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Introduction to the Problem and Its Significance

Understanding the behavior of functions over specific regions is fundamental in multivariable calculus, especially when dealing with optimization problems, double integrals, and geometric interpretations. The function $F(x, y) = x + y^2 - 6x$ presents an interesting case for analysis over a defined triangular region D with vertices at (6, 0), (0, 6), and (0, -6). Such problems often appear in fields like physics, engineering, and economics, where determining maxima, minima, or integral values over complex regions provides critical insights.

In this article, we will explore the nature of the function F(x, y), analyze the geometric properties of the region D, and perform detailed calculations such as finding the maximum and minimum values of F over D, setting up double integrals, and understanding the implications of these computations. Our goal is to provide a comprehensive, step-by-step guide to solving such problems, emphasizing clarity and depth.

Understanding the Function $F(x, y) = x + y^2 - 6x$

Simplification and Interpretation of the Function

The given function F(x, y) can be rewritten to facilitate analysis:

$$
F(x, y) = x + y^2 - 6x = -5x + y^2
$$

This simplified form reveals that F(x, y) is a quadratic function in y and linear in x, with a negative coefficient for x.

Key observations:
- The function is quadratic in y with a parabola opening upward (since the coefficient of y^2 is positive).
- Along the x-axis, the function decreases as x increases (because of the $-5x$ term).
- The function's extremal values over D depend on the interplay between x and y within the boundary region.

Graphical Interpretation

Visualizing F(x, y) helps understand where the maximum and minimum values occur:

- For a fixed x, F(x, y) is a parabola in y.
- The maximum or minimum of F over D may occur at vertices or along boundary segments.

Analyzing the Region D: The Triangular Region

Vertices and Boundaries

The region D is a closed triangle with vertices:
- A: (6, 0)
- B: (0, 6)
- C: (0, -6)

Plotting these points:
- Point A at (6, 0) lies on the x-axis.
- Point B at (0, 6) lies on the positive y-axis.
- Point C at (0, -6) lies on the negative y-axis.

The triangle's sides connect these points:
- AB: from (6, 0) to (0, 6)
- AC: from (6, 0) to (0, -6)
- BC: from (0, 6) to (0, -6)

Equations of the Boundary Lines

Calculating the equations of the sides:
- Line AB:
- Slope $m_{AB} = (6 - 0)/(0 - 6) = 6/(-6) = -1$
- Equation: $y - 0 = -1(x - 6) \Rightarrow y = -x + 6$

- Line AC:
- Slope $m_{AC} = (-6 - 0)/(0 - 6) = -6/(-6) = 1$
- Equation: $y - 0 = 1(x - 6) \Rightarrow y = x - 6$

- Line BC:
- Vertical line at $x = 0$, connecting (0, 6) and (0, -6)

In summary:
- The boundary lines are:
- AB: $y = -x + 6$, for $0 \leq x \leq 6$
- AC: $y = x - 6$, for $0 \leq x \leq 6$
- BC: $x = 0$, with y between -6 and 6

Setting up the Problem: Maxima, Minima, and Integrals

Finding the Maximum and Minimum of F(x, y) over D

To find the extrema of F(x, y) on D, consider:
- Critical points inside D: where the gradient of F is zero.
- Boundary points: along the edges of D.

Calculating Critical Points in the Interior

Gradient of F:
$$
\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y} \right) = (-5, 2y)
$$
Set $\nabla F = (0, 0)$:
$$
-5 = 0 \quad \text{and} \quad 2y = 0
$$
The first is impossible; hence, no critical points inside D.

Conclusion: The extrema occur on the boundary.

Evaluating F at the Vertices

Calculate F at each vertex:
- At (6, 0):
$$
F(6, 0) = -5(6) + 0^2 = -30
$$
- At (0, 6):
$$
F(0, 6) = -5(0) + 36 = 36
$$
- At (0, -6):
$$
F(0, -6) = -5(0) + 36 = 36
$$

Thus, at vertices:
- Minimum value is -30 at (6, 0).
- Maximum value is 36 at (0, 6) and (0, -6).

Checking Along Boundary Lines

Since the extremal values are at vertices, but boundary analysis can confirm whether other points yield higher or lower values:

- Line AB ($y = -x + 6$), $0 \leq x \leq 6$:
$$
F(x, y) = -5x + y^2 = -5x + (-x + 6)^2

\]
Expand:

$$
F(x) = -5x + x^2 - 12x + 36 = x^2 - 17x + 36
$$

Find extrema by derivative:

$$
\frac{dF}{dx} = 2x - 17 = 0 \Rightarrow x = \frac{17}{2} = 8.5
$$

But x is in [0, 6], so check endpoints:

- At $x=0$:

$$
F(0) = 0 - 0 + 36 = 36
$$

- At $x=6$:

$$
F(6) = 36 - 102 + 36 = -30
$$

- At $x=6$:

$$
y = -6 + 6= 0
$$

Corresponds to vertex (6, 0), which we already evaluated.

Since the critical point $x=8.5$ is outside the interval, the maximum on AB is at $x=0$, with $F=36$, matching the vertex value.

- Line AC ($y = x - 6$), $0 \leq x \leq 6$:

$$
F(x) = -5x + (x - 6)^2 = -5x + x^2 - 12x + 36 = x^2 -17x +36
$$

Same as ABC, so maximum at $x=0$:

$$
F(0) = 36
$$

and minimum at $x=6$:

$$
F(6) = -30
$$

- Line BC ($x=0$), $y \in [-6, 6]$:

$$
F(0, y) = 0 + y^2 = y^2
$$

Since $y \in [-6, 6]$:
- Minimum $F=0$ at $y=0$
- Maximum $F=36$ at $y= \pm 6$

Summary:
- The maximum value of F is 36, achieved at points (0, 6) and (0, -6).
- The minimum value is -30, achieved at (6, 0).

Double Integral of F(x, y) over Region D

Beyond extrema, evaluating the double integral of F over D provides insights into the average value and total accumulation across the region

Frequently Asked Questions

What is the function F(x, y) defined as in the problem?

The function F(x, y) is defined as $F(x, y) = x + y^2 - 6x$.

What are the vertices of the triangular region D?

The vertices of the region D are (6, 0), (0, 6), and (0, -6).

How can the region D be described geometrically?

Region D is a closed triangle with vertices at (6, 0), (0, 6), and (0, -6).

What is the main goal when analyzing the function F(x, y) over region D?

The main goal is typically to find the maximum and minimum values of F(x, y) within the region D.

Which methods can be used to find the extrema of F(x, y) on the boundary of D?

Methods include parametrizing each boundary segment and applying the Extreme Value Theorem, as well as checking critical points inside D.

How do you find critical points of F(x, y) inside the region D?

Set the partial derivatives of F with respect to x and y to zero and solve for (x, y).

What are the partial derivatives of F(x, y)?

The partial derivatives are $\partial F/\partial x = 1 - 6$ and $\partial F/\partial y = 2y$.

Are there any critical points inside the region D?

Since $\partial F/\partial x = -5$ (constant) and $\partial F/\partial y = 2y$, setting $\partial F/\partial y = 0$ gives $y=0$, but $\partial F/\partial x \neq 0$; thus, no critical points inside D.

How do you evaluate F(x, y) along the boundary segments of D?

Parametrize each boundary segment (lines between vertices), substitute into F(x, y), and find extrema by analyzing the resulting functions.

What is the significance of evaluating F on the boundary of D?

Since the critical points inside D do not give extrema, the maximum and minimum values occur on the boundary, so analyzing the boundary is essential.

Additional Resources

Suppose F(x, Y) = X + Y^2 - 6x and D Is The Closed Triangular Region With Vertices (6,0), (0,6), And (0,-6). This problem invites us into the fascinating world of multivariable calculus, where we analyze functions over specific regions in the plane. In particular, understanding how to evaluate such functions over a triangular region involves a combination of geometric insight, coordinate transformations, and calculus techniques like double integration. Whether you're preparing for an exam or deepening your understanding of multivariable calculus, this comprehensive guide aims to clarify the process step-by-step.

Introduction to the Problem

Suppose we are given a function:

$$F(x, y) = x + y^2 - 6x$$

which simplifies to:

$$F(x, y) = -5x + y^2$$

and a region D, which is a closed triangular region with vertices at $(6, 0)$, $(0, 6)$, and $(0, -6)$.

The goal could be to evaluate an integral of the form:

$$
\iint_D F(x, y) \, dA
$$

or to analyze the maximum and minimum values of $F(x, y)$ over D. While the specific problem isn't fully specified, the structure suggests a focus on double integrals over the region D, along with understanding the geometric and algebraic properties involved.

Understanding the Region D

Vertices and Shape

The vertices:

- $A = (6, 0)$
- $B = (0, 6)$
- $C = (0, -6)$

form a triangle in the plane. To analyze integrals over D, we need to understand its boundaries.

Plotting and Visualizing D

- The point $A = (6, 0)$ lies on the x-axis, 6 units to the right of the origin.
- The point $B = (0, 6)$ is on the y-axis, 6 units above the origin.
- The point $C = (0, -6)$ is on the y-axis, 6 units below the origin.

Connecting these points:

- Line AB: passes through $(6, 0)$ and $(0, 6)$.
- Line AC: passes through $(6, 0)$ and $(0, -6)$.
- Line BC: connects $(0, 6)$ and $(0, -6)$, i.e., the y-axis segment.

Equations of the Boundary Lines

1. Line AB:

Using points $(6,0)$ and $(0,6)$:

$$
\text{slope} = \frac{6 - 0}{0 - 6} = -1
$$

Equation:

$$
y - 0 = -1(x - 6) \implies y = -x + 6
$$

2. Line AC:

Using points $(6, 0)$ and $(0, -6)$:

$$
\text{slope} = \frac{-6 - 0}{0 - 6} = \frac{-6}{-6} = 1
$$

Equation:

$$
y - 0 = 1(x - 6) \implies y = x - 6
$$

3. Line BC:

Between $(0, 6)$ and $(0, -6)$:

$$
x = 0
$$

This is the y-axis segment between y = 6 and y = -6.

Region D Summary

The triangle is bounded by:

- The line $AB: y = -x + 6$ (connecting $(6,0)$ and $(0,6)$)
- The line $AC: y = x - 6$ (connecting $(6,0)$ and $(0,-6)$)
- The y-axis segment $x=0$, from $y=-6$ to 6

Setting Up the Double Integral

Choice of Integration Order

Given the boundaries, a natural approach is to integrate with respect to x first, then y, or vice versa.

Option 1: Integrate with respect to x first, then y.

- For a fixed y, find the corresponding x-values.

From the boundary lines:

- For y between -6 and 6:

$$
x \text{ varies between } x_{left} \text{ and } x_{right}
$$

- Left boundary ($x=0$) when y is between -6 and 6.

- Right boundary:

- For $0 \le y \le 6$, the right boundary is along AB:

$$
y = -x + 6 \implies x = 6 - y
$$

- For $-6 \le y \le 0$, the right boundary is along AC:

$$
y = x - 6 \implies x = y + 6
$$

Thus, the region can be split into two parts:

1. For $0 \le y \le 6$:

$$
x \text{ from } 0 \text{ to } 6 - y
$$

2. For $-6 \le y \le 0$:

$$
x \text{ from } 0 \text{ to } y + 6
$$

Final integral setup:

$$
\iint_D F(x, y) \, dA = \int_{-6}^0 \int_{0}^{y+6} F(x, y) \, dx \, dy + \int_{0}^6 \int_{0}^{6 - y} F(x, y) \, dx \, dy
$$

Transforming Coordinates for Simplification

While the above setup works, sometimes switching to different coordinates can simplify the integration. For example, consider a linear change of variables aligned with the boundaries.

Introducing New Variables

Define:

$$
u = x \quad \text{and} \quad v = y
$$

but since the boundaries are linear, perhaps a substitution aligned with the lines:

$$
u = y + x \quad \text{and} \quad v = y - x
$$

or similar, could linearize the region. However, given the symmetry and simplicity, sticking with the current setup might be more straightforward.

Computing the Double Integral

Step 1: Evaluate the inner integrals

Recall:

$$
F(x, y) = -5x + y^2
$$

For each part, perform the integration:

Part 1: $y \in [-6, 0]$, $x \in [0, y+6]$

$$
\int_{-6}^0 \left(\int_0^{y+6} (-5x + y^2) \, dx \right) dy
$$

Inner integral:

$$
\int_0^{y+6} (-5x + y^2) \, dx = \left[-\frac{5}{2}x^2 + y^2 x \right]_0^{y+6}
$$

Evaluate at $x = y+6$:

$$
-\frac{5}{2}(y+6)^2 + y^2 (y+6)
$$

At $x=0$, integral is 0.

Part 2: $y \in [0, 6]$, $x \in [0, 6 - y]$

$$
\int_0^{6} \left(\int_0^{6 - y} (-5x + y^2) \, dx \right) dy
$$

Inner integral:

$$
\left[-\frac{5}{2}x^2 + y^2 x \right]_0^{6 - y}
$$

Evaluate at $x=6 - y$:

$$
-\frac{5}{2}(6 - y)^2 + y^2 (6 - y)
$$

Calculating the Integrals

Part 1 Calculation

$$
I_1 = \int_{-6}^0 \left(-\frac{5}{2}(y+6)^2 + y^2 (y+6) \right) dy
$$

Expand:

$$
(y+6)^2 = y^2 + 12 y + 36
$$

So,

$$
I_1 = \int_{-6}^0 \left(-\frac{5}{2} (y^2 + 12 y + 36) + y^2 (y+6) \right) dy
$$

Distribute:

$$
I_1 = \int_{-6}^0 \left(-\frac{5}{2} y^2 - 30 y - 90 + y^3 + 6 y^2 \right) dy
$$

Combine

Suppose F X Y X Y2 6x And D Is The Closed Triangular Region With Vertices 6 0 0 6 And 0 6

Suppose F X Y X Y2 6x And D Is The Closed Triangular Region With Vertices 6 0 0 6 And 0 6

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