The 10-g Bullet Having A Velocity Of 800 M>s Is Fired Into The Edge Of The 5-kg Disk As Shown. Determine

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Understanding the physics behind the impact of a high-velocity bullet on a rotating disk is essential for applications ranging from mechanical engineering to safety analysis. In this comprehensive guide, we will explore how to analyze such a collision, determine the resulting angular velocity of the disk, and interpret the physical implications of the interaction. This article aims to provide a detailed, step-by-step approach to solving this problem, ensuring clarity and depth for students, engineers, and enthusiasts alike.

Introduction to the Problem

The scenario involves a small, dense projectile—specifically, a 10-gram bullet—fired at a high velocity of 800 meters per second, striking the edge of a large, stationary disk weighing 5 kilograms. The key questions revolve around how this impact affects the disk's motion, especially its angular velocity post-collision.

Understanding this problem involves concepts from linear momentum, angular momentum, conservation laws, and rotational dynamics. Before delving into calculations, it’s essential to define the key parameters and assumptions.

Key Parameters and Assumptions

- Mass of the bullet, $m_b = 10\, \text{g} = 0.01\, \text{kg}$
- Velocity of the bullet before impact, $v_b = 800\, \text{m/s}$
- Mass of the disk, $M_d = 5\, \text{kg}$
- Radius of the disk, R (assumed or given)
- Initial angular velocity of the disk, $\omega_i = 0$ (assumed stationary)
- Impact is perfectly elastic or inelastic (assumption based on problem

statement)

- Frictional and other dissipative forces are negligible during impact

Note: The actual radius R of the disk is critical for calculations involving angular momentum. If the problem provides a specific radius, it should be used; otherwise, a typical value or symbolic notation should be employed.

Understanding the Physics: Conservation Laws and Dynamics

The core principles involved in analyzing this problem are:

Linear Momentum Conservation

Since the bullet strikes the edge of the disk, the linear momentum transferred influences the disk's motion. If the impact is purely tangential and frictionless, the linear momentum imparted to the disk's edge converts into rotational motion.

Angular Momentum Conservation

The impact imparts angular momentum to the disk. Assuming no external torques act during the collision, the total angular momentum before and after impact remains conserved.

Impulse-Momentum Relationship

The collision imparts an impulse J to the disk, related to the change in angular momentum:

$$
J = \Delta p = m_b v_b
$$

for the bullet, assuming it comes to rest relative to the disk or transfers all its momentum.

Step-by-Step Solution Approach

To determine the final angular velocity of the disk after impact, follow these steps:

1. Establish the Impact Conditions

- The bullet strikes the disk at the edge, tangentially.
- The initial velocity of the bullet is v_b.
- The initial angular velocity of the disk, $\omega_i = 0$.

2. Calculate the Impulse Delivered to the Disk

Assuming a perfectly inelastic collision where the bullet embeds into the disk (or transfers all momentum):

$$
J = m_b v_b
$$

This impulse is applied tangentially at the edge of the disk.

3. Determine the Change in Angular Momentum of the Disk

The impulse produces a change in the disk's angular momentum:

$$
\Delta L = J \times R
$$

where R is the radius of the disk.

Thus,

$$
\Delta L = m_b v_b R
$$

Since the initial angular momentum is zero (assuming the disk was stationary), the final angular momentum of the disk is:

$$
L_{final} = \Delta L = m_b v_b R
$$

\]

4. Calculate the Final Angular Velocity \(\omega_f\)

The angular momentum of a solid disk rotating about its center is:

\[
L = I \omega
\]

where \(I \) is the moment of inertia:

\[
I = \frac{1}{2} M_d R^2
\]

Rearranging for \(\omega_f\):

\[
\omega_f = \frac{L_{final}}{I} = \frac{m_b v_b R}{\frac{1}{2} M_d R^2} =
\frac{2 m_b v_b}{M_d R}
\]

Final formula:

\[
\boxed{
\omega_f = \frac{2 m_b v_b}{M_d R}
}
\]

This expression provides the angular velocity of the disk after impact, in radians per second.

Numerical Example

Suppose the radius of the disk is \(R = 0.5\, \text{m} \). Plugging in the known values:

\[
m_b = 0.01\, \text{kg}
\]
\[
v_b = 800\, \text{m/s}
\]
\[
M_d = 5\, \text{kg}

\]

$$
R = 0.5\, \text{m}
$$

Calculate ω_f:

$$
\omega_f = \frac{2 \times 0.01 \times 800}{5 \times 0.5} = \frac{16}{2.5} = 6.4\, \text{rad/s}
$$

Thus, the disk begins to rotate at approximately 6.4 radians per second after being struck.

Physical Interpretation and Practical Considerations

Understanding the result involves interpreting what this angular velocity means in real-world scenarios.

Effect of Impact Parameters

- Mass of the Bullet: A heavier or faster bullet imparts more angular momentum, increasing the disk's spin.
- Radius of the Disk: Larger disks (greater R) receive more angular momentum but also have higher moments of inertia, leading to lower angular velocities for the same impulse.
- Impact Location: Impact at the edge maximizes the transferred angular momentum; impacts closer to the center transfer less angular momentum.

Energy Considerations

While angular momentum is conserved during the collision, kinetic energy is generally not conserved due to energy losses (deformation, heat, sound). The rotational kinetic energy after impact is:

$$
KE_{rot} = \frac{1}{2} I \omega_f^2
$$

which, for the example:

\[

$$
KE_{rot} = \frac{1}{2} \times \frac{1}{2} M_d R^2 \times (6.4)^2
$$

Calculations can be performed to assess how much energy is transferred into rotational motion.

Applications and Implications

Understanding such impact dynamics is crucial in various fields:

- **Mechanical Engineering:** Design of rotating machinery and impact-resistant components.
- **Ballistics:** Predicting projectile behavior and impact effects.
- **Safety Engineering:** Assessing the consequences of high-velocity impacts.
- **Industrial Processes:** Material testing involving high-speed projectiles.

Advanced Topics and Further Considerations

While the above provides a foundational approach, real-world scenarios may involve more complex factors:

1. Elastic vs. Inelastic Collisions

- Elastic collisions conserve kinetic energy.
- Inelastic collisions involve energy loss, affecting the final velocities.

2. Friction and Damping Effects

- Friction at the contact point can influence the transfer of momentum.
- Damping forces may dissipate energy during impact.

3. Rotational Friction and Air Resistance

- These forces reduce the rotational velocity over time, especially important in long-term analysis.

4. Multiple Impacts and Continuous Rotation

- Repeated impacts or ongoing forces could alter the rotation over time, requiring dynamic analysis.

Conclusion

Analyzing the impact of a high-velocity bullet on a rotating disk involves applying principles of linear and angular momentum, impulse-momentum relationships, and rotational dynamics. By assuming ideal conditions and applying the derived formulas, we can estimate the resulting angular velocity of the disk accurately. This understanding not only enhances theoretical knowledge but also informs practical engineering applications where impact forces and rotational motion are critical factors.

In summary, the key takeaway is that the angular velocity imparted to the disk depends directly on the mass and velocity of the projectile, the radius of the disk, and the distribution of mass within the disk itself. Mastery of these concepts enables engineers and scientists to predict system behavior under high-impact conditions effectively.

Frequently Asked Questions

What is the initial velocity of the 10-g bullet before it hits the 5-kg disk?

The initial velocity of the bullet is 800 m/s.

How can the impact of the bullet on the disk be analyzed using conservation of momentum?

By applying the principle of conservation of linear momentum, assuming no external forces, the combined velocity after impact can be calculated from the initial velocities and masses.

What is the significance of the bullet's velocity in determining if it penetrates or causes rotation of the disk?

The velocity determines the kinetic energy transferred to the disk, which influences whether the disk will rotate, deform, or be penetrated upon impact.

How do we calculate the angular velocity of the disk after the collision?

Using conservation of angular momentum, the angular velocity can be found by equating the initial linear momentum of the bullet to the rotational momentum of the disk after impact.

What assumptions are typically made in solving such collision problems?

Assumptions often include a perfectly inelastic or elastic collision, no external forces during impact, and the disk being initially at rest.

How does the mass of the disk influence its response to the bullet's impact?

A larger mass results in less acceleration or rotation for the same impact force, making the disk more resistant to movement or deformation.

What is the role of the impact point on the disk in the resulting motion?

The impact point determines the torque applied, affecting whether the disk starts to spin and at what angular velocity.

Can the kinetic energy of the bullet be calculated for this problem?

Yes, using the formula $KE = 0.5\ m\ v^2$, where m is the mass of the bullet and v is its velocity.

What additional information is needed to fully solve the problem?

Details such as the point of impact on the disk, whether the collision is elastic or inelastic, and the initial state of the disk are needed for a complete solution.

How does the conservation of energy relate to this collision problem?

In real collisions, some kinetic energy may be lost as heat, sound, or deformation; thus, energy conservation applies only in ideal elastic collisions, not necessarily in inelastic impacts.

Additional Resources

The 10-g Bullet Having A Velocity Of 800 m/s Is Fired Into The Edge Of The 5-kg Disk As Shown. Determine – this problem encapsulates fundamental principles of physics, particularly the conservation of momentum and rotational dynamics. Understanding how a projectile interacts with a rotational body like a disk is essential in fields ranging from mechanical engineering to ballistics. In this comprehensive guide, we'll walk through the steps to analyze such an impact, explore the physics concepts involved, and demonstrate how to compute the resulting motion of both the bullet and the disk.

Introduction

When a high-velocity projectile strikes a rotating or stationary disk, the physical interactions can become quite complex. These interactions involve conservation of momentum, impulse, rotational motion, and sometimes energy transfer. By carefully analyzing these principles, we can determine the disk's angular velocity post-impact, the bullet's velocity after impact (if it continues), or other quantities of interest.

In this scenario, a 10-gram bullet traveling at 800 m/s strikes the edge of a 5-kg disk. The specific question is to determine the resulting motion—most likely the angular velocity of the disk after impact, or perhaps the final velocity of the bullet, depending on the context provided.

Key Concepts and Principles

Before diving into the calculations, it's important to understand the core physics principles involved:

1. Conservation of Linear Momentum
- When no external horizontal forces act during the brief collision, the total linear momentum before impact equals the total after.

2. Conservation of Angular Momentum
- If the disk is free to rotate, the angular momentum about its center is conserved during the interaction, assuming no external torques act during the impact.

3. Impulse-Momentum Theorem
- Impulse imparted during collision changes the momentum of objects involved:

Impulse = Change in momentum = Force × time

4. Rotational Dynamics
- The disk's moment of inertia (I) determines how it responds to an applied

torque:

$$I = (1/2) \times m \times r^2 \text{ (for a solid disk)}$$

5. Impact at the Edge
- Striking at the edge introduces angular effects, as the point of impact is at radius r, producing torque and angular velocity changes.

Step-by-Step Analysis

Let's proceed step-by-step, assuming typical problem data, and outlining how to compute the unknown quantities.

Step 1: Identify Known Data
- Bullet mass, $m_b = 10$ g = 0.01 kg
- Bullet velocity, $v_b = 800$ m/s
- Disk mass, $m_d = 5$ kg
- Disk is initially at rest (unless specified otherwise)
- Radius of the disk, r (assumed or given)

Note: If the radius isn't given, a typical value (say, r = 0.5 m) can be assumed or specified in the problem.

Step 2: Determine the System's Initial Momentum
- The initial linear momentum of the bullet:

$$p_{b\_initial} = m_b \times v_b = 0.01 \text{ kg} \times 800 \text{ m/s} = 8 \text{ kg·m/s}$$

- The disk's initial momentum is zero if initially at rest.

Step 3: Impact Dynamics

Scenario 1: The bullet embeds into the disk (inelastic collision)
- The bullet sticks to the disk, and they move together afterward.
- Conservation of linear momentum applies:

$$m_b \times v_b = (m_b + m_d) \times v_{final}$$

- Solve for the final velocity:

$$v_{final} = (m_b \times v_b) / (m_b + m_d)$$

Scenario 2: The bullet bounces or causes rotation without embedding
- More complex, involving conservation of angular momentum.

Assuming the simpler case of embedding (common in such problems), proceed with the inelastic collision model.

Step 4: Calculate Final Velocity of the Disk and Bullet

Using the inelastic collision assumption:

$$v_{final} = (0.01 \text{ kg} \times 800 \text{ m/s}) / (0.01 \text{ kg} + 5 \text{ kg}) \approx 8 / 5.01 \approx 1.5968 \text{ m/s}$$

This is the linear velocity of both the disk and the embedded bullet immediately after impact.

Step 5: Determine the Angular Velocity of the Disk

Since the impact occurs at the edge:

- The linear velocity at the edge relates to angular velocity (ω):

$$v_{edge} = r \times \omega$$

- Rearranged:

$$\omega = v_{edge} / r$$

Assuming the bullet hits at the edge (r = 0.5 m):

$$\omega = 1.5968 \text{ m/s} / 0.5 \text{ m} \approx 3.1936 \text{ rad/s}$$

This is the angular velocity of the disk after impact.

Step 6: Confirming the Conservation of Angular Momentum

Alternatively, if the initial angular momentum is to be conserved, and the disk is initially stationary:

- The initial angular momentum:

$$L_{initial} = m_b \times v_b \times r$$

- The final angular momentum:

$$L_{final} = I \times \omega$$

Where the moment of inertia for a solid disk:

$I = (1/2) \times m_d \times r^2 = (1/2) \times 5\ kg \times (0.5\ m)^2 = 0.625\ kg\cdot m^2$

Set equal:

$$m_b \times v_b \times r = I \times \omega$$

Plug in values:

$$0.01\ kg \times 800\ m/s \times 0.5\ m = 0.625\ kg\cdot m^2 \times \omega$$

Calculate:

$$4\ kg\cdot m^2/s = 0.625\ kg\cdot m^2 \times \omega$$

Solve for ω:

$$\omega = 4 / 0.625 = 6.4\ rad/s$$

This indicates a higher angular velocity if angular momentum is conserved directly at the point of impact, which may suggest the impact causes the disk to spin faster than the linear momentum calculation indicates.

Note: The discrepancy highlights that the actual impact process and whether the bullet embeds or bounces affects the calculations. The problem's context or additional data (e.g., whether the bullet sticks, whether the disk is free to rotate, or whether external forces act) determines which model to use.

Additional Considerations

1. Energy Transfer and Losses
- Impact often involves energy losses due to deformation, heat, sound, or other dissipative effects.
- If the impact is perfectly elastic, kinetic energy is conserved; if perfectly inelastic, some energy is lost.

2. External Forces and Friction
- External torques or frictional forces can influence the final angular velocity.
- For idealized problems, these are often neglected.

3. Final Velocity of the Bullet
- If the bullet does not embed, its velocity after impact could be computed using conservation laws, considering whether it bounces or slips.

Summary of Key Steps to Solve Similar Problems

1. Identify data: Masses, velocities, radii, initial conditions.
2. Determine the impact type: Elastic, inelastic, or partially inelastic.
3. Apply conservation laws:
- Momentum (linear or angular)
- Impulse-momentum theorem
4. Calculate the relevant quantities:
- Final velocities (linear and angular)
- Rotational quantities (moment of inertia, angular velocity)
5. Consider energy conservation if applicable
6. Check assumptions: initial conditions, impact point, and whether external forces are negligible.

Final Remarks

Analyzing the collision between a projectile and a disk requires a careful understanding of both linear and rotational motion principles. By applying the conservation of momentum—either linear or angular—you can predict the post-impact velocities and rotations. Remember to verify your assumptions, especially regarding whether the collision is elastic or inelastic, and whether the object embeds or rebounds.

This problem exemplifies the importance of foundational physics principles in real-world applications, from designing rotating machinery to understanding ballistic impacts. Mastering these concepts enables engineers and physicists to analyze complex interactions accurately and develop innovative solutions.

Disclaimer: For precise answers, always refer to the specific problem diagram and data provided, including the radius of the disk and initial conditions. The above analysis assumes typical values and standard impact scenarios.

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