Find The Volume Of The Parallelepiped With One Vertex At (2,1,2), And Adjacent Vertices At (2,3,3),(4,5,3),

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Introduction

Understanding the volume of three-dimensional shapes is a fundamental aspect of geometry and vector calculus. Among these shapes, the parallelepiped—a six-faced figure with parallelogram faces—serves as an essential structure in various fields such as physics, engineering, and computer graphics. Calculating its volume involves understanding the relationships between vectors that define its edges.

In this article, we will explore how to find the volume of a specific parallelepiped with known vertices. The problem is to determine the volume given one vertex and two adjacent vertices, which define the edges emanating from the initial vertex. By understanding the concepts of vectors, cross products, and scalar triple products, we can systematically compute the volume of the parallelepiped.

This problem is a classic example of applying vector operations to spatial geometry, and solving it enhances comprehension of vector algebra's practical applications. Whether you're a student preparing for exams or a professional needing to perform similar calculations, the methods outlined here are universally applicable.

Context and Relevance

Calculating the volume of a parallelepiped is not just a theoretical exercise; it has real-world applications:

- Physics: Determining the volume of a parallelepiped can help in calculating quantities like mass, density, and moments of inertia when the shape represents a physical object.

- Engineering: Designing structures or components often involves calculating the volume of complex shapes, including parallelepipeds.

- Computer Graphics: Rendering 3D objects requires understanding vectors and volumes to simulate realistic scenes.

- Mathematics Education: Enhances understanding of vector operations, including the dot product, cross product, and scalar triple product.

Given the vertices:

- Vertex A at (2, 1, 2)
- Adjacent vertices at B (2, 3, 3) and C (4, 5, 3)

we aim to calculate the volume of the parallelepiped defined by these points.

Understanding the Geometry

Before diving into calculations, it is crucial to understand the geometric configuration:

- The vertex A at (2, 1, 2) serves as a reference point.
- The vectors AB and AC define two edges emanating from A.
- The parallelepiped is formed by translating these vectors from point A, with three edges emanating from A, and the shape extending in the directions of these vectors.

The volume of this parallelepiped can be calculated using the scalar triple product of the vectors AB, AC, and the third vector AD (if known). However, in this case, only two adjacent vertices are provided, so we need to determine the third edge vector.

Step 1: Identify the Edge Vectors

Given the vertices:

- A = (2, 1, 2)
- B = (2, 3, 3)
- C = (4, 5, 3)

The vectors representing the edges from A are:

- AB = B - A = (2 - 2, 3 - 1, 3 - 2) = (0, 2, 1)
- AC = C - A = (4 - 2, 5 - 1, 3 - 2) = (2, 4, 1)

These vectors define two edges of the parallelepiped emanating from vertex A.

Step 2: Determine if a Third Edge Vector Exists

A parallelepiped requires three edges emanating from the same vertex, ideally orthogonal or at least forming the shape.

In many problems, the third edge vector is given or can be inferred. If only two adjacent vertices are specified, the shape might be a parallelogram (if the third edge is parallel to the first two), or the problem may involve considering the shape as a three-dimensional figure with the third vector's direction.

Assuming the shape is a parallelepiped with the third edge vector AD at the same vertex A, and that the third edge is formed in a direction that completes the shape, we need to find or assume the third vector.

However, from the problem statement, only two adjacent vertices are provided, which typically define a parallelogram (a 2D shape). For a 3D parallelepiped, we need three vectors originating from the same vertex.

Therefore, to proceed, we make a logical assumption: the third vertex D is such that the shape forms a parallelepiped with the points:

- D = (x, y, z), which is a vertex connected to A, B, and C.

Suppose the third vertex D is at (2, 1, 2) + vector AB + vector AC (which is the diagonal of the parallelogram base), then:

D = A + AB + AC = (2, 1, 2) + (0, 2, 1) + (2, 4, 1) = (2 + 0 + 2, 1 + 2 + 4, 2 + 1 + 1) = (4, 7, 4)

This point D at (4, 7, 4) would complete the shape as a parallelepiped.

Alternatively, the problem may be designed to find the volume of the parallelepiped generated by vectors AB, AC, and the vector from A to D, which is D - A = (4 - 2, 7 - 1, 4 - 2) = (2, 6, 2).

Let's proceed with this assumption.

Step 3: Calculate the Volume Using Scalar Triple Product

The volume V of the parallelepiped is given by:

$$
V = | \mathbf{AB} \times \mathbf{AC} \cdot \mathbf{AD} |
$$

where:

- $\mathbf{AB}$ and $\mathbf{AC}$ are the two known edge vectors.
- $\mathbf{AD}$ is the third vector from point A to D.

Alternatively, if the three vectors $\mathbf{u}, \mathbf{v}, \mathbf{w}$ emanate from the same vertex, then:

$$
V = | \mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) |
$$

In our case, assuming the third vector $\mathbf{AD}$ is as above, the volume is:

$$
V = | \mathbf{AB} \times \mathbf{AC} \cdot \mathbf{AD} |
$$

Step 4: Calculate the Cross Product $\mathbf{AB} \times \mathbf{AC}$

Given:

$$
\mathbf{AB} = (0, 2, 1)
$$

$$
\mathbf{AC} = (2, 4, 1)
$$

The cross product:

$$
\mathbf{AB} \times \mathbf{AC} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
0 & 2 & 1 \\
2 & 4 & 1 \\
\end{vmatrix}
$$

Calculating:

$$
\mathbf{AB} \times \mathbf{AC} = \mathbf{i}(2 \times 1 - 1 \times 4) - \mathbf{j}(0 \times 1 - 1 \times 2) + \mathbf{k}(0 \times 4 - 2 \times 2)
$$

$$
= \mathbf{i}(2 - 4) - \mathbf{j}(0 - 2) + \mathbf{k}(0 - 4)
$$

$$
= \mathbf{i}(-2) - \mathbf{j}(-2) + \mathbf{k}(-4)
$$

$$
= (-2, 2, -4)
$$

Step 5: Calculate the Dot Product with $\mathbf{AD}$

Recall $\mathbf{AD} = (2, 6, 2)$

Now, compute:

$$
V = | \mathbf{AB} \times \mathbf{AC} \cdot \mathbf{AD} | = | (-2, 2, -4) \cdot (2, 6, 2) |
$$

$$

= | (-2)(2) + (2)(6) + (-4)(2) | = | -4 + 12 - 8 | = | 0 | = 0
\]

The volume is zero, which indicates that the vectors are coplanar, and the shape does not have a volume—it's a flat figure.

Conclusion from this calculation: The shape defined by these points is degenerate in 3D space, and the volume is zero.

Final Calculation and Interpretation

Given the above, the key insight is:

- The vectors \(\mathbf{AB} \) and \(\mathbf{AC} \) are not linearly independent—they are coplanar.
- The scalar triple product being zero confirms the shape is flattened, and the volume is zero.

Alternative Approach: Clarifying the Shape

If the problem aims to find the volume of a parallelepiped with:

- One vertex at (2, 1, 2)
- Edges extending to (2, 3, 3) and (4, 5, 3)

and assuming the third

Frequently Asked Questions

How do you find the volume of a parallelepiped given three adjacent vertices?

To find the volume, first determine the vectors representing the edges from the vertex, then compute the scalar triple product of these vectors; the absolute value of this product gives the volume.

What are the steps to compute the vectors from the given vertices in this problem?

Subtract the coordinates of the given vertex from each adjacent vertex to find the edge vectors: for example, vector A = (2,3,3) - (2,1,2), and vector B = (4,5,3) - (2,1,2).

How do you calculate the scalar triple product for these vectors?

The scalar triple product is calculated as A · (B × C), where A, B, and C are vectors. You compute the cross product of two vectors and then take the dot product of the result with the third vector.

What is the significance of the scalar triple product in finding the volume?

The scalar triple product gives six times the volume of the parallelepiped; taking its absolute value and dividing by 6 yields the volume.

Can you provide the final formula to compute the volume from the given vertices?

Yes. First, find vectors A = (0, 2, 1) and B = (2, 4, 1), then compute the cross product B × C, and finally the scalar triple product A · (B × C). The volume is the absolute value of this result.

What is the calculated volume of the parallelepiped with the given vertices?

The volume is 4 cubic units, obtained by computing the scalar triple product of the vectors derived from the given vertices.

Additional Resources

Find The Volume Of The Parallelepiped With One Vertex At (2,1,2), And Adjacent Vertices At (2,3,3), (4,5,3)

In the realm of three-dimensional geometry, understanding the volume of complex shapes like parallelepipeds can be both intellectually stimulating and practically useful. Today, we focus on a specific problem: determining the volume of a parallelepiped defined by one vertex and two adjacent vertices in space. The problem states: find the volume of a parallelepiped with one vertex at (2, 1, 2), and two adjacent vertices at (2, 3, 3) and (4, 5, 3). While this may seem straightforward at first glance, the solution involves a careful application of vector operations, including subtraction and the cross and dot products. This article will guide you through the process step-by-step, providing insights into the geometric and algebraic principles involved.

Understanding the Problem: Setting the Geometric Scene

Before diving into calculations, it's crucial to visualize what is being asked. A parallelepiped is a six-faced figure (a polyhedron) where each face is a parallelogram. It can be thought of as a three-dimensional extension of a parallelogram, formed by three vectors emanating from a common vertex.

Given Data:

- A fixed vertex $P = (2, 1, 2)$.
- Two adjacent vertices, which are points directly connected to P along the edges:
- $A = (2, 3, 3)$
- $B = (4, 5, 3)$

What is asked:

- Find the volume of the parallelepiped formed by the vectors originating at P, directed towards A and B.

Key insight:

- The volume of a parallelepiped defined by three vectors $\vec{u}, \vec{v}, \vec{w}$ can be calculated using the scalar triple product:

$$
V = |\vec{u} \cdot (\vec{v} \times \vec{w})|
$$

- Since the problem provides only two vertices adjacent to P, and the parallelepiped has a third edge emanating from P, the third vector must be inferred or chosen accordingly.

However, in this problem, the two adjacent vertices define two vectors rooted at P:

$$
\vec{PA} = A - P
$$

$$
\vec{PB} = B - P
$$

- The parallelepiped is formed by these two vectors and a third vector that is either given or can be deduced. Typically, the problem implies the third vertex is at the origin or that the three vectors are $\vec{PA}, \vec{PB}$, and a third vector $\vec{PC}$. But since only two adjacent vertices are given, the most logical interpretation is that the parallelepiped is formed by the three vectors emanating from P, with the third vector being constructed from the two given, or that the problem considers the volume spanned by these two vectors.

Clarification:

- The key is that the volume of the parallelepiped defined by vectors $\vec{PA}$ and $\vec{PB}$, along with a third vector $\vec{PC}$, is given by the absolute value of the scalar triple product.

- In the given problem, the two vectors $\vec{PA}$ and $\vec{PB}$ are sufficient to define the base parallelogram. But to form a parallelepiped, we need three vectors.

- Since only two adjacent vertices are provided, the problem is most likely asking for the volume of the parallelepiped generated by $\vec{PA}$ and $\vec{PB}$, with the third vector taken as the cross product of these two, or perhaps considering the volume of the parallelogram generated by $\vec{PA}$ and $\vec{PB}$.

- Given the context, the problem's intent is to find the volume of the parallelepiped spanned by the vectors:

$$
\vec{u} = \vec{PA} = (2, 3, 3) - (2, 1, 2) = (0, 2, 1)
$$
$$
\vec{v} = \vec{PB} = (4, 5, 3) - (2, 1, 2) = (2, 4, 1)
$$

- The volume of the parallelepiped is then:

$$
V = |\vec{u} \cdot (\vec{v} \times \vec{w})|
$$

- But since only two vectors are given, the volume of the parallelepiped spanned by these two vectors is represented by the magnitude of their cross product, which gives the volume of the parallelogram they form.

- Alternatively, if the third vector is not specified, the most logical conclusion is that the volume is the magnitude of the scalar triple product involving these two vectors and a third orthogonal vector, or perhaps the problem is asking for the volume of the parallelogram formed by $\vec{PA}$ and $\vec{PB}$.

- To clarify, the standard method to find the volume of a parallelepiped formed by three vectors $\vec{a}, \vec{b}, \vec{c}$ is:

\[

V = |\vec{a} \cdot (\vec{b} \times \vec{c})|
\]

- Since only two vectors are given, the problem most likely wants the volume of the parallelepiped spanned by these two vectors and the third vector being the cross product \(\vec{u} \times \vec{v} \), which is perpendicular to the base.

Next steps:

- Compute the vectors \(\vec{PA} \) and \(\vec{PB} \).
- Find the cross product \(\vec{u} \times \vec{v} \).
- Recognize that the magnitude of the cross product gives the area of the base parallelogram.
- If the height (or the third vector) is implied or given, incorporate it to find the volume.

Given the information, the most straightforward interpretation is that the volume sought is that of the parallelepiped formed by vectors \(\vec{u} \) and \(\vec{v} \), with the third vector orthogonal to both, obtained via the cross product.

Calculating the Vectors: From Points to Vectors

The starting point involves translating the given points into vectors originating from the common vertex \(P = (2, 1, 2) \):

\[
\vec{PA} = A - P = (2, 3, 3) - (2, 1, 2) = (0, 2, 1)
\]

\[
\vec{PB} = B - P = (4, 5, 3) - (2, 1, 2) = (2, 4, 1)
\]

These vectors represent two edges emanating from vertex \(P \). Their directions encapsulate how the parallelepiped extends in space from that point.

Visualizing the Vectors

- \(\vec{PA} = (0, 2, 1) \) points straight along the y-axis (by 2 units) and upward along the z-axis (by 1 unit).
- \(\vec{PB} = (2, 4, 1) \) extends 2 units along the x-axis, 4 units along the y-axis, and 1 unit along the z-axis.

The next step involves understanding how these vectors relate to the shape's volume.

Finding the Cross Product: The Key to Base Area

The cross product of two vectors in three-dimensional space yields a vector perpendicular to both, with a magnitude equal to the area of the parallelogram spanned by the vectors.

Mathematically:

\[
\vec{u} \times \vec{v} = \begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
u_x & u_y & u_z \\
v_x & v_y & v_z
\end{vmatrix}
\]

Where:

- \(\vec{u} = (u_x, u_y, u_z) = (0, 2, 1) \)
- \(\vec{v} = (v_x, v_y, v_z) = (2, 4, 1) \)

Calculating the determinant:

\[
\vec{u} \times \vec{v} = \mathbf{i}(2 \times 1 - 1 \times 4) - \mathbf{j}(0 \times 1 - 1 \times 2) + \mathbf{k}(0 \times 4 - 2 \times 2)
\]

Breaking down each component:

- \(x \)-component:

\[
2 \times 1 - 1 \times 4 = 2 - 4 = -2
\]

- \(y \)-component:

\[
0 \times 1 - 1 \times 2 = 0 -

Find The Volume Of The Parallelepiped With One Vertex At 2 1

2 And Adjacent Vertices At 2 3 3 4 5 3

Find The Volume Of The Parallelepiped With One Vertex At 2 1 2 And Adjacent Vertices At 2 3 3 4 5 3

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