# Sing Polar Coordinates, Evaluate The Integral Which Gives The Area Which Lies In The First Quadrant Between

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Understanding how to evaluate areas bounded by curves is a fundamental aspect of calculus, especially when dealing with complex geometrical regions. When dealing with regions in the first quadrant, converting to polar coordinates often simplifies the integral computation, making the evaluation more straightforward. This article explores the concept of using polar coordinates to evaluate the area enclosed between curves in the first quadrant, focusing on setting up and solving the relevant integrals.

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## Introduction to Polar Coordinates and Area Calculation

### What Are Polar Coordinates?

Polar coordinates provide a way to represent points in the plane based on their distance from a fixed point (the origin) and the angle from a fixed direction (usually the positive x-axis).

- Definition: A point $P$ in the plane is represented as $(r, \theta)$, where:
- $r$: the distance from the origin to $P$,
- $\theta$: the angle between the positive x-axis and the line segment $OP$.

- Conversion formulas:

$$x = r \cos \theta, \quad y = r \sin \theta$$

- Range of variables:
- $r \geq 0$,
- $0 \leq \theta \leq 2\pi$.

In the context of the first quadrant, $r \geq 0$ and $0 \leq \theta \leq \frac{\pi}{2}$.

### Why Use Polar Coordinates for Area Calculation?

- Simplifies the integration when dealing with curves like circles, spirals, or other polar functions.
- Converts complex planar regions into manageable bounds defined by functions $r = f(\theta)$.

## Area in Polar Coordinates

The area $A$ enclosed by a curve $r = f(\theta)$ from $\theta = \alpha$ to $\theta = \beta$ is given by:

$$
A = \frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta
$$

This formula is derived from the fact that an infinitesimal sector with angle $d\theta$ and radius $r$ has an area approximately $\frac{1}{2} r^2 d\theta$.

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# Setting Up the Area Integral in the First Quadrant

Suppose we are interested in finding the area enclosed between two curves $r = r_1(\theta)$ and $r = r_2(\theta)$ within the first quadrant.

## Identifying the Region

- The region is bounded by the curves $r_1(\theta)$ and $r_2(\theta)$,
- For $\theta$ within some interval $[\alpha, \beta]$,
- The region lies entirely in the first quadrant, so $0 \leq \theta \leq \frac{\pi}{2}$,
- The curves may intersect at certain points, which will determine the bounds of integration.

## General Approach to Set Up the Integral

1. Determine the bounds for $\theta$: Find the angles where the curves intersect or where the region starts and ends.
2. Express the area as an integral: Use the polar area formula for the region between the two curves:

$$
A = \frac{1}{2} \int_{\theta_1}^{\theta_2} \left[ r_2(\theta)^2 - r_1(\theta)^2 \right] d\theta
$$

3. Ensure the functions are correctly ordered: $r_2(\theta)$ should be the outer curve, and $r_1(\theta)$ the inner one, over the interval of integration.

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# Example: Calculating Area Between Two Polar Curves

# in the First Quadrant

Let's illustrate the procedure with a concrete example.

## Given Curves

- $r = 2 \sin \theta$,
- $r = 1$,
- for $0 \leq \theta \leq \frac{\pi}{2}$.

The goal is to find the area enclosed between these two curves in the first quadrant.

## Step 1: Find Intersection Points

Set $r = 2 \sin \theta$ equal to $r = 1$:

$$
2 \sin \theta = 1 \Rightarrow \sin \theta = \frac{1}{2}
$$

$$
\theta = \arcsin \frac{1}{2} = \frac{\pi}{6}
$$

Since $\sin \theta$ is positive in $[0, \pi/2]$, the intersection occurs at $\theta = \frac{\pi}{6}$.

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## Step 2: Determine the Region and Bounds of Integration

- For $0 \leq \theta \leq \frac{\pi}{6}$:
- $2 \sin \theta \leq 1$, so the circle $r=1$ is outside.
- For $\frac{\pi}{6} \leq \theta \leq \frac{\pi}{2}$:
- $2 \sin \theta \geq 1$, so the curve $r=2 \sin \theta$ is outside.

Thus, the region enclosed is between $\theta = 0$ and $\theta = \frac{\pi}{2}$, bounded by the curves:

- $r = 2 \sin \theta$ (outer curve in some parts),
- $r=1$ (outer curve in other parts).

But to compute the total area, we need to split the integral at $\theta = \frac{\pi}{6}$:

$$
A = \frac{1}{2} \left[ \int_0^{\pi/6} (1^2 - (2 \sin \theta)^2) d\theta + \int_{\pi/6}^{\pi/2} ((2 \sin \theta)^2 - 1^2) d\theta \right]
$$

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## Step 3: Write the Integral Expression

$$
A = \frac{1}{2} \left[ \int_0^{\pi/6} \left( 1 - 4 \sin^2 \theta \right) d\theta + \int_{\pi/6}^{\pi/2} \left( 4 \sin^2 \theta - 1 \right) d\theta \right]
$$

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## Step 4: Evaluate the Integrals

Recall the integral of $\sin^2 \theta$:

$$
\int \sin^2 \theta \, d\theta = \frac{\theta}{2} - \frac{\sin 2\theta}{4} + C
$$

Applying this:

- For the first integral:

$$
I_1 = \int_0^{\pi/6} (1 - 4 \sin^2 \theta) d\theta = \int_0^{\pi/6} 1 \, d\theta - 4 \int_0^{\pi/6} \sin^2 \theta \, d\theta
$$

$$
= \left[ \theta \right]_0^{\pi/6} - 4 \left[ \frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi/6}
$$

$$
= \frac{\pi}{6} - 4 \left( \frac{\pi/6}{2} - \frac{\sin (\pi/3)}{4} \right)
$$

$$
= \frac{\pi}{6} - 4 \left( \frac{\pi}{12} - \frac{\sqrt{3}/2}{4} \right)
$$

$$
= \frac{\pi}{6} - 4 \left( \frac{\pi}{12} - \frac{\sqrt{3}}{8} \right)
$$

$$
= \frac{\pi}{6} - \left( \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right)
$$

\[

= \frac{\pi}{6} - \frac{\pi}{3} + \frac{\sqrt{3}}{2} = - \frac{\pi}{6} + \frac{\sqrt{3}}{2}
\]

- For the second integral:

\[
I_2 = \int_{\pi/6}^{\pi/2} (4 \sin^2 \theta - 1) d\theta = 4 \int_{\pi/6}^{\pi/2} \sin^2 \theta \, d\theta - \int_{\pi/6}^{\pi/2} 1 \, d\theta
\]

\[
= 4 \left[ \frac{\theta}{

# Frequently Asked Questions

## What is the process to convert a Cartesian double integral into polar coordinates when finding the area in the first quadrant?

To convert a Cartesian double integral into polar coordinates, identify the region's boundaries in terms of r and θ, replace x and y with r cos θ and r sin θ, and include the Jacobian r dr dθ. For the first quadrant, θ ranges from 0 to π/2, and r varies according to the given boundary curves.

## How do you determine the limits of integration in polar coordinates for a region bounded in the first quadrant?

Determine the boundary curves in Cartesian coordinates, convert them into polar form to find the limits of r as functions of θ or vice versa, and set θ from 0 to π/2 to cover the first quadrant. The limits of r are typically from 0 up to the boundary curve defined in polar form.

## What is the general formula for the area of a region in polar coordinates?

The area A of a region in polar coordinates is given by the double integral A = ∫∫_R r dr dθ, where R is the region in the r-θ plane, with θ and r limits defined by the boundary curves.

## How does the symmetry of the region in the first quadrant simplify the evaluation of the integral?

Symmetry allows you to often evaluate the area over a smaller segment and then multiply by the number of symmetric parts, or directly set the limits in the first quadrant from 0 to π/2, simplifying the integration process.

## What are common boundary curves used when evaluating

## areas in the first quadrant with polar coordinates?

Common boundary curves include circles (r = constant), lines (θ = constant), and more complex curves like r = f(θ), such as cardioids, lemniscates, or lemniscates, that enclose the region of interest.

## How do you handle intersecting boundary curves when setting up the polar integral in the first quadrant?

Find the intersection points in polar form to determine the limits of r for each θ, and split the integral if necessary to account for different boundary segments, ensuring the entire region is covered without overlaps.

## Can you provide an example of evaluating the area in the first quadrant between two polar curves?

Yes. For example, to find the area between r = 1 and r = 2 in the first quadrant, set θ from 0 to π/2, and r from 1 to 2. The integral becomes A = $\int_0$^{π/2} $\int_1$^{2} r dr dθ, which evaluates to (π/2) ( (2)^2/2 - (1)^2/2 ) = (π/2) (2 - 0.5) = (π/2) 1.5 = (3π)/4.

## What are the key steps to evaluate the integral that gives the area in the first quadrant between given polar curves?

The key steps are: (1) Convert boundary equations into polar form if necessary, (2) determine the limits of θ (usually 0 to π/2 for the first quadrant), (3) find the corresponding r limits based on the boundary curves, (4) set up the double integral with r dr dθ, and (5) compute the integral to find the area.

## Why is it important to understand the shape and boundaries of the region before setting up the polar integral?

Understanding the shape and boundaries ensures accurate limits of integration, prevents missing parts of the region, and allows correct setup of the integral, leading to precise calculation of the area.

# Additional Resources

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Introduction to Polar Coordinates and Area Calculations

The use of polar coordinates offers a powerful framework for analyzing regions bounded by curves that are naturally expressed in terms of angles and radii. Unlike Cartesian coordinates (x, y), polar coordinates (r, θ) describe a point's position via a radius from the origin and an angle from the positive x-axis. This approach simplifies the process of calculating areas enclosed by curves that are symmetric or circular in nature.

In particular, when dealing with regions in the first quadrant—where both x and y are positive—polar coordinates often streamline the integration process, especially for curves involving circles, cardioids, spirals, or other radially defined boundaries.

The goal here is to evaluate the integral that gives the area enclosed in the first quadrant between specified curves, making full use of the polar coordinate system.

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## Fundamentals of Polar Coordinates

### Definition and Conversion

- Polar Coordinates $(r, \theta)$:
- $r$: the distance from the origin to the point.
- $\theta$: the angle measured counterclockwise from the positive x-axis.

- Conversion between Cartesian and Polar:
- $x = r \cos \theta$
- $y = r \sin \theta$
- $r = \sqrt{x^2 + y^2}$
- $\theta = \arctan(y/x)$

### Range for First Quadrant

- $r \geq 0$
- $0 \leq \theta \leq \pi/2$

### Curves in Polar Coordinates

- Circles centered at the origin: $r = a$
- Circles offset from the origin: $r = 2a \cos \theta$ or $r = 2a \sin \theta$
- Lemniscates, cardioids, and spirals: various functions of $r(\theta)$

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## Area in Polar Coordinates: Theoretical Foundation

### Differential Area Element

The differential area element in polar coordinates is:

$$
dA = \frac{1}{2} r^2 d\theta
$$

However, more precisely, the area element in Cartesian coordinates, when converted, becomes:

$$
dA = r dr d\theta
$$

But for the purpose of integrating over a region bounded by a curve $r = r(\theta)$, the integral simplifies to:

$$
A = \int_{\theta_1}^{\theta_2} \frac{1}{2} r^2 d\theta
$$

This formula is derived from the sector area of a circle with radius $r$, which is $\frac{1}{2} r^2 \theta$.

General Area Calculation

Suppose a curve is described by $r = r(\theta)$, the area enclosed between $\theta = \alpha$ and $\theta = \beta$ is:

$$
\boxed{
A = \frac{1}{2} \int_{\alpha}^{\beta} [r(\theta)]^2 d\theta
}
$$

This formula is valid for simple, well-behaved curves.

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Setting up the Problem: First Quadrant Region Between Curves

Suppose we are given two curves in polar form, say:

- $r = r_1(\theta)$
- $r = r_2(\theta)$

and we are asked to find the area in the first quadrant between them. The typical approach involves:

1. Identifying the intersection points of the two curves in the first quadrant, i.e., solving $r_1(\theta) = r_2(\theta)$ for $\theta$ in $[0, \pi/2]$.

2. Determining the interval(s) of $\theta$ over which the region exists.

3. Integrating the difference of the squared radii over these interval(s):

$$
A = \frac{1}{2} \int_{\theta_{a}}^{\theta_{b}} \left( r_1^2(\theta) - r_2^2(\theta) \right) d\theta
$$

This accounts for the area enclosed between the two curves.

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Practical Example: The Area Between Two Circles in the First Quadrant

Let's explore a concrete example to illustrate the process:

Curves:

- $r = a$ (a circle centered at the origin with radius $a$)
- $r = b$ (another circle with radius $b$), with $a < b$

Region:

We want the area in the first quadrant between these circles.

Step-by-step Solution:

1. Find intersection points:

Since both are circles centered at the origin, they intersect where:

$$
a = b \Rightarrow \text{only at } r = a \text{ or } r= b
$$

But these are concentric circles, so they do not intersect unless $a = b$.

Instead, suppose the second curve is a circle of radius $b$ but only within a certain angular sector, such as:

$$
r = b \quad \text{for } 0 \leq \theta \leq \pi/2
$$

and the first is $r = a$.

In this case, the area between the circles in the first quadrant is simply the sector of the larger circle minus the sector of the smaller circle:

$$
A = \frac{1}{2} \times (\text{area of sector of radius } b) - \frac{1}{2} \times (\text{area of sector of radius } a)
$$

which simplifies to:

$$
A = \frac{1}{2} \times \left( b^2 - a^2 \right) \times \frac{\pi}{2}
$$

since the angles go from $0$ to $\pi/2$.

---

Complex Region: An Example with Non-Concentric Curves

To demonstrate the flexibility of polar integration, consider a more complex region:

- $r = 2 \sin \theta$
- $r = 1$

These curves intersect where:

$$
2 \sin \theta = 1 \Rightarrow \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}
$$

since $\sin \theta$ is positive in the first quadrant.

Step 1: Determine the bounds of $\theta$

- $\theta$ runs from 0 to $\pi/2$.
- The region between the curves in the first quadrant is from $\theta = 0$ to $\theta = \pi/6$, where $r = 2 \sin \theta$ is outside $r=1$, and from $\theta = \pi/6$ to $\pi/2$, where $r=1$ is outside $r=2 \sin \theta$.

Step 2: Set up the integral

The area in the first quadrant enclosed between these curves is:

$$
A = \frac{1}{2} \int_{0}^{\pi/6} [2 \sin \theta]^2 d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} 1^2 d\theta
$$

which simplifies to:

$$
A = \frac{1}{2} \int_{0}^{\pi/6} 4 \sin^2 \theta d\theta + \frac{1}{2} \int_{\pi/6}^{\pi/2} 1 d\theta
$$

Step 3: Calculate the integrals

- First integral:

$$
\frac{1}{2} \times 4 \int_{0}^{\pi/6} \sin^2 \theta d\theta = 2 \int_{0}^{\pi/6} \sin^2 \theta d\theta
$$

- Second integral:

\[
\frac{1}{2} \times (\pi/2 - \pi/6) = \frac{1}{2} \times \left( \frac{\pi}{2} - \frac{\pi}{6} \right) =
\frac{1}{2} \times \frac{\pi}{3} = \frac{\pi}{6}
\]

Using the identity:

\[
\sin^2 \theta = \frac{1 - \cos 2\theta}{2}
\]

the first integral becomes:

\[
2 \int_{0}^{\pi/6} \frac{1 - \cos 2\theta}{2} d\theta = \int_{0}^{\pi/6} (1 - \cos 2\theta) d\theta
\]

which evaluates to:

\[
\left[ \theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/6} = \left( \frac{\pi}{6} - \frac{\sin (\pi/3)}{2}
\right) - (0 - 0) = \frac{\pi}{6} - \frac{\sqrt{3}/2}{2} = \

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