Order The Steps To Solve The Equationlog(x2 - 15) = Log(2x) Form 1 To 5.x2 - 2x - 15 = 0Potential Solutions

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Understanding how to solve logarithmic equations is a fundamental skill in algebra, especially when dealing with equations involving multiple logarithmic expressions. The equation given, $\log(x^2 - 15) = \log(2x)$, is a classic example where logarithmic properties can be applied to simplify and solve for the variable x. This article provides a comprehensive, step-by-step guide to solving this specific logarithmic equation, emphasizing the importance of proper solution validation, domain considerations, and potential solutions. Whether you are a student preparing for exams or a mathematics enthusiast looking to deepen your understanding, mastering this process will enhance your problem-solving toolkit.

Introduction to Logarithmic Equations

Logarithmic equations are equations where the variable appears inside a logarithm. These equations often arise in real-world contexts such as exponential growth, decay problems, and in various scientific applications. Solving these equations requires understanding the properties of logarithms and the domain restrictions that come with them.

The basic properties of logarithms include:

- Product Rule: $\log_b(xy) = \log_b(x) + \log_b(y)$
- Quotient Rule: $\log_b(x/y) = \log_b(x) - \log_b(y)$
- Power Rule: $\log_b(x^k) = k \log_b(x)$
- Change of Base: $\log_b(x) = \log_k(x) / \log_k(b)$

In algebra, the most common base for logarithms is 10 (common logarithm) or e (natural logarithm), but the principles are applicable across bases as long as the base is positive and not equal to 1.

Understanding the Given Equation

The specific equation under consideration is:

$$\log(x^2 - 15) = \log(2x)$$

This equation involves two logarithmic expressions set equal to each other. To solve for x, we will use properties of logarithms and algebraic techniques.

Key considerations:

- The logs are equal when their arguments are equal, provided those arguments are within the domain.
- The arguments of the logarithms must be positive because the logarithm function is only defined for positive real numbers.

Step-by-Step Solution Process

Step 1: Understand the Domain Restrictions

Before manipulating the equation, identify the domain restrictions to ensure the solutions are valid.

- For $\log(x^2 - 15)$: $x^2 - 15 > 0 \Rightarrow x^2 > 15 \Rightarrow x > \sqrt{15}$ or $x < -\sqrt{15}$
- For $\log(2x)$: $2x > 0 \Rightarrow x > 0$

Combined domain restrictions:

- $x > 0$ (from $\log(2x)$)
- $x > \sqrt{15}$ (~3.872) or $x < -\sqrt{15}$ (from $\log(x^2 - 15)$)

Since x must satisfy both, the feasible domain is:

$x > \sqrt{15}$ (approximately $x > 3.872$)

The negative values are invalid because 2x would be negative or zero, making $\log(2x)$ undefined.

Step 2: Set the Logarithmic Arguments Equal

Since log functions are equal and their bases are the same, their arguments must be equal:

$x^2 - 15 = 2x$

This simplifies the equation from logarithmic form to algebraic form.

Step 3: Rearrange the Equation

Bring all terms to one side to set the quadratic to zero:

$x^2 - 2x - 15 = 0$

This is a quadratic equation in standard form.

Step 4: Solve the Quadratic Equation

Use the quadratic formula:

$x = [-b \pm \sqrt{}(b^2 - 4ac)] / (2a)$

Where $a = 1$, $b = -2$, $c = -15$.

Calculate the discriminant:

$D = b^2 - 4ac = (-2)^2 - 4\ 1\ (-15) = 4 + 60 = 64$

Calculate the roots:

$x = [2 \pm \sqrt{}64] / 2$

x = [2 ± 8] / 2

- For the positive root:

x = (2 + 8) / 2 = 10 / 2 = 5

- For the negative root:

x = (2 - 8) / 2 = (-6) / 2 = -3

Step 5: Validate Solutions Against Domain Restrictions

Recall the domain restrictions: x > □15 (~3.872)

- x = 5: satisfies x > 3.872 □ Valid

- x = -3: does not satisfy x > 3.872, and also, 2 (-3) = -6, which makes log(2x) undefined □ Invalid

Conclusion: The only potential solution is x = 5.

Potential Solutions and Final Check

After identifying x = 5 as the candidate solution, verify it by substituting back into the original equation:

log($(5)^2$ - 15) = log(2 5)

Calculate each side:

- Left side: log(25 - 15) = log(10)
- Right side: log(10)

Since both sides are equal, $x = 5$ satisfies the original equation.

Summary of the Solution Steps

1. Identify domain restrictions:
- $x >$ □15
- $x > 0$
2. Set the logarithmic arguments equal:
- $x^2 - 15 = 2x$
3. Rearranged to quadratic form:
- $x^2 - 2x - 15 = 0$
4. Solve the quadratic:
- Roots: $x = 5$ and $x = -3$
5. Validate solutions against domain restrictions:
- Only $x = 5$ is valid
6. Verify the solution by substitution:
- Confirmed $x = 5$ satisfies the original equation

Additional Tips for Solving Logarithmic Equations

- Always check the domain restrictions after solving algebraic equations involving logarithms.
- Remember that the logarithm of a negative number or zero is undefined.
- When setting the arguments equal, be cautious of extraneous solutions that might not satisfy the original equation.
- Use the quadratic formula for quadratic equations, but consider factoring if possible to simplify calculations.

Conclusion

Solving the logarithmic equation $\log(x^2 - 15) = \log(2x)$ involves a systematic approach: understanding the domain restrictions, transforming the equation into a quadratic form, solving for potential solutions, and validating these solutions against the original constraints. The only valid solution in this case is $x = 5$, which satisfies all conditions and makes the original logarithmic expressions valid.

Mastering these steps enhances your ability to tackle similar logarithmic equations efficiently. Remember, always verify your solutions to avoid extraneous roots, especially when dealing with the properties of logarithms and their domains. With practice, solving such equations becomes an intuitive process, empowering you to handle more complex algebraic and logarithmic problems confidently.

Frequently Asked Questions

What is the first step to solve the equation $\log(x^2 - 15) = \log(2x)$?

The first step is to recognize that if $\log(A) = \log(B)$, then $A = B$, provided A and B are within the domain. So, set $x^2 - 15 = 2x$.

How do you set up the quadratic equation from the given logarithmic equation?

By equating the insides of the logs: $x^2 - 15 = 2x$, then rearranging to standard quadratic form: $x^2 - 2x - 15 = 0$.

What steps are involved in solving the quadratic equation $x^2 - 2x - 15 = 0$?

Factor the quadratic: $(x - 5)(x + 3) = 0$, then set each factor equal to zero: $x - 5 = 0$ and $x + 3 = 0$, resulting in potential solutions $x = 5$ and $x = -3$.

How do you determine if the potential solutions are valid in the original logarithmic equation?

Check the domain restrictions: since the logs are $\log(x^2 - 15)$ and $\log(2x)$, both arguments must be positive. For $x = 5$, $x^2 - 15 = 25 - 15 = 10 > 0$ and $2(5) = 10 > 0$, so valid. For $x = -3$, $x^2 - 15 = 9 - 15 = -6$ (invalid), and $2(-3) = -6$ (invalid).

What is the final solution to the equation after checking the domain restrictions?

The only valid solution is $x = 5$, since it satisfies the domain conditions and the original equation.

What is the importance of checking the domain when solving logarithmic equations?

Checking the domain ensures that the solutions are valid because logarithms are only defined for positive arguments; solutions that make the arguments negative or zero are invalid.

Additional Resources

Order The Steps To Solve The Equation $\log(x^2 - 15) = \log(2x)$ Form 1 To 5.$x^2 - 2x - 15 = 0$ Potential Solutions

Understanding how to solve logarithmic equations is an essential skill in algebra, particularly when dealing with equations like $\log(x^2 - 15) = \log(2x)$. These problems often involve multiple steps, including applying logarithmic properties, algebraic manipulation, and checking for extraneous solutions. In this comprehensive review, we will explore the systematic approach to solving such equations, specifically focusing on the form $\log(x^2 - 15) = \log(2x)$ and how to derive solutions from the quadratic form $x^2 - 2x - 15 = 0$. We will break down each step involved, discuss potential pitfalls, and analyze the solutions' validity.

Understanding the Logarithmic Equation

Before delving into the solution steps, it's crucial to understand the structure of the given equation:

$\log(x^2 - 15) = \log(2x)$

This equality of logarithms suggests that the arguments of the logs are equal, provided they are within the domain of the logarithmic function (i.e., arguments are positive). The key properties we will leverage include:

- Logarithmic Property: $\log(a) = \log(b)$ implies $a = b$, provided $a > 0$ and $b > 0$.
- Domain Restrictions: Both arguments must be positive:
- $x^2 - 15 > 0$
- $2x > 0$

Establishing these conditions is essential before proceeding to solve the algebraic equation.

Step-by-Step Approach to Solving the Equation

The process involves orderly steps to transform and solve the equation efficiently.

Step 1: Set the Arguments Equal

Since $\log(x^2 - 15) = \log(2x)$, and assuming both arguments are within the domain, we equate:

$x^2 - 15 = 2x$

This step relies on the logarithmic property that $\log A = \log B$ implies $A = B$, provided $A, B > 0$.

Step 2: Solve the Resulting Quadratic Equation

Rearranged, the quadratic becomes:

$x^2 - 2x - 15 = 0$

This quadratic can be factored, completed the square, or solved using the quadratic formula.

Quadratic Formula:

$x = [2 \pm \sqrt{(4 + 60)}] / 2$

$= [2 \pm \sqrt{64}] / 2$

$= [2 \pm 8] / 2$

Calculating the two solutions:

- $x = (2 + 8) / 2 = 10 / 2 = 5$
- $x = (2 - 8) / 2 = -6 / 2 = -3$

Step 3: Check Domain Restrictions

Recall the original domain conditions:

- $x^2 - 15 > 0 \Rightarrow x^2 > 15 \Rightarrow x > \sqrt{15}$ or $x < -\sqrt{15}$
- $2x > 0 \Rightarrow x > 0$

Combining these:

- For $x > 0$: $x > \sqrt{15}$ (~3.87)
- For $x < 0$: $x < -\sqrt{15}$ (~-3.87)

Now, check the solutions:

- $x = 5$: Since $5 > \sqrt{15}$, it satisfies the domain. Also, $2(5) = 10 > 0$, so $\log(2x)$ is defined. Check the original arguments:

- $x^2 - 15 = 25 - 15 = 10 > 0$
- $2x = 10 > 0$

Both are valid. So $x = 5$ is a potential solution.

- $x = -3$: Since $-3 > -\sqrt{15}$ (~ -3.87), but is it greater than $-\sqrt{15}$? Yes, because $-3 > -3.87$, so $x = -3$ is within the domain for $\log(2x)$ only if $2x > 0$:

$2(-3) = -6$, which is less than 0, so $\log(2x)$ is undefined at $x = -3$. Therefore, $x = -3$ must be discarded.

Conclusion: Only $x = 5$ is valid.

Additional Steps and Considerations

While the above steps lead to the solution, there are other considerations and potential alternative steps, especially if the equation takes different forms.

Potential Solutions in Different Forms

Suppose the equation is given in different forms, such as:

- Different quadratic forms: e.g., $x^2 + 2x - 15 = 0$
- Higher degree equations: e.g., $x^4 - 16 = 0$
- Equations involving absolute values or other functions

In such cases, the order of steps might change, but the core principles remain:

- Isolate the logarithmic expression
- Use properties of logarithms to reduce the equation
- Solve the resulting algebraic equation
- Check for extraneous solutions by verifying the domain restrictions

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Common Pitfalls and How to Avoid Them

When solving logarithmic equations, several common mistakes can occur:

- Ignoring the domain restrictions: Always verify that the solutions satisfy the arguments' positivity.
- Assuming log(a) = log(b) implies a = b without domain considerations: Ensure arguments are positive.
- Overlooking extraneous solutions: Solutions from algebraic manipulation may not be valid in the original equation.
- Incorrect quadratic solving: Be cautious with signs and discriminant calculations.

Tips to avoid these pitfalls:
- Write down the domain restrictions explicitly.
- Substitute potential solutions back into the original equation.
- Use the quadratic formula carefully, double-checking calculations.

Features and Pros/Cons of the Solution Method

Features:

- Systematic approach that combines properties of logarithms with algebraic solving
- Clear steps that can be applied to similar equations
- Emphasizes importance of domain restrictions

Pros:

- Straightforward and reliable for equations reducible to algebraic forms

- Helps develop critical thinking about domain and extraneous solutions
- Applicable to a wide range of logarithmic and algebraic problems

Cons:

- May become cumbersome with more complex or higher-degree equations
- Requires careful attention to detail, especially in checking domains
- Not always directly applicable if the logarithmic arguments are complicated or involve variables inside functions

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Conclusion

Solving the equation $\log(x^2 - 15) = \log(2x)$ involves a structured process rooted in understanding logarithmic properties and domain restrictions. The steps include equating the arguments of the logs, transforming the equation into a quadratic form, solving for the variable, and verifying solutions against the domain constraints. In this case, we found that $x = 5$ is the valid solution, satisfying all necessary conditions. Such problems highlight the importance of logical order, careful algebraic manipulation, and thorough verification. Mastery of this process not only enhances problem-solving skills but also deepens understanding of the interplay between algebra and logarithms, essential tools in mathematics.

In summary:

- Always verify domain restrictions before accepting solutions.
- Use properties of logarithms to simplify equations.
- Solve resulting algebraic equations with care.

- Check all potential solutions within the original context to avoid extraneous answers.

- Practice with various forms to build confidence and adaptability in solving logarithmic equations.

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