For The Following Exercises, Consider Points P(-1,3), Q (1,5), And R(-3,7). Determine The Requested Vectors

Understanding Vector Calculations in Coordinate Geometry

For The Following Exercises, Consider Points P(-1,3), Q (1,5), And R(-3,7). Determine The Requested Vectors. When working with points in a coordinate plane, vectors are fundamental tools used to understand direction, magnitude, and relationships between points. This article provides a comprehensive guide to calculating various vectors based on the given points, including position vectors, displacement vectors, and more advanced vector operations.

Basic Concepts of Vectors in the Coordinate Plane

What Is a Vector?

A vector is a quantity that has both magnitude (length) and direction. In a coordinate plane, vectors are often represented as arrows originating from the origin or from one point to another. The coordinates of the vector indicate its direction and length.

Position Vectors

The position vector of a point is a vector that starts at the origin (0,0) and ends at that point. For example, the position vector of point P(-1, 3) is $\vec{OP}$ = <-1, 3>.

Displacement Vectors

Displacement vectors indicate the change from one point to another. They are calculated by subtracting the coordinates of the initial point from the terminal point.

Calculating the Basic Vectors

1. Position Vectors of Points P, Q, and R

- **Position vector of P (-1, 3):** $\overrightarrow{OP} = \langle -1, 3 \rangle$
- **Position vector of Q (1, 5):** $\overrightarrow{OQ} = \langle 1, 5 \rangle$
- **Position vector of R (-3, 7):** $\overrightarrow{OR} = \langle -3, 7 \rangle$

2. Displacement Vectors Between Points

Displacement vectors are calculated by subtracting the coordinates of the starting point from the ending point:

1. **Vector from P to Q (PQ):** $\overrightarrow{PQ} = Q - P = \langle 1 - (-1), 5 - 3 \rangle = \langle 2, 2 \rangle$
2. **Vector from P to R (PR):** $\overrightarrow{PR} = R - P = \langle -3 - (-1), 7 - 3 \rangle = \langle -2, 4 \rangle$
3. **Vector from Q to R (QR):** $\overrightarrow{QR} = R - Q = \langle -3 - 1, 7 - 5 \rangle = \langle -4, 2 \rangle$

Advanced Vector Operations

3. Magnitude of Vectors

The magnitude (or length) of a vector $\langle x, y \rangle$ is given by:

Magnitude = $\sqrt{x^2 + y^2}$

- **Magnitude of PQ:** $\sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.83$
- **Magnitude of PR:** $\sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47$
- **Magnitude of QR:** $\sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} \approx 4.47$

4. Dot Product of Vectors

The dot product helps determine the angle between two vectors and is calculated as:

$$\vec{A} \cdot \vec{B} = x_1x_2 + y_1y_2$$

- **Dot product of PQ and PR:** (2)(-2) + (2)(4) = -4 + 8 = 4
- **Dot product of PQ and QR:** (2)(-4) + (2)(2) = -8 + 4 = -4

5. Cross Product and Area of Triangle

The cross product in two dimensions gives a scalar value related to the area of the triangle formed by the points:

$$\text{Area} = \frac{1}{2} |x_1y_2 - y_1x_2|$$

- **Area of triangle PQR:** $\frac{1}{2} |(-1)(5 - 7) + 1(7 - 3) + (-3)(3 - 5)|$
- Calculating step-by-step:

$$\text{Area} = \frac{1}{2} |(-1)(-2) + 1(4) + (-3)(-2)|$$
$$= \frac{1}{2} |2 + 4 + 6|$$
$$= \frac{1}{2} |12|$$
$$= 6 \text{ square units}$$

Practical Applications of Vector Calculations

Navigation and Positioning

Vectors are essential in navigation systems, allowing for the calculation of shortest paths, directions, and distances between points on a map or in space.

Physics and Engineering

Understanding displacement, force vectors, and velocity relies heavily on vector operations. These calculations help engineers design structures, analyze forces, and simulate physical systems.

Computer Graphics and Animation

Vectors are fundamental in rendering images, animations, and 3D modeling.

They help define the position, movement, and orientation of objects in virtual space.

Summary of Key Vector Calculations for Points P, Q, and R

- Position vectors: <-1, 3>, <1, 5>, <-3, 7>
- Displacement vectors: <2, 2> (PQ), <-2, 4> (PR), <-4, 2> (QR)
- Magnitudes: approximately 2.83, 4.47, and 4.47 units respectively
- Dot products: 4 (PQ · PR), -4 (PQ · QR)
- Area of triangle PQR: 6 square units

Conclusion

Calculating vectors between points in the coordinate plane is a fundamental skill in mathematics, physics, engineering, and computer science. By understanding position vectors, displacement vectors, magnitudes, dot products, and areas, students and professionals can analyze spatial relationships effectively. The points P(-1,3), Q(1,5), and R(-3,7) serve as excellent examples to practice these essential vector operations and deepen one’s understanding of coordinate geometry.

Frequently Asked Questions

What is the vector PQ given points P(-1,3) and Q(1,5)?

The vector PQ is obtained by subtracting the coordinates of P from Q: (1 - (-1), 5 - 3) = (2, 2). So, PQ = (2, 2).

How do you find the vector PR for points P(-1,3) and R(-3,7)?

Subtract the coordinates of P from R: (-3 - (-1), 7 - 3) = (-2, 4). Therefore, PR = (-2, 4).

What is the vector RQ using points R(-3,7) and Q(1,5)?

Calculate RQ by subtracting R from Q: (1 - (-3), 5 - 7) = (4, -2). So, RQ = (4, -2).

How can I find the vector QP from points Q(1,5) and P(-1,3)?

Subtract P from Q: (-1 - 1, 3 - 5) = (-2, -2). Thus, QP = (-2, -2).

Are the vectors PQ and QP opposites?

Yes, PQ = (2, 2) and QP = (-2, -2) are opposite vectors, as they have the same magnitude but opposite directions.

What is the significance of calculating these vectors in coordinate geometry?

Calculating vectors helps determine directions and magnitudes between points, which is essential for understanding relationships such as parallelism, collinearity, and movement in space.

Can these vectors be used to find the angle between the points?

Yes, by using the dot product of two vectors, you can calculate the angle between them with the formula: $\cos \theta = (A \cdot B) / (|A| |B|)$.

How do these vector calculations aid in solving geometric problems involving points?

They allow you to analyze the relative positions, directions, and distances between points, enabling you to solve problems related to lines, angles, and shapes in coordinate geometry.

Additional Resources

Vectors in Focus: An In-Depth Analysis of Points P(-1,3), Q(1,5), and R(-3,7)

Introduction to Vectors and Their Significance in Coordinate Geometry

In the realm of coordinate geometry and vector analysis, understanding how to work with points and the vectors they form is fundamental. Vectors serve as the building blocks for representing direction and magnitude in space, and they are essential in a variety of mathematical, physical, and engineering applications. When given specific points, such as P(-1,3), Q(1,5), and R(-3,7), the challenge lies in determining the relevant vectors that connect these points, analyze their properties, and interpret the geometric relationships they depict.

In this article, we will explore the process of calculating various vectors associated with these points, understand their significance, and provide comprehensive insights into their applications. Whether you are a student seeking to sharpen your understanding or an enthusiast interested in the intricacies of vector analysis, this detailed review aims to clarify each step involved in working with these points.

Understanding the Basic Concepts of Vectors

Before delving into calculations, it’s essential to review some core concepts:

- Position Vectors: Represent the location of a point relative to the origin, denoted as \(\vec{OP}\) for a point P(x, y).
- Directed Line Segments (Vectors): The vector from point A to point B, denoted as \(\vec{AB}\), signifies the displacement from A to B.
- Vector Components: In a 2D plane, vectors are expressed as \(\langle x_2 - x_1, y_2 - y_1 \rangle\), representing the change in x and y coordinates.

Calculating Basic Vectors: From Points to Vectors

The first step involves translating the given points into vectors that represent specific directions or displacements within the coordinate plane.

1. Position Vectors of P, Q, and R

Position vectors indicate the location of each point relative to the origin (0,0):

- $\vec{OP} = \langle -1, 3 \rangle$
- $\vec{OQ} = \langle 1, 5 \rangle$
- $\vec{OR} = \langle -3, 7 \rangle$

These vectors are straightforward, directly corresponding to the point's coordinates.

2. Vectors Connecting the Points

Understanding the relationships between points requires calculating the vectors that connect them:

- Vector from P to Q ($\vec{PQ}$):

$$
\vec{PQ} = \vec{OQ} - \vec{OP} = \langle 1 - (-1), 5 - 3 \rangle = \langle 2, 2 \rangle
$$

This vector indicates that moving from P to Q involves a displacement of 2 units in the x-direction and 2 units in the y-direction.

- Vector from P to R ($\vec{PR}$):

$$
\vec{PR} = \vec{OR} - \vec{OP} = \langle -3 - (-1), 7 - 3 \rangle = \langle -2, 4 \rangle
$$

- Vector from Q to R ($\vec{QR}$):

$$
\vec{QR} = \vec{OR} - \vec{OQ} = \langle -3 - 1, 7 - 5 \rangle = \langle -4, 2 \rangle
$$

- Vector from R to P ($\vec{RP}$):

$$
\vec{RP} = \vec{OP} - \vec{OR} = \langle -1 - (-3), 3 - 7 \rangle = \langle 2, -4 \rangle
$$

Summary of connecting vectors:

From	To	Vector Components	Description
P	Q	$\langle 2, 2 \rangle$	Displacement from P to Q
P	R	$\langle -2, 4 \rangle$	Displacement from P to R
Q	R	$\langle -4, 2 \rangle$	Displacement from Q to R
R	P	$\langle 2, -4 \rangle$	Displacement from R to P

Analyzing the Magnitude and Direction of Vectors

Beyond just calculating vectors, understanding their magnitude (length) and direction provides deeper insights into the geometric relationships.

1. Magnitude of Vectors

The magnitude $|\vec{v}|$ of a vector $\vec{v} = \langle x, y \rangle$ is calculated via the Pythagorean theorem:

$$
|\vec{v}| = \sqrt{x^2 + y^2}
$$

Applying this to the vectors:

- $\vec{PQ}$:

$$
|\vec{PQ}| = \sqrt{2^2 + 2^2} = \sqrt{4 + 4} = \sqrt{8} \approx 2.83
$$

- $\vec{PR}$:

$$
|\vec{PR}| = \sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47
$$

- $\vec{QR}$:

$$
|\vec{QR}| = \sqrt{(-4)^2 + 2^2} = \sqrt{16 + 4} = \sqrt{20} \approx 4.47
$$

- $\vec{RP}$:

$$
|\vec{RP}| = \sqrt{2^2 + (-4)^2} = \sqrt{4 + 16} = \sqrt{20} \approx 4.47
$$

Observation: The vectors $\vec{PR}$, $\vec{QR}$, and $\vec{RP}$ have equal magnitudes, indicating some symmetry in the relationships among these points.

2. Direction of Vectors

The direction of a vector is often expressed as an angle relative to the positive x-axis, computed using:

$$
\theta = \arctan \left(\frac{y}{x} \right)
$$

Applying this:

- $\vec{PQ} = \langle 2, 2 \rangle$:

$$
\theta_{PQ} = \arctan \left(\frac{2}{2} \right) = \arctan(1) = 45^\circ
$$

- $\vec{PR} = \langle -2, 4 \rangle$:

$$
\theta_{PR} = \arctan \left(\frac{4}{-2} \right) = \arctan(-2) \approx -63.43^\circ
$$

Since the x-component is negative and y-component positive, the vector points toward the second quadrant, so:

$$
\theta_{PR} \approx 180^\circ - 63.43^\circ = 116.57^\circ
$$

- $\vec{QR} = \langle -4, 2 \rangle$:

$$
\theta_{QR} = \arctan \left(\frac{2}{-4} \right) = \arctan(-0.5) \approx -26.57^\circ
$$

In the second quadrant (x negative, y positive):

$$
\theta_{QR} \approx 180^\circ - 26.57^\circ = 153.43^\circ
$$

- $\vec{RP} = \langle 2, -4 \rangle$:

$$
\theta_{RP} = \arctan \left(\frac{-4}{2} \right) = \arctan(-2) \approx -63.43^\circ
$$

Since the x-component is positive and y-component negative, the vector points toward the fourth quadrant:

$$
\theta_{RP} \approx -63.43^\circ
$$

Summary of directions:

Vector	Magnitude	Approximate Direction (degrees)	Quadrant
$\vec{PQ}$	2.83	45°	First
$\vec{PR}$	4.47	116.57°	Second
$\vec{QR}$	4.47	153.43°	Second
$\vec{RP}$	4.47	-63.43°	Fourth

Applications and Geometric Interpretations

The vectors calculated provide a rich source of geometric insights:

- Parallelism: Since $\vec{PR}$, $\vec{QR}$, and $\vec{RP}$ have similar magnitudes but different directions, they potentially form geometric figures

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