Given The Following Quadratic Function In Standard Form F(x)= -2(x - 1) + 16 Answer The Questions Below.All

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Understanding quadratic functions is fundamental in algebra, especially when analyzing their properties, graphs, and applications. The given function, F(x) = -2(x - 1) + 16, appears to be presented in a form that resembles the vertex form, which makes understanding its graph and key features more straightforward. In this article, we will explore this quadratic function comprehensively by converting it to standard form, identifying its key characteristics, graphing it, and answering relevant questions that deepen our understanding.

Interpreting the Given Function

Before delving into calculations, it's essential to recognize the structure of the function provided.

Understanding the Expression

The function is written as:
- F(x) = -2(x - 1) + 16

At first glance, this resembles the vertex form of a quadratic function, which is generally:
- y = a(x - h)^2 + k

However, the given expression appears linear in form because it lacks the square term. But considering the context and typical quadratic function forms, it is likely that there is a typo or omission, and perhaps the intended function is:
- F(x) = -2(x - 1)^2 + 16

This correction makes sense because a quadratic function in vertex form should include a squared term. Assuming this correction, the function becomes:

F(x) = -2(x - 1)^2 + 16

This form makes the analysis meaningful and aligns with standard quadratic function properties.

Converting to Standard Form

The first step is to expand the function into the standard quadratic form: y = ax^2 + bx + c.

Step-by-Step Expansion

Starting with:
- $F(x) = -2(x - 1)^2 + 16$

Expand the squared term:
- $(x - 1)^2 = x^2 - 2x + 1$

Multiply through by -2:
- $-2(x^2 - 2x + 1) = -2x^2 + 4x - 2$

Add the constant 16:
- $F(x) = -2x^2 + 4x - 2 + 16$

Simplify:
- $F(x) = -2x^2 + 4x + 14$

Standard form:
- $F(x) = -2x^2 + 4x + 14$

This quadratic function opens downward (because the coefficient of x^2, -2, is negative), and its graph is a parabola.

Key Features of the Quadratic Function

Understanding the key features such as vertex, axis of symmetry, y-intercept, and x-intercepts helps in graphing and analyzing the function.

Vertex of the Parabola

Since the function is in vertex form, the vertex is directly given by (h, k):

- Vertex: $(h, k) = (1, 16)$

This indicates the parabola's highest point (since it opens downward) is at $x=1$ with $y=16$.

Axis of Symmetry

The axis of symmetry passes through the vertex:

- Axis of symmetry: $x = 1$

This vertical line divides the parabola into two mirror-image halves.

Y-Intercept

To find the y-intercept, set $x=0$:

$F(0) = -2(0 - 1)^2 + 16 = -2(1)^2 + 16 = -2 + 16 = 14$

- Y-intercept: (0, 14)

X-Intercepts

To find x-intercepts, set $F(x) = 0$ and solve:

$0 = -2x^2 + 4x + 14$

Divide through by -2 to simplify:

$0 = x^2 - 2x - 7$

Now, solve using the quadratic formula:

$x = [2 \pm \sqrt{((-2)^2 - 4(1)(-7))}]/(21)$

$x = [2 \pm \sqrt{(4 + 28)}]/2$

$x = [2 \pm \sqrt{32}]/2$

$\sqrt{32} = \sqrt{(162)} = 4\sqrt{2}$

Thus,

$x = [2 \pm 4\sqrt{2}]/2$

Simplify numerator:

$x = [2/2] \pm [4\sqrt{2}/2] = 1 \pm 2\sqrt{2}$

- X-intercepts:

$x = 1 + 2\sqrt{2} \approx 1 + 2(1.414) \approx 1 + 2.828 \approx 3.828$

$x = 1 - 2\sqrt{2} \approx 1 - 2.828 \approx -1.828$

Coordinates:

- (3.828, 0)
- (-1.828, 0)

Graphing the Quadratic Function

Armed with the key features, plotting the function becomes straightforward.

Steps to Graph

1. Plot the vertex at (1, 16).
2. Draw the axis of symmetry at x=1.
3. Plot the y-intercept at (0, 14).
4. Plot the x-intercepts at approximately (-1.828, 0) and (3.828, 0).
5. Sketch the parabola opening downward through these points, ensuring symmetry about x=1.

The graph will show a downward-opening parabola with its highest point at (1, 16).

Answering Common Questions About the Function

Let's address some typical questions related to this quadratic function.

1. What is the maximum or minimum value of the function?

Since the parabola opens downward (coefficient of x^2 is negative), the function has a maximum at its vertex.

- Maximum value: 16 at x=1

2. What is the domain and range of the function?

- Domain: All real numbers, since quadratic functions are defined everywhere:

Domain: (-∞, ∞)

- Range: Since the parabola opens downward and has a maximum at y=16,

Range: (-∞, 16]

3. How can we interpret the quadratic in real-world contexts?

Quadratic functions often model situations like projectile motion, profit maximization, or area optimization. For example, if this function represented the height of an object over time, the maximum height would be 16 units at time $x=1$.

4. How does the coefficient of x^2 affect the graph?

- The coefficient (-2) determines the parabola's opening direction (downward) and the "width" of the parabola. Larger absolute values produce narrower parabolas, while smaller ones produce wider ones.

5. How do transformations affect the graph?

The original vertex form indicates the parabola is shifted right by 1 unit and upward by 16 units. The negative coefficient causes a reflection over the x-axis.

Applications and Importance of Understanding Quadratic Functions

Quadratic functions like $F(x) = -2(x - 1)^2 + 16$ are vital in various fields:

- Physics: Modeling projectile trajectories
- Engineering: Designing parabolic reflectors or antennas
- Economics: Maximizing profit or minimizing cost
- Biology: Population growth models with quadratic growth limits

Mastering the transformation, graphing, and analysis of quadratic functions equips students and professionals to solve complex problems efficiently.

Conclusion

In summary, the quadratic function $F(x) = -2(x - 1)^2 + 16$ reveals significant insights once expanded and analyzed. Its vertex at (1, 16), x-intercepts at approximately (-1.828, 0) and (3.828, 0), and y-intercept at (0, 14) provide a full picture of its graph. Recognizing the effects of the coefficients and transformations helps in understanding its behavior and applications. Whether for academic pursuits or real-world modeling, a solid grasp of quadratic functions enables effective problem-solving and analysis.

Understanding these features and calculations not only clarifies the properties of this particular function but also builds foundational skills applicable to a broad range of quadratic problems encountered in mathematics and beyond.

Frequently Asked Questions

What is the vertex of the quadratic function F(x) = -2(x - 1)^2 + 16?

The vertex is at (1, 16).

Is the parabola opening upwards or downwards in the given quadratic function?

The parabola opens downward because the coefficient of the squared term is negative (-2).

What is the axis of symmetry for the quadratic function F(x) = -2(x - 1)^2 + 16?

The axis of symmetry is the vertical line x = 1.

What is the maximum value of the quadratic function?

The maximum value of F(x) is 16, which occurs at x = 1.

How do you find the y-intercept of the quadratic function?

Substitute x = 0 into the function: F(0) = -2(0 - 1)^2 + 16 = -2(1) + 16 = 14. So, the y-intercept is at (0, 14).

What is the domain and range of the quadratic function F(x) = -2(x - 1)^2 + 16?

The domain is all real numbers (-∞, ∞), and the range is (-∞, 16], since the maximum value is 16.

Additional Resources

Given The Following Quadratic Function In Standard Form F(x) = -2(x - 1) + 16, understanding how to analyze and interpret quadratic functions is a fundamental skill in algebra. Whether you're a student preparing for exams, a teacher designing lesson plans, or a math enthusiast eager to deepen your understanding, breaking down the components of a quadratic function helps clarify its behavior and characteristics. In this comprehensive guide, we'll explore the various aspects of the given quadratic function, interpret its features, and answer key questions to enhance your grasp of quadratic functions in standard form.

Understanding the Quadratic Function: F(x) = -2(x - 1) + 16

Before diving into specific questions, it's essential to recognize that the given function is written in a form similar to the vertex form of a quadratic, but with a slight variation. The function appears as:

$F(x) = -2(x - 1) + 16$

At first glance, this resembles the vertex form:

$F(x) = a(x - h)^2 + k$

However, the given function lacks the quadratic term (the squared component). It appears to be a linear function, but the context suggests it is meant to be analyzed as a quadratic, perhaps with a typo or misinterpretation. To clarify, we'll assume the function is intended as:

$F(x) = -2(x - 1)^2 + 16$

This is a common quadratic form, where:

- $a = -2$
- $h = 1$
- $k = 16$

If this assumption aligns with your problem, we can proceed with analyzing this quadratic function. If not, please clarify, but for the purpose of this guide, we'll analyze $F(x) = -2(x - 1)^2 + 16$.

Breaking Down the Quadratic Function

1. Standard and Vertex Forms

Quadratic functions can be written in various forms:

- Standard form: $F(x) = ax^2 + bx + c$
- Vertex form: $F(x) = a(x - h)^2 + k$

The vertex form is particularly useful for identifying the vertex and transformations directly.

Given our assumed function:

$F(x) = -2(x - 1)^2 + 16$

- Vertex (h, k): (1, 16)
- Leading coefficient (a): -2

2. Identifying Key Components

- Vertex: The point where the parabola reaches its maximum or minimum.
- Axis of symmetry: The vertical line that passes through the vertex.
- Direction of opening: Determined by the sign of a.
- Width and shape: Influenced by the absolute value of a.
- Y-intercept: The point where the parabola crosses the y-axis.

- X-intercepts (roots): The points where the parabola crosses the x-axis.

Analyzing the Key Features

Vertex

Since the function is in vertex form, the vertex is directly read as (h, k) = (1, 16).

- Interpretation: The parabola reaches its maximum value of 16 at x = 1. Because a = -2 (negative), the parabola opens downward, confirming that the vertex is a maximum point.

Axis of Symmetry

- The vertical line x = h = 1 serves as the axis of symmetry.
- Implication: The parabola is symmetric about this line, meaning points equidistant from x = 1 on either side will have the same y-value.

Direction of Opening

- Since a = -2 < 0, the parabola opens downward.
- Consequence: The maximum point is at the vertex, and the parabola extends downward indefinitely on both sides.

Width and Shape

- The absolute value of a affects the "width" of the parabola.
- |a| = 2 indicates a relatively narrow parabola compared to a parabola with a smaller absolute value.
- Implication: The parabola is steep, and the y-values decrease rapidly as you move away from the vertex along the x-axis.

Finding the Intercepts

Y-Intercept

- To find the y-intercept, substitute x = 0 into the function:

F(0) = -2(0 - 1)^2 + 16

F(0) = -2(1)^2 + 16 = -2(1) + 16 = -2 + 16 = 14

- Y-intercept: (0, 14)

X-Intercepts (Roots)

- To find x-intercepts, set F(x) = 0:

-2(x - 1)^2 + 16 = 0

- Rearranged:

$-2(x - 1)^2 = -16$

$(x - 1)^2 = 8$

- Take the square root of both sides:

$x - 1 = \pm\sqrt{8} = \pm 2\sqrt{2}$

- Solve for x:

$x = 1 \pm 2\sqrt{2}$

- X-intercepts:

- $x = 1 + 2\sqrt{2} \approx 1 + 2(1.414) \approx 1 + 2.828 \approx 3.828$

- $x = 1 - 2\sqrt{2} \approx 1 - 2.828 \approx -1.828$

Graphical Summary

- The parabola opens downward with a vertex at (1, 16).
- It crosses the y-axis at (0, 14).
- It crosses the x-axis at approximately (-1.828, 0) and (3.828, 0).
- The parabola is symmetric about x = 1.

Key Questions and Detailed Answers

Q1: What is the vertex of the quadratic function, and what does it tell us?

Answer:
The vertex of the quadratic function $F(x) = -2(x - 1)^2 + 16$ is at (1, 16). This point indicates the maximum value of the parabola because the parabola opens downward. The y-value of 16 at x=1 is the highest point the parabola reaches, representing the maximum of the function.

Q2: How does the parabola open, and how does the coefficient a influence its shape?

Answer:
Since a = -2, which is negative, the parabola opens downward. The magnitude of a (2) affects how "steep" or "narrow" the parabola appears. A larger absolute value of a results in a narrower parabola, while a smaller absolute value makes it wider. In this case, a = -2 means the parabola is relatively steep, descending quickly from the vertex.

Q3: What are the x-intercepts, and how are they calculated?

Answer:

The x-intercepts are the points where the parabola crosses the x-axis, where $F(x) = 0$. To find these:

- Set the function equal to zero:

$-2(x - 1)^2 + 16 = 0$

- Rearrange:

$(x - 1)^2 = 8$

- Take the square root:

$x - 1 = \pm\sqrt{8} = \pm 2\sqrt{2}$

- Solve for x:

$x = 1 \pm 2\sqrt{2}$

- Numerically, these are approximately $x \approx -1.828$ and $x \approx 3.828$.

Q4: What is the y-intercept, and how is it determined?

Answer:
The y-intercept occurs when $x = 0$:

$F(0) = -2(0 - 1)^2 + 16 = -2(1) + 16 = 14$

- So, the y-intercept is at (0, 14).

Q5: How does the axis of symmetry relate to the vertex?

Answer:
The axis of symmetry is the vertical line $x = h = 1$. It passes through the vertex, dividing the parabola into mirror-image halves. Points equidistant from $x = 1$ will have the same y-value, highlighting the symmetry of the parabola.

Practical Applications and Significance

Understanding the features of a quadratic function like $F(x) = -2(x - 1)^2 + 16$ is crucial in various real-world contexts:

- Physics: Parabolas describe projectile motion, where the vertex indicates the highest point.
- Economics: Quadratic models can depict profit maximization or cost minimization scenarios.
- Engineering: Designing parabolic reflectors or antennas relies on understanding parabola properties.
- Statistics: Quadratic functions are used in modeling relationships with a peak or trough.

Final Thoughts

Analyzing a quadratic function involves unraveling its components—vertex, axis of symmetry, intercepts, and opening direction—to fully understand its behavior. The function F(x) = -2(x - 1)^2 + 16 exemplifies a parabola with a maximum at (1, 16), opening downward, with clear intercepts on both axes. Mastering these concepts enables students and professionals to interpret quadratic models effectively, making informed predictions and decisions based on their graphs.

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