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Understanding complex electrical circuits involving sinusoidal voltage sources is essential for electrical engineers and students alike. When analyzing such circuits, it becomes vital to determine various parameters like current flow, phase relationships, power consumption, and impedance. In this comprehensive article, we will explore how to analyze a circuit with the given voltage sources:

- Va(t) = 60 Cos (40,000t) Volts
- Vb(t) = 90 Sin (40,000t + 180°) Volts

We will systematically guide you through the steps of calculating the key electrical quantities, transforming the sinusoidal functions into their phasor forms, and interpreting the results for practical applications.

Understanding the Given Voltage Sources

Before diving into calculations, it’s crucial to understand the nature of the given voltage sources:

1. Sinusoidal Voltage Functions

Both Va(t) and Vb(t) are sinusoidal voltages with the same angular frequency, which suggests they are sinusoidally varying signals at the same frequency but with different initial phases and amplitudes.

2. Voltage Source Parameters

Parameter	Va(t)	Vb(t)
Amplitude	60 V	90 V
Frequency (ω)	40,000 rad/sec	40,000 rad/sec
Phase Shift	0°	180° (or π radians)

3. Representation in Phasor Form

Converting time-domain signals into phasor (complex exponential) form simplifies analysis:

- Va(t) = 60 Cos(ωt) translates to a phasor:
$\underline{V}_a = 60\angle 0^\circ$ V

- Vb(t) = 90 Sin(ωt + 180°)

Recall that $\sin(\theta) = \cos(\theta - 90^\circ)$, so:

$$
V_b(t) = 90 \sin(\omega t + 180^\circ) = 90 \cos(\omega t + 180^\circ - 90^\circ) = 90 \cos(\omega t + 90^\circ)
$$

But since $\sin(\theta + 180^\circ) = -\sin(\theta)$, alternatively, we can express Vb(t) as:

$$
V_b(t) = 90 \sin(\omega t + 180^\circ) = -90 \sin(\omega t)
$$

In phasor form, $\sin(\omega t + 180^\circ)$ corresponds to a phasor with amplitude 90 V and phase 180°, or equivalently,:

$$
\underline{V}_b = 90\angle 180^\circ \text{ V}
$$

Phasor Analysis of the Voltages

Phasors are complex numbers representing the magnitude and phase of sinusoidal signals, enabling easier addition, subtraction, and multiplication in AC circuit analysis.

1. Phasor Representations

Voltage Source	Phasor Form	Explanation
Va(t)	$\underline{V}_a = 60\angle 0^\circ$ V	Reference phase
Vb(t)	$\underline{V}_b = 90\angle 180^\circ$ V	Opposite phase (since 180° shift)

2. Implications

- The two voltages are at the same frequency but are phase-shifted by 180°.
- The amplitude of Vb(t) is larger, which can influence current and power calculations in the circuit.

Analyzing the Circuit: Step-by-Step Approach

To accurately analyze the circuit, follow these systematic steps:

1. Identify Circuit Components

Note the circuit's topology (resistors, inductors, capacitors), which are not specified here but are essential for a complete analysis.

2. Convert All Voltages and Currents into Phasor Domain

- Use the phasor forms for voltages.
- Express circuit elements in their impedance form:
- Resistor R : $Z_R = R$
- Inductor L : $Z_L = j\omega L$
- Capacitor C : $Z_C = \frac{1}{j\omega C}$

3. Apply Circuit Laws

- Use Kirchhoff’s Voltage Law (KVL) and Kirchhoff’s Current Law (KCL) in the phasor domain.
- Write equations relating voltages and currents.

4. Calculate Circuit Impedance and Currents

- Determine total impedance seen by each source.
- Find branch currents I_a and I_b.

5. Find Power and Power Factor

- Calculate real power:
$$
P = V_{rms} \times I_{rms} \times \cos(\phi)
$$
- Determine reactive power and apparent power:
$$
Q = V_{rms} \times I_{rms} \times \sin(\phi)
$$
$$
S = V_{rms} \times I_{rms}
$$
- Use the phase difference ϕ between voltage and current.

Calculating the Key Electrical Parameters

Given the generality of the circuit components, let's consider typical calculations based on the voltage sources.

1. RMS Values of Voltages

- For $Va(t) = 60 \cos(\omega t)$, RMS value:

$$
V_{a,\text{rms}} = \frac{60}{\sqrt{2}} \approx 42.43\, \text{V}
$$

- For $Vb(t) = 90 \sin(\omega t + 180°)$, RMS value:

$$
V_{b,\text{rms}} = \frac{90}{\sqrt{2}} \approx 63.64\, \text{V}
$$

2. Phase Difference and Its Effect

The phase difference between Va and Vb is 180°, which significantly influences the circuit's behavior, especially in power calculations.

3. Calculating Current in the Circuit

Suppose the circuit includes a known impedance Z. The complex current $\underline{I}$ supplied by each source can be calculated as:

$$
\underline{I}_a = \frac{\underline{V}_a}{Z}
$$

$$
\underline{I}_b = \frac{\underline{V}_b}{Z}
$$

The total current and power can then be derived based on these.

Power Calculations in the Circuit

Power analysis is crucial for understanding energy transfer and efficiency.

1. Real Power (P)

$$
P = V_{rms} \times I_{rms} \times \cos(\phi)
$$

\]

where ϕ is the phase angle between voltage and current.

2. Reactive Power (Q)

$$
Q = V_{rms} \times I_{rms} \times \sin(\phi)
$$

3. Apparent Power (S)

$$
S = V_{rms} \times I_{rms}
$$

4. Power Factor

$$
\text{Power Factor} = \cos(\phi)
$$

Understanding these parameters helps optimize circuit design, reduce energy losses, and improve efficiency.

Practical Applications and Optimization

Analyzing sinusoidal circuits with multiple sources is vital in power systems, communication circuits, and signal processing.

1. Power System Design

- Ensuring proper phase alignment minimizes reactive power.
- Balancing voltage amplitudes and phases reduces losses.

2. Signal Processing

- Understanding phase shifts assists in modulation and demodulation techniques.
- Accurate calculations improve the fidelity of transmitted signals.

3. Circuit Optimization

- Adjusting impedance values can optimize power transfer.
- Using reactive components to counteract phase differences enhances power factor.

Conclusion

Analyzing circuits with sinusoidal voltage sources like Va(t) = 60 Cos(40,000t) V and Vb(t) = 90 Sin(40,000t + 180°) V requires converting time-domain signals into phasor form, understanding phase relationships, and applying circuit laws in the complex domain. By calculating RMS values, impedance, and power parameters, engineers can optimize circuit performance, improve energy efficiency, and ensure reliable operation.

This methodology is fundamental for electrical engineering applications ranging from power distribution to communication systems. Mastery of these analysis techniques forms the backbone of designing and maintaining efficient, high-performance electrical circuits.

Keywords for SEO Optimization:

- Sinusoidal voltage analysis
- Phasor circuit analysis
- Power calculation in AC circuits
- Voltage sources with phase shift
- Impedance in AC circuits
- RMS voltage and current
- Power factor correction
- AC circuit fundamentals
- Electrical circuit analysis tutorial

Frequently Asked Questions

What are the given voltage functions in the circuit?

The given voltage functions are Va(t) = 60 Cos(40,000t) V and Vb(t) = 90 Sin(40,000t + 180°) V.

How can the sinusoidal voltage Vb(t) be expressed in cosine form?

Using the identity Sin(θ) = Cos(θ - 90°), Vb(t) can be written as 90 Cos(40,000t + 180° - 90°) V, which simplifies to 90 Cos(40,000t + 90°) V.

What is the phase difference between Va(t) and Vb(t)?

The phase difference is 90°, since Va(t) is a cosine with 0° phase and Vb(t) is a cosine with 90° phase shift.

How do you find the instantaneous voltage difference between Va(t) and Vb(t)?

Subtract Vb(t) from Va(t): Vdiff(t) = Va(t) - Vb(t). Substituting the expressions gives Vdiff(t) = 60 Cos(40,000t) - 90 Cos(40,000t + 90°).

What is the resulting expression for Vdiff(t)?

Using the cosine addition formula, Vdiff(t) = 60 Cos(40,000t) - 90 Sin(40,000t), since Cos(θ + 90°) = -Sin(θ).

How can Vdiff(t) be simplified into a single cosine function?

Express Vdiff(t) as a single sinusoid: Vdiff(t) = R Cos(40,000t + φ), where R = $\sqrt{(60^2 + 90^2)}$ and φ = arctangent of (−90/60).

What are the magnitude and phase angle of Vdiff(t)?

The magnitude R = $\sqrt{(60^2 + 90^2)}$ = $\sqrt{(3600 + 8100)}$ = $\sqrt{11700}$ ≈ 108.17 V. The phase angle φ = arctangent(−90/60) ≈ arctangent(−1.5) ≈ −56.31°.

What is the final expression for the voltage difference Vdiff(t)?

Vdiff(t) ≈ 108.17 Cos(40,000t − 56.31°) V.

How is this analysis useful in circuit analysis or design?

Understanding the phase difference and resultant voltage helps in analyzing power flow, impedance, and resonance conditions in AC circuits.

Additional Resources

Given the Following Circuit With Va(t) = 60 Cos (40,000t) V And Vb(t) = 90 Sin (40,000t + 180) V. Calculate – this is a classic problem in alternating current (AC) circuit analysis that involves understanding how sinusoidal voltages interact within a circuit. Whether you're an electrical engineering student, a practicing engineer, or an enthusiast diving into AC circuit analysis, mastering how to analyze such signals is fundamental. In this article, we will provide a comprehensive, step-by-step guide to analyze the given circuit and perform the necessary calculations, including converting waveforms, finding phase relationships, and calculating parameters like current, power, and impedance.

Understanding the Problem

Before plunging into calculations, it’s essential to understand what is being asked and the given data:

- Va(t) = 60 Cos(40,000t) V
- Vb(t) = 90 Sin(40,000t + 180) V

These are two sinusoidal voltage sources with the same angular frequency but different forms and phase shifts. The goal is to analyze the circuit — which could involve calculating the current flowing through elements, the power delivered, or the phase relationships between voltages and currents.

Key Concepts in AC Circuit Analysis

Sinusoidal Voltages and Currents

In AC analysis, voltages and currents are often sinusoidal functions of time, expressed as:

- Cosine form: $V(t) = V_m \cos(\omega t + \phi)$
- Sine form: $V(t) = V_m \sin(\omega t + \phi)$

Where:
- V_m is the amplitude (peak value)
- ω is the angular frequency
- ϕ is the phase angle

Converting Between Sine and Cosine

Since the given voltages are in different forms, converting them into a common form simplifies analysis:

- Cosine to Sine:
$\cos(\theta) = \sin(\theta + 90^\circ)$

- Sine to Cosine:
$\sin(\theta) = \cos(\theta - 90^\circ)$

Phasor Representation

To analyze AC circuits, it's often easier to convert time-domain waveforms to their phasor equivalents:

- Phasor of a cosine wave: $V = V_m \angle \phi$
- Phasor of a sine wave: $V = V_m \angle (\phi - 90^\circ)$

Phasor analysis allows for straightforward addition, subtraction, and multiplication of sinusoidal signals.

Step 1: Convert Voltages to Phasor Form

Voltage $Va(t)$

Given:
$Va(t) = 60 \cos(40,000 t)$

This is already in cosine form with zero phase shift:

- Amplitude: $V_{a,m} = 60\,V$
- Phase: 0°

Phasor representation:
$\mathbf{V}_a = 60 \angle 0^\circ\, V$

Voltage $Vb(t)$

Given:
$Vb(t) = 90 \sin(40,000 t + 180^\circ)$

Convert sine to cosine:

$$
Vb(t) = 90 \sin(40,000 t + 180^\circ) = 90 \cos(40,000 t + 180^\circ - 90^\circ) = 90 \cos(40,000 t + 90^\circ)
$$

Alternatively, note that:

$$
\sin(\theta) = \cos(\theta - 90^\circ)
$$

So,

$$
Vb(t) = 90 \sin(40,000 t + 180^\circ) = 90 \cos(40,000 t + 180^\circ - 90^\circ) = 90 \cos(40,000 t + 90^\circ)
$$

\]

Phasor form:

- Amplitude: $V_{b,m} = 90\,V$
- Phase: 90°

Thus:

$$
\mathbf{V}_b = 90 \angle 90^\circ\, V
$$

Step 2: Analyze the Phase Relationship

Now, with the phasors:

- $\mathbf{V}_a = 60 \angle 0^\circ$
- $\mathbf{V}_b = 90 \angle 90^\circ$

The phase difference:

$$
\Delta \phi = 90^\circ - 0^\circ = 90^\circ
$$

This indicates that V_b leads V_a by 90 degrees.

Step 3: Circuit Configuration and Components

While the problem statement at first glance seems to only specify voltages, typically, such problems involve:

- Series or parallel components (resistors, inductors, capacitors)
- The need to find currents, impedance, or power

Assuming a generic circuit, common analyses include:

Possible scenarios:

- Series RLC circuit:
Resistance R, inductance L, capacitance C

- Multiple sources connected to a network

To proceed, we need to specify the circuit elements or assume a standard configuration. For this tutorial, let's assume the following scenario:

> A series circuit with a resistor R, an inductor L, and a capacitor C, with the two voltage sources $Va(t)$ and $Vb(t)$ applied across the series network.

Step 4: Find the Equivalent Phasor Voltage

In a circuit with two voltage sources in series, the total phasor voltage V_{total} is the sum of individual phasors:

$$
\mathbf{V}_{total} = \mathbf{V}_a + \mathbf{V}_b
$$

Calculating:

$$
\mathbf{V}_a = 60 \angle 0^\circ
$$
$$
\mathbf{V}_b = 90 \angle 90^\circ
$$

Expressed in rectangular form:

- $\mathbf{V}_a = 60 + j0$
- $\mathbf{V}_b = 0 + j90$

Sum:

$$
\mathbf{V}_{total} = (60 + 0) + j(0 + 90) = 60 + j90
$$

Magnitude:

$$
|V_{total}| = \sqrt{60^2 + 90^2} = \sqrt{3600 + 8100} = \sqrt{11700} \approx 108.17\,V
$$

Phase:

$$
\angle V_{total} = \arctan \left(\frac{90}{60} \right) = \arctan(1.5) \approx 56.31^\circ
$$

Therefore:

$$
\mathbf{V}_{total} \approx 108.17 \angle 56.31^\circ\, V
$$

Step 5: Calculate Circuit Impedance and Current

Suppose the circuit has known impedance:

$$
Z = R + j \left(\omega L - \frac{1}{\omega C} \right)
$$

The current phasor:

$$
\mathbf{I} = \frac{\mathbf{V}_{total}}{Z}
$$

Without specific R, L, C, we cannot compute a numeric value. For demonstration, assume:

- $R = 10\, \Omega$
- $L = 10\, \text{mH}$
- $C = 10\, \mu \text{F}$

Calculate impedance:

$$
\omega = 40,000\, \text{rad/sec}
$$

$$
X_L = \omega L = 40,000 \times 10 \times 10^{-3} = 400\, \Omega
$$

$$
X_C = \frac{1}{\omega C} = \frac{1}{40,000 \times 10 \times 10^{-6}} = \frac{1}{0.4} = 2.5\, \Omega
$$

Total impedance:

$$
Z = 10 + j (400 - 2.5) = 10 + j 397.5\, \Omega
$$

Magnitude:

\[

|Z| = \sqrt{10^2 + 397.5^2} \approx 397.58\, \Omega
\]

Phase:

\[
\phi_Z = \arctan \left(\frac{397.5}{10} \right) \approx 87.14^\circ
\]

Step 6: Compute the Current

Phasor current:

\[
\mathbf{I} = \frac{108.17 \angle 56.31^\circ}{397.58 \angle 87.14^\circ}
\]

Divide magnitudes:

\[
I_m = \frac{108.17}{397.58} \approx 0.272\,A
\]

Subtract angles:

\[
\phi_I = 56.31^\circ - 87.14^\circ \approx -30.83^\

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