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In An Interval Whose Length Is Z Seconds, A Body Moves ($32z+2z^2$) ft. Which Of The Following Is The Average

Understanding the concept of average speed in physics is fundamental for analyzing motion. When a body moves within a specific time interval, calculating its average speed provides insights into its overall motion pattern. In this article, we explore the problem: given an interval of Z seconds during which a body covers a distance of ($32z + 2z^2$) feet, how do we determine the average speed? We will delve into the mathematical formulation, interpret the problem, and provide step-by-step solutions to find the average speed, along with relevant explanations and tips.

Understanding the Problem Statement

Before solving the problem, it’s essential to understand the key components:

- Interval Length: The time during which the body moves, denoted as Z seconds.
- Distance Covered: The total distance traveled in that interval, given as ($32z + 2z^2$) feet.
- Objective: To find the average speed over this interval.

The question can be summarized as: What is the average speed of the body during the Z seconds?

Key Concepts and Definitions

Average Speed

Average speed is defined as the total distance traveled divided by the total time taken. Mathematically, it is expressed as:

$$
\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time}}
$$

where:
- Total Distance is measured in feet (or any other length unit).

- Total Time is measured in seconds (or any other time unit).

Variables and Units

- Z: Duration of the interval in seconds.
- Distance: $32z + 2z^2$ feet.
- Average Speed (V_{avg}): To be calculated in feet per second (ft/sec).

Step-by-Step Solution

Let's break down the process into clear steps.

Step 1: Express the Total Distance

The total distance covered during the interval is already given as:

$$
D(z) = 32z + 2z^2 \text{ feet}
$$

Step 2: Understand the Time Interval

The time interval is Z seconds.

Step 3: Calculate the Average Speed

Using the formula:

$$
V_{avg} = \frac{D(z)}{Z} = \frac{32z + 2z^2}{z}
$$

since the total time is Z seconds.

Step 4: Simplify the Expression

Dividing numerator by Z:

$$
V_{avg} = \frac{32z}{z} + \frac{2z^2}{z} = 32 + 2z
$$

Result:

$$
\boxed{
\text{Average Speed} = 32 + 2z \text{ ft/sec}
}
$$

This formula indicates that the average speed depends linearly on Z.

Interpreting the Result

The derived expression, Average Speed = 32 + 2z ft/sec, provides a direct way to compute the average speed for any given value of Z.

- As Z increases, the average speed increases linearly.
- When Z approaches zero, the average speed approaches 32 ft/sec, which makes sense because, at an infinitesimally small time, the body covers a very small distance, but the instantaneous speed approaches the initial component of the function.

Practical Applications and Examples

Let’s consider some specific values of Z to see how the average speed varies:

- **Z = 1 second:**
  $$
  V_{avg} = 32 + 2(1) = 34 \text{ ft/sec}
  $$

- **Z = 5 seconds:**
  $$
  V_{avg} = 32 + 2(5) = 42 \text{ ft/sec}
  $$

- **Z = 10 seconds:**
  $$
  V_{avg} = 32 + 2(10) = 52 \text{ ft/sec}
  $$

These examples demonstrate how the average speed increases with time, consistent with the quadratic component in the distance formula.

Understanding the Context of the Problem

This problem is typical in kinematics, especially when analyzing uniformly accelerated motion or variable speeds over time. The distance function $s(z) = 32z + 2z^2$ suggests an acceleration component, as the quadratic term indicates increasing velocity over time.

Relating Distance and Velocity

- The term $32z$ can be seen as the initial velocity component times time.
- The term $2z^2$ indicates acceleration, as the distance covered increases quadratically with time.

Additional Insights: Instantaneous vs. Average Speed

While the problem focuses on average speed over the interval, it’s also valuable to understand instantaneous speed at any point Z.

- Instantaneous Speed at time Z:

$$
v(z) = \frac{d}{dz} s(z) = \frac{d}{dz} (32z + 2z^2) = 32 + 4z
$$

- Comparison:

At any moment Z, the instantaneous speed is $32 + 4z$ ft/sec, which is twice the average speed at that instant (since the average speed over an interval is generally less than or equal to the instantaneous speed at the end of the interval).

This relationship emphasizes how acceleration affects the body's motion.

Mathematical Considerations and Extensions

The problem offers a foundation for exploring more advanced concepts:

1. Derivatives and Velocity

- The derivative of the distance function gives the velocity function:

\[

$$
v(z) = 32 + 4z
$$

- This shows that velocity increases linearly with time, indicating acceleration.

2. Calculus Applications

- Calculus allows us to analyze the body's motion more thoroughly, including acceleration, jerk, and displacement over any sub-interval.

3. Graphical Representation

- Plotting the distance function $s(z) = 32z + 2z^2$ versus time Z provides visual insight into how the body accelerates.
- Similarly, plotting the velocity function $v(z) = 32 + 4z$ against time illustrates increasing speed.

Summary and Key Takeaways

- The average speed over an interval of Z seconds, during which a body covers a distance $32z + 2z^2$ feet, is:

$$
\boxed{
V_{avg} = 32 + 2z \text{ ft/sec}
}
$$

- The formula indicates a linear increase of average speed with respect to Z.
- The problem exemplifies fundamental principles in kinematics, such as the relationship between distance, time, velocity, and acceleration.
- Understanding these concepts is vital for solving real-world problems involving motion analysis.

Final Thoughts

This problem serves as a practical example of how mathematical functions describe real-world motion. By exploring the relationship between distance and time through derivatives and algebraic manipulation, we gain deeper insights into the body's movement dynamics. Whether applied in physics, engineering, or everyday life, mastering the calculation of average speed and understanding instantaneous velocity are essential skills.

Remember: Always analyze the functions involved, simplify expressions carefully, and interpret the results within the context of the problem to ensure a comprehensive understanding of motion phenomena.

Frequently Asked Questions

What is the formula for the distance traveled by the body in an interval of Z seconds?

The distance traveled is given by ($32z + 2z^2$) feet.

How do you calculate the average speed of the body over the interval of Z seconds?

The average speed is the total distance divided by the time interval, so it is $(32z + 2z^2) / z = 32 + 2z$ feet per second.

What is the expression for the average velocity of the body during the interval?

If the motion is uniform, the average velocity is the total displacement over Z seconds, which can be derived from the given distance function; in this case, it simplifies to the average speed as ($32 + 2z$).

How does the length of the interval Z affect the average speed of the body?

Since the average speed is ($32 + 2z$), increasing Z results in a higher average speed, showing a linear relationship between Z and average speed.

Which of the following options best represents the average speed: 32 + 2z, 16 + z, or 64 + 4z?

The correct average speed is $32 + 2z$ feet per second, matching the derived formula for the average speed over the interval.

Additional Resources

Kinematics: Analyzing the Average Velocity of a Moving Body over a Given Interval

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Introduction

In the realm of physics, understanding motion is fundamental. Whether analyzing the trajectory of a projectile or the acceleration of a vehicle, the key to unlocking these dynamics lies in grasping the concepts of displacement, time, and velocity. An intriguing problem often posed in physics involves calculating the average velocity of a body moving over a specific interval, especially when the displacement is expressed as a function of the interval's length.

Today, we delve into a classic problem: "In an interval whose length is Z seconds, a body moves ($32Z + 2Z^2$) feet. Which of the following is the average?" This question not only tests our understanding of average velocity but also offers an opportunity to explore the underlying principles of motion, functions of time, and the calculus-based approach to kinematics.

Understanding the Problem

The Given Data

- Time interval: Z seconds (where $Z > 0$)
- Displacement (s): $s(Z) = 32Z + 2Z^2$ feet

The Objective

- To find the average velocity of the body over the interval of length Z seconds.

Clarifying Key Concepts

What Is Average Velocity?

Average velocity is defined as the total displacement divided by the total time taken. Mathematically,

$$
\text{Average Velocity} = \frac{\text{Displacement over the interval}}{\text{Time interval}}
$$

In the context of the problem, since the displacement is expressed as a function of Z, the average velocity over an interval of Z seconds is:

$$
V_{avg} = \frac{s(Z)}{Z}
$$

where:

- $s(Z)$ is the displacement after Z seconds,
- Z is the interval length in seconds.

Why Is This Important?

Understanding this concept allows us to analyze how the body's motion behaves over different time intervals, especially when the displacement isn't linear but quadratic, indicating acceleration.

Step-by-Step Solution Approach

1. Expressing the Displacement Function

Given:

$$
s(Z) = 32Z + 2Z^2
$$

This function indicates that displacement depends on the interval length Z, with both linear and quadratic components.

2. Calculating the Average Velocity

By the definition of average velocity:

$$
V_{avg} = \frac{s(Z)}{Z} = \frac{32Z + 2Z^2}{Z}
$$

Simplifying:

$$
V_{avg} = \frac{32Z}{Z} + \frac{2Z^2}{Z} = 32 + 2Z
$$

Thus, the average velocity over an interval of Z seconds is:

$$
\boxed{
V_{avg} = 32 + 2Z \text{ ft/sec}
}
$$

3. Interpreting the Result

The expression $32 + 2Z$ reflects how the average velocity changes with the length of the interval Z. This linear relationship suggests that as Z increases, the average velocity also increases, consistent with an accelerating body.

Deeper Insights: Connecting to Instantaneous Velocity

While the problem explicitly asks for the average velocity over an interval, it's insightful to consider the instantaneous velocity at any point, which is the derivative of displacement with respect to time:

$$
v(t) = \frac{ds}{dt}
$$

Given:

$$
s(t) = 32t + 2t^2
$$

Differentiating:

$$
v(t) = \frac{d}{dt}(32t + 2t^2) = 32 + 4t
$$

At $t = Z$, the instantaneous velocity is:

$$
v(Z) = 32 + 4Z
$$

Notice that:

- The instantaneous velocity at time Z is $32 + 4Z$,
- The average velocity over the interval Z is $32 + 2Z$.

This relationship shows that the average velocity is exactly half of the instantaneous velocity at the endpoint Z:

$$
V_{avg} = \frac{v(Z)}{2}
$$

which is consistent with the properties of uniformly accelerated motion where displacement involves quadratic terms.

Visualizing the Motion

Graph of Displacement vs. Time

Plotting $s(t) = 32t + 2t^2$:

- The graph is a parabola opening upwards,
- The slope at any point t (instantaneous velocity) is $32 + 4t$,
- The displacement increases more rapidly as time advances because of the quadratic term.

Graph of Velocity vs. Time

Plotting $v(t) = 32 + 4t$:

- A straight line with slope 4,
- Velocity increases linearly with time, indicating constant acceleration.

Practical Examples and Applications

Example 1: Estimating Displacement over Different Intervals

Suppose Z = 5 seconds:

$$
V_{avg} = 32 + 2 \times 5 = 32 + 10 = 42 \text{ ft/sec}
$$

Total displacement:

$$
s(5) = 32 \times 5 + 2 \times 25 = 160 + 50 = 210 \text{ ft}
$$

Average velocity:

$$
V_{avg} = \frac{210}{5} = 42 \text{ ft/sec}
$$

Similarly, for Z = 10 seconds:

$$
V_{avg} = 32 + 20 = 52 \text{ ft/sec}
$$

Displacement:

$$
s(10) = 32 \times 10 + 2 \times 100 = 320 + 200 = 520 \text{ ft}
$$

Average velocity:

$$
V_{avg} = \frac{520}{10} = 52 \text{ ft/sec}
$$

These calculations confirm the derived formula and demonstrate how average velocity increases with longer intervals.

Summary of Findings

Interval (Z seconds)	Displacement $s(Z)$ (ft)	Average Velocity V_{avg} (ft/sec)
5	210	42
10	520	52
15	$32(15) + 2(15)^2 = 480 + 450 = 930$	$930 / 15 = 62$

The key takeaway is that the average velocity over an interval of Z seconds for the given motion is $32 + 2Z$ ft/sec, a linear function of Z.

Final Thoughts: Broader Implications

Understanding how displacement relates to time and how average velocity can be derived from these relationships is vital in physics and engineering. The problem underscores the importance of functions in describing motion, especially when acceleration is involved. The linear increase of average velocity with respect to the interval length Z demonstrates the presence of constant acceleration, as the velocity function's derivative (acceleration) remains constant at 4 ft/sec^2.

This analysis not only helps in solving specific problems but also deepens conceptual understanding, enabling practitioners to interpret motion data accurately, design systems, and predict future states of moving bodies.

Conclusion

In conclusion, when given a displacement function of the form $s(Z) = 32Z + 2Z^2$, the average velocity over an interval of Z seconds is straightforward to compute:

$$
\boxed{
\text{Average velocity} = \frac{s(Z)}{Z} = 32 + 2Z
$$

```
}
\]
```

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This formula provides an immediate insight into how the body's motion evolves
over different time intervals, reflecting the effects of acceleration and the
dynamic nature of motion. Whether used in academic exercises or real-world
applications, this understanding forms the backbone of kinematic analysis.
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