Suppose Thatf(x)=(84x)ex. Note: Several Parts Of This Problem Require Answers Entered In Interval Notation.

Suppose That f(x) = (84x) e^x. Note: Several Parts Of This Problem Require Answers Entered In Interval Notation.

When analyzing the function $f(x) = (84x) e^x$, it's essential to understand its behavior, including its domain, range, critical points, concavity, and asymptotic behavior. This comprehensive exploration offers insights into calculus concepts such as derivatives, critical points, intervals of increase/decrease, concavity, and the end behavior of the function. These concepts are fundamental in understanding how the function behaves across its domain and how to interpret its graph accurately.

Understanding the Function $f(x) = (84x) e^x$

The function $f(x) = (84x) e^x$ is a product of a linear function $84x$ and an exponential function e^x. The exponential component dominates the growth as x increases, but the linear multiplier influences the overall shape, zeros, and critical points of the function.

Domain of $f(x)$

Since both components, $84x$ and e^x, are defined for all real numbers, the domain of $f(x)$ is:

$$
\boxed{(-\infty, \infty)}
$$

Range of $f(x)$

To determine the range, we need to analyze the function's critical points, end behavior, and whether the function attains maximums or minimums.

Calculus Analysis of $f(x) = (84x) e^x$

First Derivative $f'(x)$

The first derivative helps identify critical points, where the function could have local maxima or minima, and determine intervals of increase or decrease.

Using the product rule:

$$
f'(x) = \frac{d}{dx} [84x \cdot e^x] = 84 \cdot e^x + 84x \cdot e^x = 84 e^x (1 + x)
$$

Critical Points

Set $f'(x) = 0$:

$$
84 e^x (1 + x) = 0
$$

Since $e^x \neq 0$ for all real x, the critical points occur where:

$$
1 + x = 0 \Rightarrow x = -1
$$

Thus, there is a single critical point at $x = -1$.

Behavior at Critical Point

To classify this critical point, analyze the second derivative or use the First Derivative Test.

Second Derivative $f''(x)$

Differentiate $f'(x)$:

$$
f''(x) = \frac{d}{dx} [84 e^x (1 + x)] = 84 \frac{d}{dx} [e^x (1 + x)]
$$

Apply product rule again:

$$
f''(x) = 84 [e^x (1 + x) + e^x \cdot 1] = 84 e^x [(1 + x) + 1] = 84 e^x (x + 2)
$$

Evaluate at $x = -1$:

$$
f''(-1) = 84 e^{-1} (-1 + 2) = 84 e^{-1} (1) > 0
$$

Since $f''(-1) > 0$, the critical point at $x = -1$ is a local minimum.

Intervals of Increase and Decrease

- For $x < -1$:

$$
f'(x) = 84 e^x (1 + x)
$$

Since $e^x > 0$, the sign depends on $1 + x$:

- When $x < -1$, $1 + x < 0$, thus $f'(x) < 0$.

Decreasing interval: $(-\infty, -1)$

- When $x > -1$:

- $1 + x > 0$, so $f'(x) > 0$.

Increasing interval: $(-1, \infty)$

Summary of Increasing/Decreasing Behavior

- Decreasing on $(-\infty, -1)$
- Increasing on $(-1, \infty)$

Critical Point and Extrema

- At $x = -1$, $f(x)$ attains a local minimum.

Calculate $f(-1)$:

$$
f(-1) = 84 \times (-1) \times e^{-1} = -84 \times \frac{1}{e} = -\frac{84}{e}
$$

This is the minimum value in the neighborhood.

End Behavior and Range

As $x \to -\infty$:

- $84x \to -\infty$
- $e^x \to 0$
- Product $f(x) = 84x e^x$:

Since $e^x \to 0$ faster than $x \to -\infty$, the product approaches 0:

$$
\lim_{x \to -\infty} 84x e^x = 0
$$

but from the negative side, because $84x$ is negative and small positive e^x:

$$
f(x) \to 0^- \quad \text{(approaching zero from below)}
$$

As $x \to \infty$:

- Both $84x$ and e^x approach infinity, so:

$$
f(x) \to +\infty
$$

Range of $f(x)$:

- The function attains all values greater than or equal to its minimum $-\frac{84}{e}$, as the function is decreasing from 0 to $-\frac{84}{e}$ when x goes from $-\infty$ to -1, and then increases from $-\frac{84}{e}$ to $+\infty$.

- The function approaches 0 from below as $x \to -\infty$, but never reaches 0; similarly, it approaches infinity as $x \to \infty$.

Thus, the range is:

$$
\boxed{\left[-\frac{84}{e}, \infty \right)}
$$

Summary of Key Features

Feature	Details
Domain	$\boxed{(-\infty, \infty)}$
Critical Point	$x = -1$
Local Minimum	$f(-1) = -\frac{84}{e}$
Intervals of Increase	$\boxed{(-1, \infty)}$
Intervals of Decrease	$\boxed{(-\infty, -1)}$
Range	$\boxed{\left[-\frac{84}{e}, \infty \right)}$

Graphical Interpretation

The graph of $f(x) = 84x e^x$ can be visualized with the following characteristics:

- Approaches zero from below as $x \to -\infty$.
- Has a minimum point at $x = -1$ with value $-\frac{84}{e}$.
- Increases without bound for $x > -1$.
- The function is continuous and differentiable everywhere, exhibiting a smooth curve with one local minimum.

Additional Calculus Applications

Solving for $f(x) = k$ in Interval Notation

Suppose we want to solve $f(x) = k$ for specific values of k. For example:

- For $k > -\frac{84}{e}$, the equation $84x e^x = k$ has:

- One solution in $(-\infty, -1)$ if $k < 0$,

- One solution in \((-1, \infty)\),
- Or possibly no solutions depending on the value of \(k\).

Since the function is increasing on \((-1, \infty)\), for each \(k \geq -\frac{84}{e}\) there is exactly one \(x\) satisfying \(f(x) = k\).

Solving Inequalities

- To find where \(f(x) \geq k\):

- If \(k > -\frac{84}{e}\), then:

\[
x \in \left[x_1, \infty \right)
\]

where \(x_1\) is the solution to \(f(x) = k\) in \((-1, \infty)\).

- If \(k \leq -\frac{84}{e}\), then:

\[
x \in (-\infty, x_2]
\]

where \(x_2\) is the solution to \(f(x) = k\) in \((-\infty, -1)\).

Solving for Critical Points and Extrema in Interval Notation

Critical point:

\[
x \in \boxed{ \{-1\} }
\]

Maximum and minimum intervals:

- No maximum (

Frequently Asked Questions

What is the function definition for f(x) in the given problem?

f(x) = (84x) e^x

How do you find the critical points of f(x) = (84x) e^x?

First, differentiate f(x) to find f'(x), set f'(x) = 0, and solve for x to find critical points.

What is the derivative f'(x) of the function f(x) = (84x) e^x?

Using the product rule, $f'(x) = 84 e^x + 84x e^x = 84 e^x (x + 1)$.

How do you determine the intervals where f(x) is increasing or decreasing?

Analyze the sign of $f'(x) = 84 e^x (x + 1)$; since $84 e^x > 0$ for all x, the sign depends on $(x + 1)$.

What is the critical point of f(x) based on the derivative f'(x) = 84 e^x (x + 1)?

Set $x + 1 = 0$, so the critical point is at $x = -1$.

At x = -1, is f(x) a maximum or minimum?

Since f'(x) changes from positive to negative at $x = -1$, it indicates a local maximum.

How do you find the maximum value of f(x) on a given interval?

Evaluate f(x) at critical points within the interval and at the endpoints; the largest value among these is the maximum.

What is the value of f(x) at the critical point x = -1?

f(-1) = (84 -1) e^{-1} = -84 / e.

In what interval notation should answers about the domain or solution intervals be entered?

Answers should be entered in standard interval notation, such as $(-\infty, \infty)$, $[-1, 2]$, etc.

Additional Resources

Suppose That f(x) = (84x) e^x. Note: Several parts of this problem require answers entered in interval notation.

An In-Depth Exploration of the Function \(f(x) = 84x e^x\)

Understanding complex functions is fundamental in calculus, as they often model real-world phenomena ranging from growth processes to physical systems. In this article, we delve into the properties, behavior, and various characteristics of the function $f(x) = 84x e^x$. We aim to provide a comprehensive, analytical overview, emphasizing concepts such as domain, range, critical points, inflection points, and asymptotic behaviors. As part of this exploration, certain conclusions will be expressed in interval notation, aligning with the problem's instructions.

---

1. Introduction to the Function

1.1 Function Definition

The function in question is:

$$
f(x) = 84x e^x
$$

Here, x is a real number, and the function combines a linear component $84x$ with an exponential component e^x. The coefficient 84 scales the function but does not affect its fundamental properties.

1.2 Context and Applications

Functions of this form appear in various scientific contexts:

- Population dynamics, modeling growth with linear modifiers.
- Radioactive decay or growth with linear factors.
- Economics, where combined linear and exponential growth factors are used to model investments or market behavior.
- Physics, especially in problems involving exponential decay or growth modulated by linear terms.

Understanding the behavior of $f(x)$ is essential for analyzing such phenomena, especially when predicting maxima, minima, or asymptotic behavior.

---

2. Domain and Range of $f(x)$

2.1 Domain

Since the function involves the exponential e^x, which is defined for all real numbers, and the linear term $84x$ is also defined everywhere, the domain is:

$$
\boxed{\text{Domain: } (-\infty, \infty)}
$$

2.2 Range

To find the range, analyze the behavior of $f(x)$ as $x \to \pm \infty$.

- As $x \to -\infty$:

$$
e^x \to 0, \quad 84x \to -\infty
$$

The product $84x e^x \to 0^-$ because $84x$ tends to $-\infty$ and e^x tends to 0; their product tends to 0 from below.

- As $x \to \infty$:

$$
84x \to \infty, \quad e^x \to \infty
$$

Thus, $f(x) \to \infty$.

- At $x=0$:

$$
f(0) = 84 \times 0 \times e^0 = 0
$$

- The function crosses zero at $x=0$, since $f(0)=0$.

Conclusion:

- The function approaches 0 from below as $x \to -\infty$, and reaches 0 at $x=0$.

- For $x > 0$, $f(x) > 0$ and increases to infinity.

- For $x<0$, $f(x) < 0$ (since $84x$ is negative, and $e^x > 0$), approaching 0 from below.

Therefore, the range is:

$$
\boxed{(-\infty, \infty)}
$$

since the function attains all real values (it is surjective over $\mathbb{R}$).

---

3. Critical Points and Extrema

3.1 Finding the Derivative

To analyze the function’s increasing and decreasing behavior, compute the first derivative

$f'(x)$:

$$
f(x) = 84x e^x
$$

Using the product rule:

$$
f'(x) = 84 \left(e^x + x e^x \right) = 84 e^x (1 + x)
$$

3.2 Critical Points

Critical points occur where $f'(x) = 0$ or where $f'(x)$ is undefined.

- Since $e^x \neq 0$ for all x, the critical points are solutions to:

$$
1 + x = 0 \Rightarrow x = -1
$$

- The derivative is defined everywhere, so no points of non-differentiability.

3.3 Analyzing $f'(x)$

- For $x > -1$:

$$
1 + x > 0 \Rightarrow f'(x) > 0
$$

The function is increasing.

- For $x < -1$:

$$
1 + x < 0 \Rightarrow f'(x) < 0
$$

The function is decreasing.

- At $x=-1$:

$$
f'(-1) = 0
$$

3.4 Nature of the Critical Point

To determine whether $x=-1$ is a maximum or minimum, examine the second derivative or use the first derivative test.

---

4. Concavity and Inflection Points

4.1 Second Derivative

Compute $f''(x)$:

$$
f'(x) = 84 e^x (1 + x)
$$

Apply the product rule again:

$$
f''(x) = 84 \left[e^x (1 + x) + e^x (1) \right] = 84 e^x \left(1 + x + 1 \right) = 84 e^x (x + 2)
$$

4.2 Concavity Analysis

- For $x > -2$:

$$
x + 2 > 0 \Rightarrow f''(x) > 0
$$

The function is concave upward.

- For $x < -2$:

$$
x + 2 < 0 \Rightarrow f''(x) < 0
$$

The function is concave downward.

- At $x=-2$:

$$
f''(-2) = 0
$$

4.3 Inflection Point

Since $f''(x)$ changes sign at $x = -2$, there is an inflection point at:

$$
x = -2

\]

The point of inflection can be found by evaluating $f(x)$ at $x=-2$:

$$
f(-2) = 84 \times (-2) \times e^{-2} = -168 e^{-2}
$$

---

5. Behavior at Infinity and Asymptotic Analysis

5.1 As $x \to -\infty$

- $f(x) \to 0^-$, approaching zero from below.

5.2 As $x \to \infty$

- $f(x) \to \infty$, increasing rapidly due to the exponential term.

5.3 Horizontal and Vertical Asymptotes

- The function does not have any vertical asymptotes, as it's defined everywhere.

- Horizontal asymptote at $y=0$ as $x \to -\infty$, since $f(x) \to 0^-$.

---

6. Summary of Key Features

Property	Behavior / Value	Explanation
Domain	$(- \infty, \infty)$	Entire real line
Range	$\mathbb{R}$	Attains all real numbers
Critical point	$x = -1$	Candidate for max/min
Nature of critical point	Local minimum	Since $f'(-1) = 0$ and $f''(-1) = 84 e^{-1} (-1 + 2) > 0$
Inflection point	$x = -2$, $f(-2) = -168 e^{-2}$	Concavity change occurs here
Increasing / decreasing intervals	Decreasing on $(-\infty, -1)$, increasing on $(-1, \infty)$	Based on sign of $f'(x)$
Concavity	Concave down on $(-\infty, -2)$, concave up on $(-2, \infty)$	Based on $f''(x)$
End behavior	Approaches 0 from below as $x \to -\infty$, diverges to $+\infty$ as $x \to \infty$	Asymptotic tendencies

---

7. Practical Applications and Interpretations

This function's behavior reveals insights into systems where linear and exponential factors interact:

- The initial decline (for \(x< -1\)) suggests a process that diminishes or stabilizes before growing rapidly.
-

[Suppose Thatf X 84x Ex Note Several Parts Of This Problem Require Answers Entered In Interval Notation]

Suppose Thatf X 84x Ex Note Several Parts Of This Problem Require Answers Entered In Interval Notation

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