Find The Value Of - At The Point (1, 1, 1) If The Equation Xy+zx-2yz = 0 Defines Z Implicitly As A Function

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Introduction

In multivariable calculus, understanding how to evaluate functions defined implicitly is crucial, especially when dealing with surfaces and their tangent planes. The problem at hand involves an implicit surface defined by the equation:

$$ xy + zx - 2 yz = 0 $$

which implicitly defines z as a function of x and y. Our goal is to find the value of the partial derivative $\frac{\partial z}{\partial x}$ (sometimes denoted as z_x) at the specific point $(1, 1, 1)$.

This process involves applying implicit differentiation with respect to x, recognizing how z depends on x, and carefully evaluating the derivative at the given point. This technique is widely used in calculus when explicit solutions are difficult or impossible to find directly.

Understanding the Problem

The surface is given by:

$$ xy + zx - 2 yz = 0 $$

which implicitly relates x, y, and z. We are told this defines z as a function of x and y, i.e.,

$$ z = z(x, y) $$

at least locally around the point $(1, 1, 1)$.

Our task is to compute:

$$
\left. \frac{\partial z}{\partial x} \right|_{(1, 1, 1)}
$$

Step 1: Confirm the Point Lies on the Surface

Before differentiating, verify that the point $(1, 1, 1)$ satisfies the original equation:

$$
xy + zx - 2 yz = 0
$$

Plugging in the values:

$$
(1)(1) + (1)(1) - 2 (1)(1) = 1 + 1 - 2 = 0
$$

Yes, the point $(1, 1, 1)$ lies on the surface, so the differentiation process is valid at this point.

Step 2: Implicit Differentiation with Respect to x

Given the relation:

$$
xy + zx - 2 yz = 0
$$

we differentiate both sides with respect to x, treating z as a function of x and y (with y held constant during partial differentiation).

The differentiation rules involved are:

- Product rule: $\frac{d}{dx}(uv) = u'v + uv'$
- Chain rule: $\frac{d}{dx} z = z_x$

Let's differentiate term by term:

1. $\frac{d}{dx}(xy) = y + x \frac{\partial y}{\partial x}$

Since y is held constant in partial derivatives, $\frac{\partial y}{\partial x} = 0$, so:

$$
\frac{d}{dx}(xy) = y
$$

2. $\frac{d}{dx}(zx) = z + x \frac{\partial z}{\partial x} = z + x z_x$

3. $\frac{d}{dx}(-2 y z) = -2 y z_x$

Putting it all together:

\[

$$
y + z + x z_x - 2 y z_x = 0
$$

Step 3: Solve for $\frac{\partial z}{\partial x}$

Group the terms involving z_x:

$$
(y + z) + (x z_x - 2 y z_x) = 0
$$

$$
(y + z) + z_x (x - 2 y) = 0
$$

Isolate z_x:

$$
z_x (x - 2 y) = - (y + z)
$$

$$
z_x = - \frac{y + z}{x - 2 y}
$$

Now, substitute the point $(x, y, z) = (1, 1, 1)$:

$$
z_x \big|_{(1, 1, 1)} = - \frac{1 + 1}{1 - 2 \times 1} = - \frac{2}{1 - 2} = - \frac{2}{-1} = 2
$$

Answer:

$$
\boxed{
\left. \frac{\partial z}{\partial x} \right|_{(1, 1, 1)} = 2
}
$$

Additional Insights and Context

Why Use Implicit Differentiation?

In many real-world scenarios, the relationship between variables is complex and can't be explicitly solved for one variable in terms of others. Implicit differentiation allows us to find derivatives without explicitly solving for the function $z(x, y)$, which may be difficult or impossible analytically.

Geometrical Interpretation

The derivative $\frac{\partial z}{\partial x}$ at a point provides the slope of the tangent plane to the surface at that point in the direction of increasing x. Knowing this slope helps in understanding the surface's local behavior, which is essential in fields like physics, engineering, and computer graphics.

Applications

- Surface analysis: Determining tangent planes, normals, and curvatures.
- Optimization problems: Finding maxima, minima, or saddle points on surfaces.
- Modeling in physics: Describing how physical quantities change in space.

Summary of the Process

Step	Description	Result
1	Verify the point lies on the surface	Confirmed (1,1,1) satisfies the equation
2	Differentiate the implicit equation w.r.t x	$y + z + x z_x - 2 y z_x = 0$
3	Rearrange to solve for z_x	$z_x = - \frac{y + z}{x - 2 y}$
4	Substitute point values	$z_x = 2$

Final Remarks

Understanding how to find the derivative of an implicitly defined function at a point is a foundational skill in calculus. It serves as a bridge to more advanced topics like multivariable calculus, differential geometry, and mathematical modeling.

The specific problem addressed here—finding $\frac{\partial z}{\partial x}$ at $(1, 1, 1)$ for the surface $xy + zx - 2 yz = 0$—demonstrates the power of implicit differentiation. By carefully differentiating and substituting, we obtained a clear value for the derivative, deepening our understanding of the surface's local geometry.

Keywords for SEO Optimization

- implicit differentiation
- partial derivatives
- surface analysis
- multivariable calculus
- implicit function theorem
- derivative of implicit functions
- calculating $\frac{\partial z}{\partial x}$
- surface derivatives at a point
- calculus problems involving surfaces
- mathematical modeling in calculus

If you need further assistance with related topics like finding $\frac{\partial z}{\partial y}$, tangent planes, or applications of the implicit function theorem, feel free to ask!

Frequently Asked Questions

How do you find the value of z at the point (1, 1, 1) given the equation xy + zx - 2yz = 0?

Substitute x=1, y=1, z=1 into the equation: (1)(1) + (1)(1) - 2(1)(1) = 1 + 1 - 2 = 0. Since the equation holds true, z=1 is the value at that point.

Does the equation xy + zx - 2yz = 0 define z explicitly at the point (1, 1, 1)?

Yes, at the point (1, 1, 1), the equation holds true, and z can be expressed explicitly as a function of x and y around that point, allowing us to find its value directly.

What steps are involved in finding the value of z at (1, 1, 1) from the given implicit equation?

First, substitute the point coordinates into the equation to verify it satisfies the relation. Then, differentiate with respect to variables if needed, or solve for z explicitly to confirm its value at that point.

Is the point (1, 1, 1) consistent with the implicit definition of z from the equation xy + zx - 2yz = 0?

Yes, substituting (1, 1, 1) into the equation yields 0, confirming that the point lies on the surface defined by the implicit equation and the value of z at this point is 1.

Can we determine the slope or rate of change of z at (1, 1, 1) from the given implicit equation?

Yes, by applying implicit differentiation to the equation with respect to x or y, we can find the partial derivatives and thus determine the rate of change of z at the point (1, 1, 1).

Additional Resources

Find The Value Of - At The Point (1, 1, 1) If The Equation Xy + zx - 2yz = 0 Defines Z Implicitly As A Function is a classic problem in multivariable calculus that tests your understanding of partial derivatives, implicit differentiation, and the application of the chain rule. This type of problem is common in calculus courses, especially when exploring how functions are implicitly defined and how to compute the derivative of one variable with respect to others at a specific point. In this article, we will walk through a detailed, step-by-step process to find the value of the partial derivative - (minus), which commonly refers to $-\partial z/\partial x$ or $-\partial z/\partial y$ depending on context, at the point (1, 1, 1), given the implicit relation X y + z x - 2 y z = 0.

Understanding the Problem

The problem states:

> Find the value of - at the point (1, 1, 1) if the equation X y + z x - 2 y z = 0 defines z implicitly as a function of x and y.

To clarify, the equation:

$$ xy + zx - 2 yz = 0 $$

is an implicit relation involving variables x, y, and z. Since it is stated that z is defined implicitly as a function of x and y, our goal is to find the partial derivative $\partial z/\partial x$ or $\partial z/\partial y$ at the point (x, y, z) = (1, 1, 1) and then interpret the "Find The Value Of -" accordingly.

Given the context, "Find The Value Of -" most likely refers to $-\partial z/\partial x$ (or possibly $-\partial z/\partial y$), but for clarity, we will focus on finding $\partial z/\partial x$ at the point, then take the negative as required.

Step 1: Recognize the Implicit Function and Variables

The key points:

- The equation relates x, y, and z.
- At the point (1, 1, 1), the relation holds true:

$$ (1)(1) + (1)(1) - 2(1)(1) = 0 \Rightarrow 1 + 1 - 2 = 0 $$

which satisfies the equation.

- Our task is to compute $\partial z/\partial x$ at this point, treating z as a function of x and y: $z = z(x, y)$.

Step 2: Differentiate the Equation Implicitly

To find $\partial z/\partial x$, we differentiate the entire relation with respect to x, treating y as constant (since we are calculating a partial derivative with respect to x).

The original relation:

$$ xy + zx - 2 y z = 0 $$

Differentiating term-by-term:

- xy: Product of x and y:

$$\frac{\partial}{\partial x}(xy) = y + x \cdot 0 = y$$

(since y is treated as constant).

- zx: Product of z and x:

$$\frac{\partial}{\partial x}(zx) = z + x \frac{\partial z}{\partial x}$$

- $-2 y z$:

$$\frac{\partial}{\partial x}(-2 y z) = -2 y \frac{\partial z}{\partial x}$$

(since y is constant).

Putting it all together:

$$y + z + x \frac{\partial z}{\partial x} - 2 y \frac{\partial z}{\partial x} = 0$$

Step 3: Solve for $\frac{\partial z}{\partial x}$

Rearranged, the equation becomes:

$$(x) \frac{\partial z}{\partial x} - 2 y \frac{\partial z}{\partial x} = - y - z$$

Factor $\partial z/\partial x$:

$$(x - 2 y) \frac{\partial z}{\partial x} = - y - z$$

Thus,

$$\frac{\partial z}{\partial x} = \frac{- y - z}{x - 2 y}$$

Step 4: Plug in the Point (1, 1, 1)

At the point $(x, y, z) = (1, 1, 1)$:

- Numerator:

$$- y - z = -1 - 1 = -2$$

- Denominator:

$$x - 2 y = 1 - 2(1) = 1 - 2 = -1$$

Therefore:

$$ \frac{\partial z}{\partial x} \bigg|_{(1,1,1)} = \frac{-2}{-1} = 2 $$

Step 5: Find the Value of the "Minus"

Since the problem asks for "Find The Value Of -", and the notation suggests the negative of the derivative, we interpret:

$$ - \frac{\partial z}{\partial x} \bigg|_{(1,1,1)} = - 2 = -2 $$

Answer: -2

Additional Considerations

Implicit Differentiation with Respect to y

If the problem also implied to find $-\partial z/\partial y$, a similar process would be followed:

Differentiate the original relation with respect to y, treating x as constant:

$$ xy + zx - 2 y z = 0 $$

- $\frac{\partial}{\partial y}(xy) = x$
- $\frac{\partial}{\partial y}(zx) = 0$ (since z depends on x; with respect to y treating x as constant, z depends on y, so:

$$ \frac{\partial}{\partial y}(zx) = x \frac{\partial z}{\partial y} $$

- $\frac{\partial}{\partial y}(-2 y z) = -2 z - 2 y \frac{\partial z}{\partial y}$

Putting it all together:

$$ x + 0 + x \frac{\partial z}{\partial y} - 2 z - 2 y \frac{\partial z}{\partial y} = 0 $$

Rearranged:

$$ (x - 2 y) \frac{\partial z}{\partial y} = 2 z - x $$

At the point (1, 1, 1):

$$ (1 - 2 \times 1) \frac{\partial z}{\partial y} = 2 \times 1 - 1 = 2 - 1 = 1 $$

$$ (1 - 2) \frac{\partial z}{\partial y} = 1 \Rightarrow -1 \times \frac{\partial z}{\partial y} = 1 $$

$$ \frac{\partial z}{\partial y} = -1 $$

The negative of this derivative:

$$ - \frac{\partial z}{\partial y} = -(-1) = 1 $$

Summary of Results

Derivative	Value at (1, 1, 1)	Negative of derivative ("-")
$\frac{\partial z}{\partial x}$	2	-2
$\frac{\partial z}{\partial y}$	-1	1

Final Thoughts

The key to solving such implicit differentiation problems is:

- Properly differentiate each term with respect to the variable of interest.
- Treat other variables as constants when differentiating with respect to one variable.
- Solve algebraically for the derivative.
- Plug in the specific point to obtain a numerical value.
- Remember to interpret the "minus" sign as per the problem's context.

This process demonstrates the power of implicit differentiation in multivariable calculus, enabling us to understand how variables relate dynamically at specific points. The approach can be extended to more complex relations, supporting a deep understanding of multivariable functions and their derivatives.

In conclusion, the value of - (presumably $-\partial z/\partial x$) at the point (1, 1, 1), given the relation XY + ZX - 2YZ = 0, is -2. This insight is critical for applications involving implicit functions, such as in physics, engineering, and economics, where understanding the local behavior of functions under constraints is essential.

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