How Do You Find A Vector Parametric Equation R(t) For The Line Through Points P=(-3,-1,1) And Q = (-8,-4,5)

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When working with lines in three-dimensional space, one common task is to find a vector parametric equation that describes the line passing through two given points. In this article, we will explore the step-by-step process of deriving the vector parametric equation \(R(t) \) for the line passing through the points \(P = (-3, -1, 1) \) and \(Q = (-8, -4, 5) \). This process involves understanding key concepts such as vectors, direction vectors, and parametric equations, and applying them systematically.

Understanding the problem and the goal is essential before diving into calculations. The goal here is to express the line as a vector function \(R(t) \), which can generate all points on the line as \(t \) varies over the real numbers. This allows us to analyze, visualize, and manipulate the line mathematically and geometrically.

In this article, we will cover the following topics:
- Basic concepts needed for deriving the parametric equation
- Step-by-step process to find the direction vector
- Constructing the parametric equation \(R(t) \)
- Visualizing the line in 3D space
- Practical applications and tips for working with parametric equations

Let's begin by reviewing some foundational concepts.

Fundamental Concepts for Deriving the Line Equation

Points and Vectors in 3D Space

In three-dimensional space (\(\mathbb{R}^3 \)), a point is represented by an ordered triple \((x, y, z)\). For example, the points \(P = (-3, -1, 1) \) and \(Q = (-8, -4, 5) \) are specific locations in space.

A vector in \(\mathbb{R}^3 \) is an entity with both magnitude and direction. It can be represented as an ordered triple as well, such as \(\vec{v} = (v_x, v_y, v_z) \).

The Line as a Set of Points

A line in 3D space can be represented as the set of all points \(R \) obtained by starting from a fixed point \(P \) on the line and moving in some fixed direction \(\vec{d} \). Mathematically, this can be expressed as:
\[

R(t) = P + t \vec{d}
\]
where:
- \(P \) is a point on the line.
- \(\vec{d} \) is the direction vector.
- \(t \) is a real parameter that scales the direction vector.

This form is called the vector parametric equation of the line.

Importance of the Direction Vector

The direction vector \(\vec{d} \) indicates the line's orientation in space. It is crucial because it determines how the line extends from the initial point \(P \).

To find \(\vec{d} \), we typically use the difference between two known points on the line:
\[
\vec{d} = Q - P
\]
This vector points from \(P \) toward \(Q \) and describes the line's direction.

Step-by-Step Process to Find the Vector Parametric Equation

Step 1: Identify the Known Points

Given points:
\[
P = (-3, -1, 1)
\]
\[
Q = (-8, -4, 5)
\]

Step 2: Find the Direction Vector \(\vec{d} \)

Calculate the difference between \(Q \) and \(P \):
\[
\vec{d} = Q - P = (-8 - (-3), -4 - (-1), 5 - 1)
\]
Simplify each component:
\[
\vec{d} = (-8 + 3, -4 + 1, 5 - 1) = (-5, -3, 4)
\]

This vector points from \(P \) to \(Q \) and indicates the direction of the line.

Step 3: Write the Vector Equation $R(t)$

Using the point P as the initial point and the direction vector $\vec{d}$, the parametric form is:

$$
R(t) = P + t \vec{d}
$$

which expands to:

$$
R(t) = (-3, -1, 1) + t (-5, -3, 4)
$$

Step 4: Express the Equation in Component Form

Breaking it down into x, y, and z components:

$$
x(t) = -3 - 5t
$$

$$
y(t) = -1 - 3t
$$

$$
z(t) = 1 + 4t
$$

Thus, the parametric equations of the line are:

$$
\boxed{
\begin{cases}
x(t) = -3 - 5t \\
y(t) = -1 - 3t \\
z(t) = 1 + 4t
\end{cases}
}
$$

Visualizing the Line in 3D Space

Understanding the geometric interpretation of the parametric equations helps in visualizing the line. As t varies over all real numbers:
- The point $R(t)$ traces the entire line.
- When $t = 0$, the point is at $P = (-3, -1, 1)$.
- When $t = 1$, the point is at $P + \vec{d} = (-3 - 5, -1 - 3, 1 + 4) = (-8, -4, 5) = Q$.

This confirms that the parametric equation correctly describes the line passing through P and Q.

Additional Tips and Variations

Using the Point-Vector Form

You can also express the parametric equation as:

$$
R(t) = P + t \vec{d}
$$

where P is any point on the line, and $\vec{d}$ is the direction vector.

Scalar Parameter Range

While t typically ranges over all real numbers for the entire line, setting specific bounds on t can describe line segments:
- For the segment between P and Q, t varies from 0 to 1.
- For the entire line, $t \in (-\infty, \infty)$.

Using Different Points as the Starting Point

You can choose either P or Q as the initial point in the parametric equation. For example, starting at Q:

$$
R(t) = Q + t (P - Q)
$$

which yields the same line, just traced in the opposite direction for negative t.

Applications of Vector Parametric Equations

Understanding how to find and use vector parametric equations is fundamental in various fields such as:
- Computer graphics and visualization
- Engineering and physics for modeling trajectories
- Geometry problems involving lines and planes
- Robotics and kinematics for motion planning
- CAD (Computer-Aided Design) modeling

Summary

To find the vector parametric equation $R(t)$ for the line passing through points $P = (-3, -1, 1)$ and $Q = (-8, -4, 5)$, follow these key steps:
1. Identify the two points.
2. Compute the direction vector $\vec{d} = Q - P = (-5, -3, 4)$.
3. Choose one point (say, P) as the initial point.

4. Write the parametric form:

$$
R(t) = P + t \vec{d}
$$

which in component form is:

$$
x(t) = -3 - 5t, \quad y(t) = -1 - 3t, \quad z(t) = 1 + 4t
$$

This process provides a complete mathematical description of the line in three-dimensional space, enabling visualization, analysis, and further application in various disciplines.

If you want to explore further, consider how this approach generalizes to lines passing through more points, lines intersecting planes, or even lines in higher-dimensional spaces. Mastering the derivation of parametric equations is a foundational skill in vector calculus and analytical geometry.

Frequently Asked Questions

What is the first step in finding the parametric equation of a line through points P and Q?

The first step is to find the direction vector by subtracting the coordinates of P from Q, i.e., Q - P.

How do you compute the direction vector for the line passing through points P=(-3,-1,1) and Q=(-8,-4,5)?

Subtract the coordinates of P from Q: (-8 - (-3), -4 - (-1), 5 - 1) which simplifies to (-5, -3, 4).

What is the general form of the vector parametric equation R(t) for a line?

R(t) = P + t d, where P is a point on the line and d is the direction vector.

Using point P and the direction vector, what is the parametric equation R(t) for this line?

R(t) = (-3, -1, 1) + t (-5, -3, 4).

How can the parametric equations be written component-wise for this line?

x(t) = -3 - 5t, y(t) = -1 - 3t, z(t) = 1 + 4t.

Why is it important to find the direction vector when deriving the parametric equation?

The direction vector indicates the line's orientation and is essential for expressing how the points move along the line as t varies.

Can the parametric equation R(t) be written using point Q instead of P?

Yes, using Q as the point, R(t) = Q + t (P - Q), which results in a similar parametric form with a different starting point.

What is the significance of the parameter t in the parametric equation?

The parameter t varies over all real numbers and determines specific points along the line.

Are there alternative ways to write the line equation besides the vector parametric form?

Yes, you can also express the line in symmetric form or as parametric equations for each coordinate separately, as shown above.

Additional Resources

How Do You Find A Vector Parametric Equation R(t) For The Line Through Points P=(-3,-1,1) And Q = (-8,-4,5)

Understanding the mathematical foundation behind deriving the vector parametric equation of a line passing through two points is fundamental in fields ranging from computer graphics and engineering to physics and advanced mathematics. This article provides a comprehensive, step-by-step exploration of the process, offering insights suitable for students, educators, and practitioners seeking to deepen their grasp of vector calculus concepts.

Introduction to Vector Parametric Equations of Lines

At the core of three-dimensional geometry lies the concept of a vector parametric equation, which describes a line in space using a parameter, usually denoted as t. Unlike the familiar Cartesian equations, parametric forms offer a flexible and intuitive way to represent lines, curves, and surfaces.

The general form of a vector parametric equation for a line in three-dimensional space is:

$$R(t) = \mathbf{r}_0 + t \mathbf{v}$$

where:
- $\mathbf{r}_0$ is a position vector to a specific point on the line,
- $\mathbf{v}$ is the direction vector indicating the line's orientation,
- t is a real parameter that varies over all real numbers.

This form provides a simple yet powerful way to generate all points on the line as t varies.

Identifying the Core Components: Points and Direction Vector

Given two distinct points in space, P and Q, the goal is to derive a vector parametric equation of the line passing through both.

Points given:

- $P = (-3, -1, 1)$
- $Q = (-8, -4, 5)$

Step 1: Find the Direction Vector $\mathbf{v}$

The direction vector is a vector that points from P to Q:

$$
\mathbf{v} = \overrightarrow{PQ} = \mathbf{Q} - \mathbf{P}
$$

Calculating component-wise:

$$
\mathbf{v} = (-8 - (-3),\ -4 - (-1),\ 5 - 1) = (-8 + 3,\ -4 + 1,\ 4) = (-5,\ -3,\ 4)
$$

This vector indicates the direction of the line.

Step 2: Choose a Point on the Line

Any point on the line can serve as $\mathbf{r}_0$. The point P is a convenient choice, but Q could also be used.

Deriving the Vector Parametric Equation

With the components identified, the parametric equation takes the form:

$$
\boxed{
R(t) = \mathbf{P} + t \mathbf{v}
}
$$

Substituting the known values:

$$
R(t) = (-3, -1, 1) + t(-5, -3, 4)
$$

Expressed component-wise:

$$
\begin{cases}
x(t) = -3 - 5t \\
y(t) = -1 - 3t \\
z(t) = 1 + 4t
\end{cases}
$$

This set of equations fully describes every point on the line as t varies over $\mathbb{R}$.

Alternative Forms and Interpretations

While the above parametric equations are straightforward, understanding their relationships with other forms enhances their utility.

Vector Form

The vector form of the line:

$$
\mathbf{R}(t) = \mathbf{r}_0 + t \mathbf{v}
$$

corresponds to:

$$
\mathbf{R}(t) = \begin{bmatrix} -3 \\ -1 \\ 1 \end{bmatrix} + t \begin{bmatrix} -5 \\ -3 \\ 4 \end{bmatrix}
$$

Symmetric Equations

Optionally, the parametric equations can be rearranged into symmetric form (assuming $\mathbf{v}$ components are non-zero):

$$
\frac{x + 3}{-5} = \frac{y + 1}{-3} = \frac{z - 1}{4} \quad \text{(for } t \text{)}
$$

which can be simplified further or used depending on the context.

Visualizing the Line in Space

Understanding the geometry of the line provides valuable insight:

- The point $P = (-3, -1, 1)$ acts as a reference point.
- The direction vector $\mathbf{v} = (-5, -3, 4)$ indicates the line's orientation in three-dimensional space.
- The parameter t scales along this direction, moving from the point P (when $t=0$) outward in both directions.

Graphically, as t varies:

- For $t=1$, the point is $R(1) = (-8, -4, 5)$, which matches point Q, confirming the line passes through both points.
- For $t=0$, the point is P.

Verification and Consistency Checks

Ensuring the derived equation accurately represents the line through P and Q is crucial.

Check at $t=1$:

$$
\begin{cases}
x(1) = -3 - 5(1) = -3 - 5 = -8 \\
y(1) = -1 - 3(1) = -1 - 3 = -4 \\
z(1) = 1 + 4(1) = 1 + 4 = 5
\end{cases}
$$

which matches point Q.

Check at $t=0$:

$$
\begin{cases}
x(0) = -3 \\
y(0) = -1 \\
z(0) = 1
\end{cases}
$$

which matches point P.

This confirms the parametric equations precisely describe the line passing through both points.

Applications and Broader Context

The process described finds widespread application in several domains:

- Computer graphics: Rendering lines and curves in 3D space.
- Physics: Describing the trajectory of particles or objects.
- Engineering: Analyzing structural elements and their spatial relationships.
- Mathematics: Foundations for more advanced topics like planes, surfaces, and vector calculus.

Understanding how to derive and manipulate these equations underpins many computational and theoretical methods.

Conclusion

Deriving the vector parametric equation of a line through two points involves a systematic process:

1. Calculating the direction vector $\mathbf{v} = \mathbf{Q} - \mathbf{P}$.
2. Selecting a point on the line ($\mathbf{r}_0$), typically one of the given points.
3. Formulating the parametric equations as $R(t) = \mathbf{r}_0 + t \mathbf{v}$.

For the points $P=(-3,-1,1)$ and $Q=(-8,-4,5)$, the resulting parametric equations are:

$$
x(t) = -3 - 5t,\quad y(t) = -1 - 3t,\quad z(t) = 1 + 4t
$$

This explicit form facilitates visualization, analysis, and further computations involving the line in three-dimensional space.

References and Further Reading

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About the Author

This article was prepared by a mathematician dedicated to elucidating complex concepts in geometry and vector calculus for learners and professionals alike. The focus remains on clarity, accuracy, and practical application.

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