Solve. Don't Forget To Check! 1/(5z^2180)+3/(z6)=6/(z+6) Select The Correct Choice Below And, If Necessary,

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Introduction

Mathematics is an essential skill that empowers students and professionals alike to analyze, interpret, and solve complex problems. One common type of problem involves solving rational equations, which feature variables in the denominators. These equations can seem intimidating at first, but with systematic approaches and careful checks, they become manageable.

In this article, we will explore how to solve the equation:

$$
\frac{1}{5z^{2180}} + \frac{3}{z6} = \frac{6}{z+6}
$$

and how to select the correct solution from a set of options. We will walk through the steps methodically, discuss common pitfalls, and emphasize the importance of verifying solutions to avoid extraneous roots. Whether you're a student preparing for exams or a professional needing a refresher, this comprehensive guide will help you master solving such rational equations.

Understanding the Equation

Before diving into solving the equation, it's crucial to understand its structure and identify potential challenges.

The Equation at a Glance

The given equation is:

$$
\frac{1}{5z^{2180}} + \frac{3}{z6} = \frac{6}{z+6}
$$

At first glance, it appears complex due to:

- The high exponent 2180 in z^{2180}.
- The expressions $z6$, which likely denote $z \times 6$.
- The structure of the equation involving multiple rational terms.

Clarifying the Notation

To proceed effectively, clarify the notation:

- Is $z6$ meant to be $z \times 6$?
Most likely, yes. So, $z6 = 6z$.

- Is $5z^{2180}$ exponentiation?

Yes, z^{2180} is z raised to the 2180th power, multiplied by 5 in the denominator.

Therefore, rewriting with clearer notation:

$$
\frac{1}{5z^{2180}} + \frac{3}{6z} = \frac{6}{z+6}
$$

Step-by-Step Solution Approach

1. Simplify the Equation

Rewrite the equation for clarity:

$$
\frac{1}{5z^{2180}} + \frac{3}{6z} = \frac{6}{z+6}
$$

Notice that:

$$
\frac{3}{6z} = \frac{1}{2z}
$$

So, the simplified form is:

$$
\frac{1}{5z^{2180}} + \frac{1}{2z} = \frac{6}{z+6}
$$

2. Find a Common Denominator or Clear Fractions

To solve rational equations, a common approach is to multiply through by the least common denominator (LCD) to clear fractions.

The denominators are:

- $5z^{2180}$
- $2z$
- $z+6$

The LCD is:

$$
\text{LCD} = \text{lcm}(5z^{2180}, 2z, z+6)
$$

Note:

- z appears in two denominators; the highest power is z^{2180}.
- The factor $z+6$ is separate — it's linear, so it will be included in the LCD.

Thus, the LCD is:

$$
\text{LCD} = 10z^{2180}(z+6)
$$

because:

- $5z^{2180}$ divides into $10z^{2180}$,
- $2z$ divides into $10z$,
- and the denominator $z+6$ remains as is.

3. Multiply Both Sides by the LCD

Multiplying through:

$$
10z^{2180}(z+6) \times \left(\frac{1}{5z^{2180}} + \frac{1}{2z} \right) = 10z^{2180}(z+6) \times \frac{6}{z+6}
$$

Distribute:

$$
\left[10z^{2180}(z+6) \times \frac{1}{5z^{2180}}\right] + \left[10z^{2180}(z+6) \times \frac{1}{2z}\right] = 6 \times 10z^{2180}
$$

Simplify each term:

- First term:

$$
10z^{2180}(z+6) \times \frac{1}{5z^{2180}} = 2(z+6)
$$

- Second term:

$$
10z^{2180}(z+6) \times \frac{1}{2z} = 5z^{2180}(z+6) \times \frac{1}{z}
$$

Note that:

$$
\frac{z^{2180}(z+6)}{z} = z^{2179}(z+6)
$$

Thus, the second term simplifies to:

$$
5 \times z^{2179} (z+6)
$$

- Right side:

$$
6 \times 10 z^{2180} = 60 z^{2180}
$$

4. Rewrite the Equation

Putting it all together:

$$

$$
2(z+6) + 5 z^{2179} (z+6) = 60 z^{2180}
$$

Factor out $(z+6)$:

$$
(z+6) \left[2 + 5 z^{2179} \right] = 60 z^{2180}
$$

Solving for z

Now, we have:

$$
(z+6) \left[2 + 5 z^{2179} \right] = 60 z^{2180}
$$

Recall that:

$$
z^{2180} = z^{2179} \times z
$$

Rewrite the right side:

$$
60 z^{2179} z
$$

So, the equation becomes:

$$
(z+6) (2 + 5 z^{2179}) = 60 z^{2179} z
$$

Analyzing Possible Solutions

Case 1: $z = 0$

Check if $z=0$ satisfies the original equation.

Substitute $z=0$:

$$
\frac{1}{5 \times 0^{2180}} + \frac{1}{2 \times 0} = \frac{6}{0+6}
$$

- $0^{2180} = 0$, so denominator $5 \times 0^{2180} = 0$, division by zero — undefined.

Similarly, the second term:

$$
\frac{1}{2 \times 0} = \frac{1}{0}
$$

Undefined.

Conclusion: $z=0$ is not a valid solution.

Case 2: $z = -6$

Substitute $z=-6$:

- Denominator in the RHS: $z+6 = 0$. The RHS becomes:

$$
\frac{6}{0}
$$

which is undefined.

Conclusion: $z=-6$ is not a solution.

Case 3: $2 + 5 z^{2179} = 0$

Set:

$$
2 + 5 z^{2179} = 0
$$

$$
5 z^{2179} = -2
$$

$$
z^{2179} = -\frac{2}{5}
$$

Since 2179 is an odd number, taking the odd root:

$$
z = \left(-\frac{2}{5} \right)^{1/2179}
$$

This yields a real solution because an odd root of a negative number is negative:

$$
z = - \left(\frac{2}{5} \right)^{1/2179}
$$

Now, check whether this solution makes sense in the original equation.

Verifying the Candidate Solution

Let's denote:

$$
z_{} = - \left(\frac{2}{5} \right)^{1/2179}
$$

Since \(z_\) is negative and not zero or \(-6\), it's a valid candidate.

1. Compute \(z+6\):

$$
z_ + 6 = 6 - \left(\frac{2}{5} \right)^{1/2179}
$$

which is positive because:

$$
\left(\frac{2}{5} \right)^{1/2179} \ll 1
$$

hence:

$$
z_ + 6 > 0
$$

2. Check the original equation:

$$
\frac{1}{5 z_^{2180}} + \frac{1}{2 z_} = \frac{6}{z_ + 6}
$$

Recall:

\[
z_^{2180} = (z_^{2179}) \times z_ = \left(- \

Frequently Asked Questions

What is the first step in solving the equation 1/(5z^2 - 180) + 3/(z6) = 6/(z + 6)?

The first step is to clarify and factor the denominators if possible, and then find a common denominator to combine the fractions.

How do I handle the term 3/(z6) in the equation?

It appears to be a typo; likely it should be 3/(6z). Recognize this and rewrite it accordingly to ensure correct calculations.

What is the importance of checking the solutions in this type of rational equation?

Checking solutions is crucial because extraneous solutions can arise from multiplying both sides by denominators, which might introduce invalid solutions.

How do I avoid missing solutions when solving rational equations?

Always consider restrictions from the denominators, simplify carefully, and substitute solutions back into the original equation to verify their validity.

What is the role of factoring denominators in solving this equation?

Factoring denominators helps identify common factors, simplify the equation, and determine restrictions on the variable.

What strategies can I use to solve equations involving multiple rational expressions?

Find a common denominator, multiply through to clear fractions, solve the resulting polynomial, and then check for extraneous solutions.

Why is it essential to check the solutions after solving the equation?

Because solutions may cancel out denominators or be invalid due to restrictions, so verifying ensures they satisfy the original equation.

What is the correct approach if the equation involves complex denominators like 5z^2 - 180?

Factor the denominator where possible, identify restrictions, multiply through by the least common denominator, and solve the resulting algebraic equation.

Additional Resources

Solve. Don't Forget To Check!
An In-Depth Examination of the Rational Equation:
\[\frac{1}{5z^{2180}} + \frac{3}{z6} = \frac{6}{z+6} \]

Introduction

Mathematics often presents us with complex rational equations that challenge even seasoned problem-solvers. The equation:

\[
\frac{1}{5z^{2180}} + \frac{3}{z6} = \frac{6}{z+6}
\]

is an intriguing example. At first glance, the notation appears straightforward, but upon closer inspection, it reveals potential ambiguities, pitfalls, and opportunities for deeper understanding. This article aims to dissect this equation comprehensively, explore its solutions, and emphasize the importance of meticulous checking—a fundamental step often overlooked in problem-solving.

Clarifying the Equation: Notational Ambiguities

Before attempting to solve, it is critical to interpret the given equation accurately. The notation as presented:

\[
\frac{1}{5z^{2180}} + \frac{3}{z6} = \frac{6}{z+6}
\]

raises questions regarding the terms z6:

- Is z6 intended as $z \times 6$ (i.e., z multiplied by 6)?

- Or is it a typographical error, perhaps meant to be z^6—z raised to the sixth power?

Possible Interpretations:

1. Interpretation 1:
$$
\frac{1}{5z^{2180}} + \frac{3}{6z} = \frac{6}{z+6}
$$
where z6 = $6z$.

2. Interpretation 2:
$$
\frac{1}{5z^{2180}} + \frac{3}{z^6} = \frac{6}{z+6}
$$
where z6 = z^6.

3. Interpretation 3:
A typographical error where the notation is inconsistent.

Given the context of typical algebraic equations, the second interpretation—z^6—seems most plausible, especially considering the pattern of exponents and common notation.

Priorities in Analysis: Choosing the Most Probable Form

Given the above, the most logical and mathematically consistent interpretation is:

$$
\frac{1}{5z^{2180}} + \frac{3}{z^6} = \frac{6}{z+6}
$$

This form involves exponential terms with large exponents and a rational term with a linear denominator. It is a challenging yet manageable rational equation.

Structural Breakdown of the Equation

The equation:

$$
\frac{1}{5z^{2180}} + \frac{3}{z^6} = \frac{6}{z+6}
$$

contains three rational expressions involving powers of z and a linear term $z+6$. The key challenges include:

- High exponents: z^{2180} and z^6.

- Potential domain restrictions: $z \neq 0$ (to avoid division by zero), $z \neq -6$.

- The interplay of exponential terms with vastly different exponents.

Strategy for Solving the Equation

To solve this equation, consider the following approach:

1. Identify the domain restrictions
- $z \neq 0$ (denominator $5z^{2180}$ invalid if $z=0$)
- $z \neq -6$ (denominator $z+6$ invalid if $z=-6$)

2. Simplify the equation
- Multiply through by the least common denominator (LCD):

$$
\text{LCD} = 5z^{2180} \times z^6 \times (z+6)
$$

- The LCD avoids fractions, allowing us to obtain a polynomial equation.

3. Express all terms with a common basis
- Recognize that z^{2180} is vastly larger than z^6 for $|z| > 1$.
- Consider substitution to reduce complexity.

4. Substitution approach
Let:

$$
t = z^6
$$

then:

$$
z^{2180} = (z^6)^{363 + \frac{2}{3}} \quad \text{(since } 2180 = 6 \times 363 + 2)
$$

but fractional exponents complicate matters; instead, perhaps focusing on the dominant terms or considering specific solutions (e.g., $z=1$, $z=-6$, etc.) might be more practical.

Attempted Solution: Exploring Special Values

Given the complexity, testing specific values of z might reveal solutions or constraints:

a) $z=1$

- Compute:

$$
\frac{1}{5 \times 1^{2180}} + \frac{3}{1^6} = \frac{6}{1+6}
$$

$$
\frac{1}{5} + 3 = \frac{6}{7}
$$

$$
0.2 + 3 = \frac{6}{7} \approx 0.857
$$

Left side:

$$
3.2
$$

Right side:

$$
0.857
$$

Not equal, so $z=1$ is not a solution.

b) $z=-6$

- Denominator $z+6=0$, undefined. So $z=-6$ is excluded.

c) $z=0$

- Denominator $5z^{2180}$ becomes zero, undefined. So $z=0$ is invalid.

d) $z \to \infty$

- As $z \to \infty$, the terms:

$$
\frac{1}{5z^{2180}} \to 0, \quad \frac{3}{z^6} \to 0
$$

- The RHS:

$$
\frac{6}{z+6} \to 0
$$

- So, at infinity, both sides tend to zero, suggesting a possible limit behavior, but not an explicit solution.

Deep Solution Approach: Transformations and Approximations

1. Recognize the dominance of terms

Because z^{2180} grows vastly faster than z^6, for large $|z|$:

$$
\frac{1}{5z^{2180}} \to 0
$$

and the equation approximates to:

$$
\frac{3}{z^6} \approx \frac{6}{z+6}
$$

which simplifies to:

$$

$$
3(z+6) \approx 6z^6
$$

or

$$
3z + 18 \approx 6z^6
$$

Dividing both sides by 3:

$$
z + 6 \approx 2z^6
$$

Rearranged:

$$
2z^6 - z - 6 \approx 0
$$

This polynomial can be solved approximately or graphically to find potential solutions.

2. Solving the approximate polynomial:

$$
2z^6 - z - 6 = 0
$$

This is a high-degree polynomial, but numerical methods or graphing might reveal approximate roots.

Numerical Exploration of the Polynomial $2z^6 - z - 6 = 0$

Using computational tools or root-finding algorithms:

- For $z=1$:

$$
2(1)^6 - 1 - 6 = 2 - 1 - 6 = -5 \neq 0
$$

- For $z=2$:

$$
2(64) - 2 - 6 = 128 - 2 - 6= 120 \neq 0
$$

- For $z=1.5$:

$$
2(1.5^6) - 1.5 - 6
$$

Calculate $1.5^6 \approx 11.39$:

$$
2 \times 11.39 - 1.5 - 6 = 22.78 - 1.5 - 6 = 15.28
$$

Positive, so root between 1 and 1.5.

- For $z=1.2$:

$$
1.2^6 \approx 2.98598 \Rightarrow 2 \times 2.98598 - 1.2 - 6 \approx 5.97 - 1.2 - 6 = -1.23
$$

Negative at 1.2, positive at 1.5, so root between 1.2 and 1.5.

Interpolating further suggests a root near approximately 1.3–1.4.

Conclusion:

Potential approximate solutions exist near $z \approx 1.3$ to 1.4.

The Critical Step: Verify Solutions and Check for Extraneous Roots

When solving rational equations, especially with high exponents, solutions found through approximation or substitution must be checked against the original equation to avoid extraneous roots.

1. Plug back approximate solutions into the original:

Test $z \approx$

Solve Don T Forget To Check 1 5z 2180 3 Z6 6 Z 6 Select The Correct Choice Below And If Necessary

Solve Don T Forget To Check 1 5z 2180 3 Z6 6 Z 6 Select The Correct Choice Below And If Necessary

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