

PLATO: Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated

PLATO: Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated is a captivating challenge that combines geometric transformations with problem-solving skills. This type of question is often encountered in advanced mathematics, particularly in the study of vectors and transformations, and is a popular format in educational platforms and competitive exams. Understanding how to approach such problems requires a clear grasp of the underlying concepts of dilation and rotation in the coordinate plane, as well as the ability to apply these transformations step-by-step to find the correct answer.

In this article, we will explore the fundamental concepts behind such problems, provide a detailed methodology for solving them, and offer tips to improve your accuracy and efficiency. Whether you're a student preparing for exams or a math enthusiast interested in geometric transformations, this comprehensive guide will serve as a valuable resource.

Understanding Geometric Transformations

Before diving into the specifics of the problem, it's essential to understand the basic types of geometric transformations involved: dilation and rotation. These transformations alter the position and size of vectors or shapes in the coordinate plane.

What Is Dilation?

Dilation, also known as resizing or scaling, is a transformation that enlarges or reduces a figure by a specific scale factor, relative to a fixed point called the center of dilation. When the center of dilation is the origin (0,0), the process simplifies to multiplying the coordinates by the scale factor.

Key points:

- The scale factor determines whether the figure enlarges (>1) or reduces (<1).
- Coordinates are multiplied by the scale factor to find the new position.
- Dilation preserves the shape of the figure but changes its size.

What Is Rotation?

Rotation involves turning a figure or vector around a fixed point, typically the origin, by a specific angle in a specified direction (clockwise or counterclockwise).

Key points:

- The rotation angle determines the degree of turn.
- The rotation preserves the size and shape of the figure.
- Coordinates are transformed using rotation formulas involving sine and cosine functions.

Step-by-Step Approach to the Problem

When faced with a problem that involves first dilating a vector and then rotating it, a systematic approach ensures accuracy and clarity.

Step 1: Identify the Initial Vector

Begin by noting the initial coordinates of the vector. For example, suppose the vector is represented by the point (x, y) .

Step 2: Apply Dilation by the Given Scale Factor

Since the problem specifies a dilation by a factor of 2.5 centered at the origin, multiply both x and y coordinates by 2.5:

- New x -coordinate: $x' = 2.5x$
- New y -coordinate: $y' = 2.5y$

This step enlarges the vector proportionally in all directions.

Step 3: Apply Rotation by the Specified Angle

Next, rotate the dilated vector by the given angle, θ degrees, typically using the rotation formulas:

- $x'' = x' \cos(\theta) - y' \sin(\theta)$
- $y'' = x' \sin(\theta) + y' \cos(\theta)$

Ensure the angle is in radians when using most mathematical functions or convert degrees to radians:

- $\text{Radians} = \text{Degrees} \times (\pi / 180)$

Use a calculator or software to obtain accurate sine and cosine values.

Step 4: Interpret and Write the Final Coordinates

The resulting coordinates (x'' , y'') after these transformations represent the final position of the vector. These are the values you will enter as your answer.

Practical Example

Let's illustrate this process with a concrete example.

Given:

- Initial vector: (4, 3)
- Dilation factor: 2.5
- Rotation angle: 90° counterclockwise

Solution:

1. Dilate the vector:

- $x' = 2.5 \cdot 4 = 10$
- $y' = 2.5 \cdot 3 = 7.5$

2. Convert rotation angle to radians:

- $\theta = 90^\circ \times (\pi / 180) = \pi / 2$ radians

3. Apply rotation formulas:

- $\cos(\pi/2) = 0$
- $\sin(\pi/2) = 1$

$$- x'' = (10)(0) - (7.5)(1) = -7.5$$

$$- y'' = (10)(1) + (7.5)(0) = 10$$

Final answer:

- Coordinates after transformations: (-7.5, 10)

Tips for Solving Vector Transformation Problems

To enhance your skills and efficiency, consider the following tips:

- **Memorize the rotation formulas:** Knowing the formulas by heart speeds up the process.
- **Practice conversions:** Be quick at converting degrees to radians and vice versa.
- **Use a calculator or software:** For complex angles, software like GeoGebra or graphing calculators can verify your results.

- **Draw diagrams:** Visual aids can help you better understand how transformations affect the vector.
- **Check your work:** Always verify if the transformed vector makes sense in the context of the problem.

Common Mistakes to Avoid

While solving these problems, some errors often occur. Being aware of them helps prevent mistakes:

- Mixing up the order of transformations; dilation should be applied before rotation if specified.
- Using degrees directly in sine and cosine functions that require radians — always convert.
- Incorrectly applying the rotation formulas, especially sign errors.
- Neglecting to double-check the signs of the final coordinates, especially after rotation.

Conclusion

Understanding how to manipulate vectors through dilation and rotation is a fundamental skill in geometry that enhances spatial reasoning and problem-solving abilities. The problem titled "PLATO: Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated" exemplifies the importance of mastering these transformations. By breaking down the process into clear, manageable steps—dilating the vector, converting angles, applying rotation formulas, and verifying your results—you can confidently tackle similar problems.

Practicing these transformations with various initial vectors and different angles will improve your proficiency and prepare you for more complex geometric challenges. Remember, visualization and systematic calculation are your best tools in mastering vector transformations. With patience and practice, you'll be able to solve such problems efficiently and accurately, enhancing your overall understanding of geometry and its applications.

Frequently Asked Questions

What transformation is applied to the vector first in the problem?

The vector is first dilated by a factor of 2.5.

After dilation, what is the next transformation performed on the vector?

The vector is then rotated.

If a vector is dilated by 2.5 times and then rotated, how does its magnitude change?

The magnitude is multiplied by 2.5, and then the vector is rotated by a certain angle.

Which mathematical operation comes first when transforming the vector in this problem?

The dilation by a factor of 2.5 occurs first, followed by rotation.

Why is understanding the order of transformations important in vector problems?

Because the order affects the final position and orientation of the vector, leading to different results if changed.

Additional Resources

PLATO: Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated

Introduction: Understanding the Foundations of Geometric Transformations in Educational Tools

In the realm of educational technology and mathematical visualization, interactive platforms such as PLATO have revolutionized the way students grasp complex concepts. One such intriguing example is the problem titled "Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated." This problem, while seemingly straightforward, embodies a confluence of fundamental geometric transformations—dilation and rotation—that are critical in understanding vectors and their transformations in plane geometry. As educators and learners delve into this problem, they are exposed to the core principles of linear transformations, vector scaling, and

rotational matrices, all through an engaging, hands-on approach.

This article explores the underlying mathematical principles behind this problem, explains the transformations involved, discusses how such problems are structured in educational platforms like PLATO, and highlights the importance of these concepts in broader mathematical and real-world contexts. Whether you are a student seeking clarity, an educator designing curricula, or a mathematician interested in visualization tools, understanding this problem offers rich insights into the mechanics of geometric transformations.

Section 1: The Significance of Geometric Transformations in Mathematics Education

Why Teach Transformations?

Geometric transformations—dilation, rotation, reflection, and translation—are foundational in understanding how figures behave under various manipulations. They serve as visual and conceptual tools that bridge algebraic operations with geometric intuition.

Educational Impact

Interactive problems that require students to apply these transformations reinforce spatial reasoning, improve problem-solving skills, and deepen understanding of vector operations. Platforms like PLATO leverage these transformations to make abstract concepts tangible, offering instant feedback and dynamic visualizations.

The Role of Vectors

Vectors are fundamental in describing geometric transformations because they encapsulate both magnitude and direction. When a vector undergoes dilation and rotation, its components change according to specific rules, which can be systematically studied using matrix operations and scalar multiplications.

Section 2: Breaking Down the Transformation - Dilation and Rotation

What Is Dilation?

Dilation, also called scaling, involves enlarging or shrinking a figure relative to a fixed point, often the origin in coordinate space. For a vector, dilation by a factor k involves multiplying its components by k .

Mathematically:

Given a vector $\vec{v} = (x, y)$, after dilation by a factor k :

$$\vec{v'} = (k \times x, k \times y)$$

\]

In our problem:

\[

$$k = 2.5$$

\]

So, the dilated vector:

\[

$$\vec{v'} = (2.5x, 2.5y)$$

\]

What Is Rotation?

Rotation involves turning a vector around a fixed point (usually the origin) by a specified angle θ . The rotation preserves the vector's length (if only rotation is applied) and changes its direction.

Mathematically:

Rotating $\vec{v} = (x, y)$ by θ degrees involves multiplying by the rotation matrix:

\[

$$R(\theta) =$$

$\begin{bmatrix}$

$$\cos \theta \quad -\sin \theta$$

$$\sin \theta \quad \cos \theta$$

$\end{bmatrix}$

\]

Result:

\[

$$\vec{v''} = R(\theta) \times \vec{v'}$$

\]

Sequence of Transformations

The problem indicates that the vector is first dilated, then rotated, implying the sequence:

\[

$\text{Original vector} \rightarrow \text{Dilate by 2.5} \rightarrow \text{Rotate by } \theta$

\]

Understanding the order is crucial because matrix operations are non-commutative; applying dilation before rotation yields a different result than doing it in the reverse order.

Section 3: Mathematical Detailing of the Transformation Process

Step 1: Applying Dilation

Suppose the initial vector is $\vec{v} = (x, y)$.

After dilation by 2.5:

$$\vec{v'} = (2.5x, 2.5y)$$

This straightforward scalar multiplication increases the magnitude of the vector by 2.5 times while maintaining its direction.

Step 2: Applying Rotation

Let the rotation angle be θ degrees. Convert to radians for calculation:

$$\theta_{\text{rad}} = \frac{\pi \theta}{180}$$

The rotation matrix:

$$R(\theta) = \begin{bmatrix} \cos \theta_{\text{rad}} & -\sin \theta_{\text{rad}} \\ \sin \theta_{\text{rad}} & \cos \theta_{\text{rad}} \end{bmatrix}$$

Applying this matrix to $\vec{v'}$:

$$\vec{v''} = R(\theta) \times \vec{v'} = \begin{bmatrix} \cos \theta_{\text{rad}} & -\sin \theta_{\text{rad}} \\ \sin \theta_{\text{rad}} & \cos \theta_{\text{rad}} \end{bmatrix} \begin{bmatrix} 2.5x \\ 2.5y \end{bmatrix}$$

Resulting in:

$$x'' = 2.5x \cos \theta_{\text{rad}} - 2.5y \sin \theta_{\text{rad}}$$
$$y'' = 2.5x \sin \theta_{\text{rad}} + 2.5y \cos \theta_{\text{rad}}$$

This final vector $\vec{v''}$ embodies both transformations.

Section 4: Practical Application - Solving the PLATO Problem

Step-by-Step Approach for Students

1. Identify the initial vector $\vec{v}$.
Usually given or represented in a diagram.
2. Perform Dilation:
Multiply both components by 2.5.
3. Determine Rotation Angle θ :
Read from the problem or diagram.
4. Construct the Rotation Matrix:
Convert θ to radians, compute $\cos \theta_{\text{rad}}$ and $\sin \theta_{\text{rad}}$.
5. Apply the Rotation Matrix:
Multiply the dilated vector by the rotation matrix to find the final components.
6. Type the Correct Answer:
The transformed vector components are input into the answer boxes.

Example:

Suppose the initial vector is $\vec{v} = (4, 3)$, and $\theta = 30^\circ$.

- Dilation:

$$\begin{aligned} &[(2.5 \times 4, 2.5 \times 3) = (10, 7.5)] \end{aligned}$$

- Rotation:

$$\begin{aligned} &[\theta_{\text{rad}} = \frac{\pi \times 30}{180} = \frac{\pi}{6}] \\ &[\cos \frac{\pi}{6} = \frac{\sqrt{3}}{2} \approx 0.8660, \quad \sin \frac{\pi}{6} = 0.5] \end{aligned}$$

- Final components:

$$\begin{aligned} &[x'' = 10 \times 0.8660 - 7.5 \times 0.5 = 8.66 - 3.75 = 4.91] \\ &[y'' = 10 \times 0.5 + 7.5 \times 0.8660 = 5 + 6.495 = 11.495] \end{aligned}$$

Thus, the answer box should be filled with approximately (4.91, 11.50).

Section 5: Broader Context and Applications

In Mathematics and Beyond

Understanding these transformations extends beyond classroom exercises. They are crucial in fields like computer graphics, robotics, physics, and engineering.

- Computer Graphics:

Scaling and rotation are fundamental operations for rendering images, animations, and modeling objects.

- Robotics:

Manipulating vectors to control movement and orientation.

- Physics:

Describing the motion of particles and rigid bodies.

- Engineering:

Analyzing stress, strain, and structural transformations.

In Education

Interactive problems like those in PLATO help students develop computational fluency and geometric intuition, essential skills for advanced mathematical studies and STEM careers.

Section 6: Conclusion - Embracing the Power of Geometric Transformations

The problem titled "Type The Correct Answer In Each Box. The Vector Is First Dilated By A Factor Of 2.5 And Then Rotated" encapsulates the elegance and utility of geometric transformations. By systematically applying dilation and rotation, students gain a deeper understanding of how vectors behave under these operations—skills that are fundamental in both theoretical mathematics and practical applications.

This exploration underscores the importance of visualizing transformations, mastering matrix operations, and appreciating the sequence of operations. As educational tools like PLATO continue to incorporate such problems, they foster a generation of learners who are not only proficient in mathematical procedures but also confident

Plato Type The Correct Answer In Each Box The Vector Is First Dilated By A Factor Of 2 5 And Then Rotated

Plato Type The Correct Answer In Each Box The Vector Is First Dilated By A Factor Of 2 5 And Then Rotated

Related Articles

- [The Following Data Values Represent A Sample. What Is The Variance Of The sample? \$X = 8\$. Use The Information](#)
- [In Exercises 27-28, The Images Of The Standard Basis Vectors For \$\mathbb{R}^3\$ Are Given For A Linear Transformation](#)
- [Read The Following Excerpts And Then Answer The Question Below.Excerpt 1:Excerpt From "Pearl Harbor Address](#)

[Back to Home](#)