Leta1, A2 A3 Be A Sequence Defined By A1 = 1 And Ak = 2ak-1 . Find A Formula For An And Prove It Is Correct

Leta1, A2 A3 Be A Sequence Defined By A1 = 1 And Ak = 2ak-1 . Find A Formula For An And Prove It Is Correct

Understanding sequences and their formulas is fundamental in mathematics, especially in areas like algebra, number theory, and discrete mathematics. In this article, we explore a particular recursive sequence defined by specific initial conditions and recurrence relations. Our goal is to derive an explicit formula for the sequence A_n and rigorously prove its correctness.

Introduction to the Sequence

Definition of the Sequence

The sequence $\{A_n\}$ is defined as follows:

- Initial term: $A_1 = 1$
- Recursive relation: $A_k = 2 \times A_{k-1}$ for all $k \geq 2$

This recursive formula indicates that each term is obtained by multiplying the previous term by 2.

Purpose of the Article

- To derive a closed-form (explicit) formula for A_n.
- To prove that the derived formula satisfies the recursive relation and initial condition.
- To understand the pattern and behavior of the sequence.

Deriving the Explicit Formula for $\{A_n\}$

Analyzing the Recursive Relation

Given the recursive relation:

$$
A_k = 2 \times A_{k-1}
$$

for $k \geq 2$, and initial condition:

$$
A_1 = 1
$$

We observe that each term after the first is obtained by multiplying the previous term by 2.

Calculating the First Few Terms

Let's compute the first few terms to identify a pattern:

- $A_1 = 1$
- $A_2 = 2 \times A_1 = 2 \times 1 = 2$
- $A_3 = 2 \times A_2 = 2 \times 2 = 4$
- $A_4 = 2 \times A_3 = 2 \times 4 = 8$
- $A_5 = 2 \times A_4 = 2 \times 8 = 16$

From these calculations, the sequence begins as:

$$
1, 2, 4, 8, 16, \ldots
$$

which suggests an exponential pattern.

Identifying the Pattern

The sequence appears to be powers of 2:

$$
A_1 = 2^0 = 1
$$
$$
A_2 = 2^1 = 2
$$
$$
A_3 = 2^2 = 4
$$
$$
A_4 = 2^3 = 8
$$

$$
A_5 = 2^4 = 16
$$

Thus, the pattern indicates:

$$
A_n = 2^{n-1}
$$

for $n \geq 1$.

Formulating the Explicit Formula

Based on the pattern observed, the explicit formula for the sequence is:

$$
\boxed{
A_n = 2^{n-1}
}
$$

This formula directly calculates any term of the sequence without recursion.

Proof of the Formula's Correctness

To validate the explicit formula, we need to demonstrate that:

1. It satisfies the initial condition $A_1 = 1$.
2. It obeys the recursive relation $A_k = 2 \times A_{k-1}$ for all $k \geq 2$.

Proof via Mathematical Induction

Mathematical induction is a powerful method to prove that a formula holds for all natural numbers.

Base Case: $n=1$

Calculate A_1 using the explicit formula:

$$
A_1 = 2^{1-1} = 2^0 = 1
$$

which matches the initial condition.

Inductive Step: Assume the formula holds for some arbitrary $n = k$, i.e.,

$$
A_k = 2^{k-1}
$$

We need to show that it holds for $n = k+1$, i.e.,

$$
A_{k+1} = 2^{k}
$$

Using the recursive relation:

$$
A_{k+1} = 2 \times A_k
$$

Substituting the induction hypothesis:

$$
A_{k+1} = 2 \times 2^{k-1} = 2^{1} \times 2^{k-1} = 2^{1 + k - 1} = 2^{k}
$$

which confirms the formula holds for $n = k+1$.

Conclusion of the Inductive Proof

Since the base case is true and the inductive step holds, by the principle of mathematical induction, the explicit formula:

$$
A_n = 2^{n-1}
$$

is valid for all integers $n \geq 1$.

Additional Properties and Applications of the Sequence

Growth Rate

The sequence $\{A_n\}$ grows exponentially, doubling with each subsequent term. This characteristic is typical of geometric sequences with common ratio $r=2$.

Relevance in Computer Science

Such sequences are fundamental in analyzing algorithms with exponential complexity or in modeling binary data structures like trees and binary representations.

Other Mathematical Contexts

- Powers of 2 appear in combinatorics, especially in counting subsets.
- They are also relevant in number theory and cryptography.

Summary and Final Remarks

The recursive sequence defined by:

- $A_1 = 1$,
- $A_k = 2 \times A_{k-1}$ for $k \geq 2$,

has an explicit, closed-form formula:

$$
\boxed{
A_n = 2^{n-1}
}
$$

This formula was derived through pattern recognition and rigorously proven via mathematical induction. It exemplifies how recursive definitions can often be transformed into explicit formulas, facilitating easier computation and deeper understanding of the sequence's behavior.

Conclusion

Understanding recursive sequences and their explicit formulas is a key skill in mathematics, providing insights into their growth, structure, and applications. The sequence $\{A_n\}$ discussed here illustrates the power of mathematical induction and pattern recognition in deriving formulas. Whether in pure mathematics, computer science, or applied fields, such sequences serve as fundamental building blocks for more complex concepts and systems.

By mastering these techniques, students and professionals alike can analyze and utilize recursive sequences effectively, opening doors to advanced mathematical reasoning and problem-solving.

Frequently Asked Questions

What is the initial value of the sequence Leta1, A2, A3, ... given A1 = 1?

The initial value of the sequence is A1 = 1.

How is each term Ak in the sequence defined?

Each term Ak is defined recursively as Ak = 2 A(k-1).

What is the pattern observed in the sequence A1, A2, A3, ...?

The pattern shows that each term is twice the previous term, indicating exponential growth.

How can we derive a formula for the nth term An of the sequence?

By recognizing the recursive pattern, we can derive An = 2^(n-1) A1, which simplifies to An = 2^{n-1}.

What is the explicit formula for An in terms of n?

The explicit formula for An is An = 2^{n-1}.

How can we prove that An = 2^{n-1} is correct for the sequence?

We can use mathematical induction: verify it for n=1, then assume it holds for n=k, and show it holds for n=k+1.

What is the base case for the induction proof of An = 2^{n-1}?

When n=1, A1 = 1, and $2^{1-1} = 2^0 = 1$, which matches the initial value.

How does the inductive step confirm the formula's correctness?

Assuming An = 2^{n-1}, then $A_{k+1} = 2 A_k = 2\ 2^{k-1} = 2^{k} = 2^{(k+1)-1}$, confirming the formula holds for n=k+1.

Why is understanding the recursive definition important for deriving the explicit formula?

Because recognizing the recursive pattern allows us to formulate a direct expression for An, simplifying analysis and calculations.

Additional Resources

In-Depth Analysis of the Leta1, A2, A3 Be Sequence: Deriving and Proving the Formula for An

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Introduction

Sequences are fundamental constructs in mathematics, serving as the backbone for understanding patterns, limits, and various properties of numbers. They often emerge in diverse fields such as calculus, combinatorics, computer science, and number theory. Among the myriad types of sequences, recursive sequences—where each term is defined in terms of previous terms—are particularly intriguing due to their intimate connection with iterative processes and their sometimes non-trivial formulas.

In this detailed review, we explore the sequence defined by the initial term and a recursive relation:

- Given: $A_1 = 1$
- Recursive relation: $A_k = 2A_{k-1}$

Our goal is to:

1. Find an explicit (closed-form) formula for A_n.
2. Prove that the formula is correct.

This journey will involve understanding the sequence's structure, hypothesizing the formula, and rigorously establishing its validity through mathematical induction.

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1. Understanding the Sequence: Definitions and Initial Terms

Before delving into the derivation, it's crucial to understand what the sequence looks like based on the initial conditions and recursive relation.

Initial Term:

$$
A_1 = 1
$$

Recursive Formula:

$$
A_k = 2A_{k-1}
$$

for $k \geq 2$.

Let's compute the first few terms to observe the pattern:

Term A_k	Calculation	Result	Comment

A_1	Given	1	Initial condition
A_2	$2A_1 = 2 \times 1$	2	Multiply previous by 2
A_3	$2A_2 = 2 \times 2$	4	Continue doubling
A_4	$2A_3 = 2 \times 4$	8	Doubling pattern continues
A_5	$2A_4 = 2 \times 8$	16	Pattern persists

From this, it’s evident that the sequence doubles at each step, starting from 1.

Explicitly, the sequence looks like:

$$
A_1 = 1,\quad A_2 = 2,\quad A_3 = 4,\quad A_4 = 8,\quad A_5 = 16,\quad \dots
$$

This pattern suggests the sequence is a geometric progression with ratio 2.

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2. Deriving the Explicit Formula for A_n

Given the pattern and recursive relation, we can hypothesize that the sequence takes the form:

$$
A_n = C \times r^{n-1}
$$

where:

- C is a constant (initial term),
- r is the common ratio.

Using initial conditions:

$$
A_1 = C \times r^{0} = C \times 1 = C
$$

Since $A_1 = 1$, it follows that:

$$
C = 1
$$

and the common ratio:

$$
r = 2
$$

\]

Therefore, the explicit formula is:

$$
\boxed{
A_n = 2^{n-1}
}
$$

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3. Verifying the Formula: Inductive Proof

To ensure the correctness of the formula, a rigorous proof is necessary. The most appropriate method here is mathematical induction, which confirms that the formula holds for all positive integers n.

Base Case:

Check $n=1$:

$$
A_1 \stackrel{?}{=} 2^{1-1} = 2^{0} = 1
$$

which matches the initial condition.

Inductive Hypothesis:

Assume the formula holds for some arbitrary positive integer $k \geq 1$:

$$
A_k = 2^{k-1}
$$

Inductive Step:

Show that the formula holds for $k+1$:

$$
A_{k+1} \stackrel{?}{=} 2^{(k+1)-1} = 2^{k}
$$

Using the recursive relation:

$$
A_{k+1} = 2A_k
$$

By the inductive hypothesis:

$$
A_{k+1} = 2 \times 2^{k-1} = 2^{1} \times 2^{k-1} = 2^{k}
$$

which matches the formula predicted for A_{k+1}.

Hence, by the principle of mathematical induction, the explicit formula:

$$
\boxed{
A_n = 2^{n-1}
}
$$

is valid for all integers $n \geq 1$.

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4. Additional Properties and Insights

Having established the explicit formula, it's beneficial to explore some properties and implications:

- Growth Rate: The sequence grows exponentially, doubling each time.
- Sum of the Sequence Terms: For the first n terms:

$$
S_n = A_1 + A_2 + \dots + A_n = 1 + 2 + 4 + \dots + 2^{n-1}
$$

which is a geometric series with ratio 2:

$$
S_n = \frac{2^{n} - 1}{2 - 1} = 2^{n} - 1
$$

- Limit Behavior: As $n \to \infty$, $A_n \to \infty$ exponentially.

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5. Broader Context and Applications

Understanding sequences like $A_n = 2^{n-1}$ is fundamental in many areas:

- Computer Science: Binary sequences, data structures like binary trees, and algorithms often rely on exponential growth patterns.
- Mathematics: Such sequences exemplify geometric progressions, serving as

foundational examples in series and convergence.
- Physics and Biology: Exponential growth models are used in population dynamics, radioactive decay, and modeling of certain physical processes.

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6. Variations and Generalizations

While this sequence is simple, exploring variations can deepen comprehension:

- Changing the initial term: For example, if $A_1 = c$, then $A_n = c \times 2^{n-1}$.
- Different ratios: Recursions like $A_k = rA_{k-1}$ produce $A_n = c \times r^{n-1}$.
- Non-constant recursive relations: For example, quadratic or Fibonacci-type relations introduce complexity and rich structures.

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7. Conclusion and Summary

- Sequence Definition: $A_1 = 1$, $A_k = 2A_{k-1}$
- Derived Explicit Formula: $\boxed{A_n = 2^{n-1}}$
- Proof Method: Mathematical induction confirms the formula's correctness.

This sequence exemplifies how recursive definitions often lead to geometric sequences, and how initial conditions determine the explicit formula. Recognizing such patterns simplifies analysis and enables broader applications across mathematics and science.

Final Remarks

Understanding and proving formulas for recursive sequences like this one not only solidifies algebraic intuition but also builds foundational skills essential for higher mathematics. The process of hypothesizing, testing, and rigorously proving such formulas embodies the essence of mathematical reasoning and problem-solving.

In essence, the sequence defined by $A_1=1$ and $A_k=2A_{k-1}$ is a classic example of a geometric progression with initial term 1 and common ratio 2, leading directly to the explicit formula $A_n = 2^{n-1}$. This elegant and straightforward result underscores the power of recursive relations and the beauty of mathematical induction.

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