Question 22 The Values Of M For Which Y=x'' Is A Solution Of Xy'' - 5xy' +8y=0 Are Select The Correct Answer.

Question 22 The Values Of M For Which Y = x^m Is A Solution Of xy'' - 5xy' + 8y = 0 Are Select The Correct Answer.

Understanding the solutions of differential equations is a fundamental aspect of advanced mathematics, particularly in fields like engineering, physics, and applied sciences. In this article, we will explore how to determine the values of the parameter \(m \) for which the function \(y = x^m \) satisfies the differential equation:

\[
xy'' - 5xy' + 8y = 0
\]

This type of problem involves substituting a proposed solution into the differential equation and solving for \(m \). Let's delve into the details step-by-step.

Understanding the Differential Equation

Before solving, it's crucial to analyze the structure of the given differential equation:

\[
xy'' - 5xy' + 8y = 0

\]

Key observations:

- The equation is a second-order linear differential equation.
- It is expressed in terms of y, its first derivative y', and second derivative y''.
- The coefficients depend on x, indicating it could be a Cauchy-Euler (or equidimensional) equation.

Recognizing the Type of Differential Equation

The form:

$$
xy'' + p(x) y' + q(x) y = 0
$$

with specific functions $p(x)$ and $q(x)$.

In our case:

$$
xy'' - 5xy' + 8y = 0
$$

which simplifies to:

\[

xy'' + (-5x) y' + 8 y = 0

\]

This suggests the equation is an equidimensional (or Cauchy-Euler) differential equation, since the coefficients are proportional to \(x \).

Assuming a Power Function Solution

Since the question asks for solutions of the form \(y = x^m \), we proceed by substituting:

\[

y = x^m

\]

where \(m \) is a constant to be determined.

Calculating Derivatives of \(y = x^m \)

Find the derivatives:

- First derivative:

\[

y' = \frac{d}{dx} x^m = m x^{m-1}

$$

- Second derivative:

$$
y'' = \frac{d}{dx} y' = m (m - 1) x^{m-2}
$$

Substituting into the Differential Equation

Plugging $y = x^m$, y', and y'' into the original differential equation:

$$
x y'' - 5 x y' + 8 y = 0
$$

becomes:

$$
x \cdot [m (m - 1) x^{m-2}] - 5 x \cdot [m x^{m-1}] + 8 \cdot x^m = 0
$$

Simplify each term:

1. $x \cdot m (m - 1) x^{m-2} = m (m - 1) x^{m-1}$

2. $-5 x \cdot m x^{m-1} = -5 m x^{m}$

3. $8 x^{m}$ remains as is.

Putting all together:

$$
m (m - 1) x^{m-1} - 5 m x^{m} + 8 x^{m} = 0
$$

Expressing All Terms with the Same Power of x

Notice the first term involves x^{m-1}, while the others involve x^{m}. To combine terms, factor out x^{m-1}:

$$
x^{m-1} \left[m (m - 1) - 5 m x + 8 x \right] = 0
$$

Alternatively, express all terms with x^{m-1}:

$$
m (m - 1) x^{m-1} - 5 m x^{m} + 8 x^{m} = 0
$$

Rewrite the last two terms:

$$
-5 m x^{m} + 8 x^{m} = x^{m} (-5 m + 8)
$$

Since $x^{m} = x^{m-1} \cdot x$, we can write:

$$
x^{m} = x^{m-1} \cdot x
$$

Thus, the entire expression becomes:

$$
x^{m-1} \left[m (m - 1) - 5 m x + 8 x \right] = 0
$$

But to factor correctly, note the dependence on x. To avoid confusion, it's standard practice to divide the entire equation by x^{m-1}, assuming $x \neq 0$, leading to the indicial equation.

Deriving the Characteristic (Indicial) Equation

Divide the entire equation by x^{m-1}:

$$
m (m - 1) - 5 m x + 8 x = 0
$$

But since this involves x explicitly, and the differential equation is homogeneous with respect to x, the standard method for Cauchy-Euler equations is to substitute $y = x^{m}$, which leads to the characteristic equation:

\[

$$
m (m - 1) - 5 m + 8 = 0
$$

Here's how:

- The derivative terms produce coefficients involving m.
- The coefficients of y' and y are proportional to x, but when substituting $y = x^m$, the dependence on x cancels out, leaving an algebraic equation in m.

Therefore, the characteristic equation is:

$$
m (m - 1) - 5 m + 8 = 0
$$

Simplify:

$$
m^2 - m - 5 m + 8 = 0
$$

$$
m^2 - 6 m + 8 = 0
$$

Solving the Quadratic Equation for m

The quadratic:

$$
m^2 - 6 m + 8 = 0
$$

Use the quadratic formula:

$$
m = \frac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 8}}{2 \times 1}
$$

Calculate discriminant:

$$
\Delta = 36 - 32 = 4
$$

Compute roots:

$$
m = \frac{6 \pm \sqrt{4}}{2} = \frac{6 \pm 2}{2}
$$

Thus, the solutions:

1. $m = \frac{6 + 2}{2} = \frac{8}{2} = 4$

2. $m = \frac{6 - 2}{2} = \frac{4}{2} = 2$

Conclusion: The Values of m and Corresponding Solutions

The values of m for which $y = x^m$ is a solution of the differential equation $xy'' - 5xy' + 8 y = 0$ are:

- $m = 2$
- $m = 4$

This means the general solution to the differential equation can be written as:

$$
y = C_1 x^{2} + C_2 x^{4}
$$

where C_1 and C_2 are arbitrary constants.

Summary of Key Steps

- Recognize the differential equation as an equidimensional (Cauchy-Euler) type.
- Assume a power function $y = x^m$.
- Compute derivatives y' and y''.
- Substitute into the original differential equation.
- Simplify to derive the characteristic equation.
- Solve the quadratic equation for m.
- Interpret the roots as the exponents in the solution.

Additional Notes and Applications

Understanding how to find solutions of differential equations of this form is vital in modeling phenomena where scaling properties are important, such as:

- Heat conduction
- Wave propagation
- Population dynamics
- Mechanical vibrations

The method demonstrated here offers a straightforward way to solve such equations analytically, especially when the solution form is a power function.

Frequently Asked Questions (FAQs)

Q1: Why do we assume $y = x^m$ for this type of differential equation?

Answer: Because the differential equation is of Cauchy-Euler (or equidimensional) type, solutions are typically power functions x^m. This substitution transforms the differential equation into an algebraic quadratic in m, simplifying the solution process.

Q2: Can the roots $m = 2$ and $m =$

Frequently Asked Questions

What is the main goal when solving Question 22 regarding the differential equation and the value of M?

The main goal is to find the values of M for which the function $y = x$ is a solution of the differential equation $xy'' - 5xy' + 8y = 0$.

How do you verify if $y = x$ is a solution for the differential equation involving parameter M?

By substituting $y = x$ and its derivatives into the differential equation and solving for M to see which values satisfy the equation.

What are the derivatives of $y = x$ that need to be substituted into the differential equation?

The first derivative $y' = 1$ and the second derivative $y'' = 0$ should be substituted into the equation.

After substitution, what form does the differential equation take to find M?

It reduces to an algebraic equation in M, which can be solved to determine the specific values of M for which $y = x$ is a solution.

What are the possible values of M that satisfy the differential equation when $y = x$?

The values of M are found to be $M = 4$, which makes $y = x$ a solution of the differential equation.

Why is it important to check the solution $y = x$ for different M values

in this problem?

Because only specific values of M will satisfy the differential equation when y = x, confirming the solution's validity for those M values.

Additional Resources

Question 22 The Values Of M For Which Y = x^m Is A Solution Of XY'' - 5XY' + 8Y = 0 Are Select The Correct Answer

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Introduction

In the realm of differential equations, especially linear second-order differential equations with variable coefficients, the quest for particular solutions often involves assuming a specific form and verifying its validity. One common approach is to test solutions of the form $y = x^m$, where m is a parameter to be determined. This method is particularly effective when dealing with Euler-Cauchy equations, which often feature powers of x as solutions.

In this detailed exploration, we examine the differential equation:

$$
XY'' - 5XY' + 8Y = 0
$$

and analyze the conditions under which the assumed solution:

$$
Y = x^m
$$

satisfies the equation. The primary goal is to identify the specific values of m (or M as sometimes denoted) for which this form is a valid solution.

Context and Significance

Understanding the solution space of differential equations is critical in mathematical modeling, physics, engineering, and related disciplines. Identifying the values of parameters that yield solutions can provide insights into the behaviors of systems modeled by such equations.

The differential equation at hand appears to be a form of an Euler-Cauchy (or equidimensional) equation, characterized by variable coefficients proportional to powers of x. Recognizing the solution form $y = x^m$ is a classic method for Euler equations, which often reduces the problem to an algebraic characteristic equation in m. The process involves substituting the assumed form into the original differential equation and solving for m.

Derivation of the Characteristic Equation

Step 1: Express derivatives of $y = x^m$

Given:

$$
Y = x^m
$$

we compute:

$$
Y' = \frac{d}{dx} (x^m) = m x^{m-1}
$$

$$
Y'' = \frac{d}{dx} (m x^{m-1}) = m (m-1) x^{m-2}
$$

Step 2: Substitute into the differential equation

The original differential equation is:

$$
XY'' - 5XY' + 8Y = 0
$$

Plugging in derivatives:

$$
x \cdot m (m-1) x^{m-2} - 5x \cdot m x^{m-1} + 8 x^{m} = 0
$$

Simplify each term:

- First term:

$$
x \cdot m (m-1) x^{m-2} = m (m-1) x^{m-1}
$$

- Second term:

$$
-5x \cdot m x^{m-1} = -5 m x^{m}
$$

- Third term:

$$
8 x^{m}
$$

Now, write the entire equation:

$$
m (m-1) x^{m-1} - 5 m x^{m} + 8 x^{m} = 0
$$

Divide through by x^{m-1} (assuming $x \neq 0$):

$$
m (m-1) - 5 m x + 8 x = 0
$$

Note that the terms involving x are problematic unless they cancel or are combined appropriately. To correctly derive the characteristic equation, note that the terms involving x^{m-1} and x^{m} suggest re-grouping:

$$
m (m-1) x^{m-1} + (-5 m + 8) x^{m} = 0
$$

Dividing the entire equation by x^{m-1}:

$$
m (m-1) + (-5 m + 8) x = 0
$$

This indicates that unless the coefficients are independent of x, this method is inconsistent unless the entire expression involves only powers of x.

But in Euler-Cauchy equations, the standard approach involves recognizing that the differential equation is homogeneous in form, and the substitution $y = x^m$ reduces the differential equation to an algebraic quadratic in m.

Therefore, the original differential equation can be rewritten to align with standard Euler form:

$$
x^2 y'' + p x y' + q y = 0
$$

Comparing with the given:

$$
x y'' - 5 y' + 8 y = 0
$$

which is not directly in the Euler form because of the coefficient of y'. To facilitate analysis, multiply through by x:

$$
x^2 y'' - 5 x y' + 8 x y = 0
$$

But this introduces additional complexity.

Alternatively, recognizing that the original differential equation is:

$$
X y'' - 5 y' + 8 y = 0
$$

and assuming $y = x^m$, the derivatives are as above, and the substitution simplifies to an algebraic characteristic equation involving m.

Correct Approach: Deriving the Characteristic Equation

Step 1: Given $y = x^m$:

$$
Y' = m x^{m-1}
$$

$$
Y'' = m (m-1) x^{m-2}
$$

Step 2: Substitute into the differential equation:

$$
x \cdot m (m-1) x^{m-2} - 5 m x^{m-1} + 8 x^{m} = 0
$$

which simplifies to:

$$

$$
m (m-1) x^{m-1} - 5 m x^{m-1} + 8 x^{m} = 0
$$

Notice the last term is $8 x^{m}$, which can be written as $8 x^{m-1} \times x$. Thus, rewriting:

$$
m (m-1) x^{m-1} - 5 m x^{m-1} + 8 x^{m-1} \times x = 0
$$

Dividing through by x^{m-1} (assuming $x \neq 0$):

$$
m (m-1) - 5 m + 8 x = 0
$$

which is only valid if the term involving x vanishes or if the equation is not meant to be simplified this way.

Key insight: For the assumed form $y = x^m$, the standard method for Euler equations involves the form:

$$
a x^2 y'' + b x y' + c y = 0
$$

which leads to a characteristic equation:

$$
a m (m-1) + b m + c = 0
$$

Therefore, to match the form, the original differential equation should be expressed as:

$$
x^2 y'' + p x y' + q y = 0
$$

which suggests multiplying the original equation by x:

$$
x (x y'' - 5 y' + 8 y) = 0
$$

$$
x^2 y'' - 5 x y' + 8 x y = 0
$$

Now, the standard Euler form is:

$$
x^2 y'' + p x y' + q y = 0
$$

with:

- $p = -5$
- $q = 8 x$

But the coefficient of y is not proportional to x, which indicates the original differential equation is not a pure Euler-Cauchy form unless we re-express or clarify the problem statement.

Clarification and Assumptions

Given the confusion, and considering typical differential equations of the form:

$$
x^2 y'' + p x y' + q y = 0
$$

and the common approach to solutions $y = x^m$, the problem likely intends the differential equation:

$$
x^2 y'' - 5 x y' + 8 y = 0
$$

which is a standard Euler-Cauchy equation with coefficients:

$$
x^2 y'' + p x y' + q y = 0
$$

and

$$
p = -5, \quad q = 8
$$

Deriving the Characteristic Equation for the Euler-Cauchy Equation

Assuming the corrected form:

$$
x^2 y'' - 5 x y' + 8 y = 0
$$

and $y = x^m$, then:

$$
Y' = m x^{m-1}
$$

$$
Y'' = m (m-1) x^{m-2}
$$

Substituting into the differential equation:

$$
x^2 \cdot m (m-1) x^{m-2} - 5 x \cdot m x^{m-1} + 8 x^{m} = 0
$$

Simplify each term:

- First term:

$$
m (m-1) x^{m}
$$

- Second term:

\[

-5 m x^{m}

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