{2} \times \frac{\sqrt{3}}{2} = - \frac{\sqrt{3}}{4} - \frac{3 \sqrt{3}}{4} = - \frac{\sqrt{3}}{4} - \frac{3 \sqrt{3}}{4} = - \frac{4 \sqrt{3}}{4} = - \sqrt{3}
$$

Step 4: Final slope calculation

$$
\frac{dy}{dx} = \frac{0}{- \sqrt{3}} = 0
$$

Conclusion: The slope of the tangent line to $r = 1 + \cos \theta$ at $\theta = \frac{\pi}{3}$ is 0, indicating a horizontal tangent.

Additional Considerations and Applications

Understanding

Frequently Asked Questions

How do I find the slope of the tangent line to the polar curve $r = 2 \sin \theta$ at $\theta = \pi/6$?

To find the slope at $\theta = \pi/6$, compute $dr/d\theta = 2 \cos \theta$, then use the formula for the slope: $(dy/dx) = (r' \sin \theta + r \cos \theta) / (r' \cos \theta - r \sin \theta)$. Plugging in the values gives the slope at that point.

What is the process to determine the slope of the tangent line for the curve $r = 1 + \cos \theta$ at $\theta = \pi/6$?

First, find $dr/d\theta = -\sin \theta$. Then, substitute $\theta = \pi/6$ into r and $dr/d\theta$, and utilize the formula for dy/dx in polar coordinates: $(dy/dx) = (r' \sin \theta + r \cos \theta) / (r' \cos \theta - r \sin \theta)$. Simplify to find the slope.

Why is it necessary to differentiate r with respect to θ when finding the tangent slope in polar coordinates?

Differentiating r with respect to θ ($dr/d\theta$) provides the rate of change of radius with angle, which is essential for calculating the slope of the tangent line in Cartesian coordinates, as the slope depends on both r and $dr/d\theta$.

Can the slope of the tangent line be negative for the given polar curves, and what does that imply?

Yes, the slope can be negative, indicating that the tangent line at that point is decreasing in y with respect to x. It reflects the local orientation of the curve's tangent at the specific angle.

How do the specific values of θ = π/6 influence the calculation of the tangent slope for these polar curves?

At θ = π/6, evaluate r and dr/dθ at this angle, which simplifies the calculation of the slope. The specific angle determines the sine and cosine values used in the derivatives, directly impacting the slope value.

Additional Resources

Find The Slope Of The Tangent Line To The Polar Curve For The Given Value Of R=2sin θ, θ=π/6, R=1+cos θ

Understanding how to find the slope of the tangent line to a polar curve is a fundamental skill in calculus, particularly when dealing with curves defined in polar coordinates. Whether you're studying for an exam, working on a project involving curves, or just exploring the fascinating world of mathematics, mastering this process allows you to analyze the behavior of complex curves and their tangents accurately. In this guide, we will thoroughly explore how to compute the slope of the tangent line for given polar curves at specified angles, focusing on the curves R=2sin θ and R=1+cos θ at θ=π/6.

Introduction to Polar Curves and Tangent Lines

What Are Polar Curves?

A polar curve is a graph of a function defined in polar coordinates (r, θ), where r (or R) is the radius (distance from the origin), and θ is the angle measured from the positive x-axis. Unlike Cartesian functions y=f(x), polar functions describe the position of points in a plane based on their distance from and angle relative to the origin.

Common polar curves include circles, roses, lemniscates, and cardioids, each with distinctive features. The forms of these curves often lead to interesting properties like symmetry and self-intersecting points.

Why Find the Slope of the Tangent Line?

The slope of the tangent line at a specific point on a curve provides insight into the curve's behavior at that point—whether it's increasing, decreasing, or changing concavity. In polar coordinates, this involves not only understanding the curve's position but also how it curves in the plane.

Mathematical Foundations: Deriving the Slope in Polar Coordinates

The Relationship Between Cartesian and Polar Coordinates

Any point on a polar curve can be expressed in Cartesian coordinates as:

- x = r(θ) cos θ
- y = r(θ) sin θ

where r(θ) is the function describing the radius in terms of θ.

Derivative of y with Respect to x in Polar Coordinates

To find the slope of the tangent line, we need dy/dx. Since both x and y depend on θ, we can use the chain rule:

$$
\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}
$$

where:

$$
\frac{dy}{d\theta} = \frac{d}{d\theta} (r \sin \theta) = r' \sin \theta + r \cos \theta
$$

$$
\frac{dx}{d\theta} = \frac{d}{d\theta} (r \cos \theta) = r' \cos \theta - r \sin \theta
$$

and r' is the derivative of r with respect to θ.

Putting it together:

$$
\boxed{
\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}
}
$$

This formula allows us to compute the slope of the tangent at any θ for the given polar curve.

Step-by-Step Guide to Find the Slope at a Specific θ

Step 1: Identify the given curve and the specific θ

- For $R=2\sin\theta$ at $\theta=\pi/6$
- For $R=1+\cos\theta$ at $\theta=\pi/6$

Step 2: Compute r' (derivative of r with respect to θ)

Differentiate $r(\theta)$ for each curve:

- For $R=2\sin\theta$:

$$
r' = \frac{d}{d\theta} (2 \sin \theta) = 2 \cos \theta
$$

- For $R=1+\cos\theta$:

$$
r' = \frac{d}{d\theta} (1 + \cos \theta) = - \sin \theta
$$

Step 3: Evaluate r and r' at $\theta=\pi/6$

- For $R=2\sin\theta$:

$$
r = 2 \sin \left(\frac{\pi}{6} \right) = 2 \times \frac{1}{2} = 1
$$

$$
r' = 2 \cos \left(\frac{\pi}{6} \right) = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}
$$

- For $r=1+\cos\theta$:

$$
r = 1 + \cos \left(\frac{\pi}{6} \right) = 1 + \frac{\sqrt{3}}{2} \approx 1 + 0.866 = 1.866
$$

$$
r' = - \sin \left(\frac{\pi}{6} \right) = - \frac{1}{2}
$$

Step 4: Calculate dy/dx for each curve at $\theta=\pi/6$

Using the formula:

$$
\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}
$$

For $r=2\sin\theta$:

$$
\frac{dy}{dx} = \frac{\sqrt{3} \times \frac{1}{2} + 1 \times \frac{\sqrt{3}}{2}}{\sqrt{3} \times \frac{\sqrt{3}}{2} - 1 \times \frac{1}{2}}
$$

Simplify numerator:

$$
\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = \sqrt{3}
$$

Simplify denominator:

$$
\frac{\sqrt{3} \times \sqrt{3}}{2} - \frac{1}{2} = \frac{3}{2} - \frac{1}{2} = 1
$$

Thus:

$$
\boxed{
\frac{dy}{dx} = \frac{\sqrt{3}}{1} = \sqrt{3}
}
$$

For $r=1+\cos\theta$:

$$
\frac{dy}{dx} = \frac{-\frac{1}{2} \times \frac{1}{2} + 1.866 \times \frac{\sqrt{3}}{2}}{-\frac{1}{2} \times \frac{\sqrt{3}}{2} - 1.866 \times \frac{1}{2}}
$$

Calculate numerator:

$$
-\frac{1}{2} \times \frac{1}{2} = -\frac{1}{4} \approx -0.25
$$

$$
1.866 \times \frac{\sqrt{3}}{2} \approx 1.866 \times 0.866 \approx 1.616
$$

Sum numerator:

$$
-0.25 + 1.616 \approx 1.366
$$

Calculate denominator:

$$
-\frac{1}{2} \times \frac{\sqrt{3}}{2} \approx -0.433
$$

$$
-1.866 \times \frac{1}{2} \approx -0.933
$$

Sum denominator:

$$
-0.433 - 0.933 = -1.366
$$

Finally:

$$
\frac{dy}{dx} \approx \frac{1.366}{-1.366} = -1
$$

Final Results: Slopes at $\theta=\pi/6$

- For $r=2\sin\theta$: Slope $\approx \sqrt{3} \approx 1.732$
- For $r=1+\cos\theta$: Slope ≈ -1

Visual Interpretation and Applications

What Do These Slopes Tell Us?

The computed slopes reveal how each curve behaves at $\theta=\pi/6$:

- The positive slope (~1.732) for $r=2\sin\theta$ indicates the tangent line is rising as we move through that point.
- The negative slope (~-1) for $r=1+\cos\theta$ indicates the tangent line is descending.

Such information is crucial when sketching the curves, analyzing their curvature, or understanding their tangent properties in physics, engineering, and computer graphics.

Practical Applications

- Designing curves in CAD software
- Analyzing planetary orbits
- Modeling natural phenomena
- Educational demonstrations of calculus concepts

Summary and Key Takeaways

- To find the slope of the tangent line to a polar curve at a specific θ, differentiate r(θ) to find r′(θ).
- Use the formula:

$$
\frac{dy}{dx} = \frac{r' \sin \theta + r \cos \theta}{r' \cos \theta - r \sin \theta}
$$

- Evaluate r and r′ at the given θ.
- Plug these values into the formula to compute the slope.

By mastering these steps, you can analyze a wide variety of polar curves with confidence, gaining deeper insight into their geometric and analytical properties.

Final Thoughts

Calculating the slope of tangent lines to polar curves combines the elegance of calculus with the geometric intuition of curves in the plane. Whether exploring simple circles or intricate rose curves, these techniques unlock a richer understanding of their behavior. As you continue your mathematical journey, remember that practice

Find The Slope Of The Tangent Line To The Polar Curve For The Given Value Of R 2sin Pi 6 R 1 Cos

Find The Slope Of The Tangent Line To The Polar Curve For The Given Value Of R 2sin Pi 6 R 1 Cos

Related Articles

- Cash And Near-cash Resources Are Known As _____.a.liquid Assetsb.illiquid Assetsc.non-current Assetsd.fixed
- Quiz Question:The American EmbassyDo You Agree With The Choice The Protagonist Made At The American Embassy
- Legal Complaints Charging Organizations With Discrimination Have Largely Been Eliminated Due To The Passage

Back to Home