The Current In An Rl Circuit Builds Up To One-third Of Its Steady State Value In 5.70 S. Find The Inductive

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Understanding the behavior of RL (Resistor-Inductor) circuits is fundamental in electrical engineering and physics, especially when analyzing transient responses. When a voltage source is suddenly applied to an RL circuit, the current does not instantaneously reach its maximum value; instead, it gradually increases over time due to the inductor's opposition to sudden changes in current. This article explores how to determine the inductance in an RL circuit based on the time it takes for the current to reach a specific fraction of its steady-state value.

Introduction to RL Circuits and Transient Response

An RL circuit consists of a resistor (R) and an inductor (L) connected in series with a voltage source (V). When the circuit is energized, the current gradually increases from zero to its maximum value, called the steady-state current, which is determined by Ohm's law:

$$ I_{ss} = \frac{V}{R} $$

However, this process takes time and is characterized by a time constant, denoted as τ (tau), which depends on the resistance and inductance:

$$ \tau = \frac{L}{R} $$

The transient response describes how the current evolves over time, especially during the initial moments after the circuit is closed.

Mathematical Description of Current Build-Up in RL Circuits

The time-dependent current in an RL circuit when a voltage source V is suddenly applied at t=0 is given by:

$$ I(t) = I_{ss} \left(1 - e^{-\frac{t}{\tau}}\right) $$

Where:

- $I(t)$ is the current at time t,
- I_{ss} is the steady-state current,
- e is Euler's number (~2.71828),
- τ is the time constant.

This exponential behavior indicates that the current approaches its maximum value asymptotically, reaching approximately 63.2% of I_{ss} at $t = \tau$.

Problem Statement and Approach

Given that:

- The current in the RL circuit builds up to one-third ($\frac{1}{3}$) of its steady-state value,
- This occurs at $t = 5.70\, \text{seconds}$,
- The goal is to find the inductance L.

The key steps involve:

1. Using the current expression to relate the fractional current to the time and time constant.
2. Solving for the time constant τ.
3. Calculating the inductance L using the relation $\tau = \frac{L}{R}$.

Step-by-Step Solution

Step 1: Express the current in terms of the steady-state current

Since the current reaches one-third of the steady-state value at $t = 5.70\, \text{s}$:

$$I(t) = \frac{1}{3} I_{ss}$$

From the exponential formula:

$$\frac{1}{3} I_{ss} = I_{ss} \left(1 - e^{-\frac{t}{\tau}}\right)$$

Dividing both sides by I_{ss}:

$$\frac{1}{3} = 1 - e^{-\frac{t}{\tau}}$$

Rearranged:

$$ e^{-\frac{t}{\tau}} = 1 - \frac{1}{3} = \frac{2}{3} $$

Step 2: Solve for the time constant τ

Taking natural logarithms on both sides:

$$ -\frac{t}{\tau} = \ln \left(\frac{2}{3} \right) $$

$$ \Rightarrow \frac{t}{\tau} = - \ln \left(\frac{2}{3} \right) $$

Plugging in $t = 5.70\, \text{s}$:

$$ \tau = \frac{t}{- \ln \left(\frac{2}{3} \right)} $$

Calculate $\ln \left(\frac{2}{3} \right)$:

$$ \ln \left(\frac{2}{3} \right) \approx \ln(2) - \ln(3) \approx 0.6931 - 1.0986 = -0.4055 $$

Thus:

$$ \tau = \frac{5.70}{- (-0.4055)} = \frac{5.70}{0.4055} $$

$$ \tau \approx 14.07\, \text{seconds} $$

Step 3: Find the inductance L

The relation between the time constant τ, resistance R, and inductance L is:

$$ \tau = \frac{L}{R} $$

However, in this problem, R is not directly provided. To find L, additional information about the circuit resistance is necessary.

Assumption: If the resistance R is known or given, then:

$$ L = R \times \tau $$

Without a given resistance, the problem cannot be fully solved unless the resistance R is provided or assumed.

Implications and Additional Considerations

The Importance of Resistance in RL Circuits

- The resistance R influences how quickly the current reaches its steady state.
- A higher resistance results in a larger time constant τ, meaning the current builds up more slowly.
- Conversely, a lower resistance results in a faster response.

Determining L with Known Resistance

Suppose the resistance R was provided or measured; then, calculating the inductance is straightforward:

$$L = R \times \tau$$

For example, if:

- $R = 50\, \Omega$,
- $\tau \approx 14.07\, \text{s}$,

then:

$$L = 50 \times 14.07 = 703.5\, \text{H}$$

This example illustrates how knowing R is crucial for determining L.

Real-World Applications of RL Circuit Analysis

RL circuits are fundamental in various electrical and electronic applications, such as:

- Filters: RL filters are used to block or pass specific frequency ranges.
- Transformers and inductors: Components that manage energy transfer and storage.
- Pulse circuits: Managing transient responses in digital and analog circuits.
- Motor control: Inductive properties influence how motors respond to voltage changes.

Understanding how to analyze transient responses and determine inductance based on time-dependent current behavior is essential for designing and troubleshooting these systems.

Summary and Key Takeaways

- The exponential growth of current in an RL circuit is characterized by the time constant $\tau = \frac{L}{R}$.
- When current reaches a specific fraction of its steady-state value, this time can be used to calculate τ.
- With τ known and resistance R specified, the inductance L can be directly computed.
- Precise knowledge of circuit parameters allows engineers and technicians to predict circuit behavior effectively.

Conclusion

In this problem, the key step was relating the fractional current value to the exponential growth function and solving for the time constant τ. Given that the current reaches one-third of its steady-state value in 5.70 seconds, the calculated τ is approximately 14.07 seconds. Without the resistance R, the inductance L cannot be explicitly determined, but the relationship $L = R \times \tau$ provides a clear pathway once R is known. This approach exemplifies the importance of understanding transient responses in RL circuits and highlights how fundamental principles of electrical engineering enable precise analysis and design of complex systems.

Note: For comprehensive problem-solving, always ensure all circuit parameters are provided. If resistance is unknown, additional measurements or circuit details are necessary to find the inductance accurately.

Frequently Asked Questions

What is the significance of the time constant in an RL circuit when the current builds up?

The time constant (τ) indicates the time taken for the current to reach approximately 63.2% of its final steady-state value in an RL circuit.

How can we determine the inductance in an RL circuit if the current reaches one-third of its steady-state value in a given time?

Use the exponential current growth formula and solve for L by relating the time to the fraction of steady-state current achieved, considering the known resistance.

What is the formula relating current growth in an RL circuit to resistance, inductance, and time?

The current at time t is given by $I(t) = I_{final} (1 - e^{-t/\tau})$, where $\tau = L/R$.

Given that the current reaches one-third of its steady-state value in 5.70 seconds, how do we find the inductance L?

Set $I(t)/I_{final} = 1/3$ and $t = 5.70$ s in the formula, then solve for L using $\tau = -t / \ln(1 - 1/3)$.

What is the resistance value needed to find the inductance in this RL circuit?

The resistance value must be known or given; without it, the inductance cannot be calculated directly, as the time constant depends on both R and L.

If the resistance is not provided, how can the inductance be determined from the given data?

Additional information about the resistance is necessary; without R, the inductance cannot be uniquely determined from the data.

Why does the current in an RL circuit build up exponentially rather than instantly?

Because of the inductance, which opposes changes in current, causing a gradual exponential increase rather than an instantaneous change.

What is a typical approximation for the time when the current reaches one-third of its steady-state value?

It occurs at approximately 0.36τ, since $1 - e^{-t/\tau} = 1/3$ leads to $t \approx 0.36\tau$.

How does the value of the inductance affect the rate at which current builds up in an RL circuit?

A larger inductance results in a longer time constant, meaning the current takes more time to reach its steady-state value.

Can the inductance be determined directly from the time it takes to reach one-third of the steady-state current without resistance data?

No, since the time constant depends on both inductance and resistance, resistance must be known to accurately calculate the inductance.

Additional Resources

Understanding the Build-Up of Current in an R-L Circuit: A Deep Dive into Inductive Reactance and Transient Response

When analyzing electrical circuits, especially those involving resistors and inductors, understanding how current evolves over time is fundamental. The problem statement—"The current in an R-L circuit builds up to one-third of its steady-state value in 5.70 seconds"—provides a classic scenario to explore the transient response of inductive circuits and how to determine the circuit's inductance (L). This discussion aims to dissect this problem comprehensively, delving into the physics, mathematics, and practical implications involved.

Fundamentals of R-L Circuits and Transient Response

An R-L circuit consists of a resistor (R) and an inductor (L) connected in series with a voltage source (V). When the circuit is initially energized, the current does not instantly reach its maximum value due to the inductor's opposition to changes in current, a phenomenon known as inductive reactance.

Key concepts:

- Steady-State Current (I_ss): The maximum current after a long time, when the inductor behaves like a short circuit.
- Transient Response: The period during which the current transitions from zero (at the instant of switching on) to its steady-state value.
- Time Constant (τ): Characteristic time that defines how quickly the current approaches its steady state, mathematically expressed as $\tau = \frac{L}{R}$.

Mathematical Foundation of the Transient Current Build-Up

The current $i(t)$ in a series R-L circuit when a voltage V is suddenly applied at $t=0$ is governed by the differential equation:

$$
V = L \frac{di(t)}{dt} + R i(t)
$$

The solution to this differential equation, assuming zero initial current, is:

$$
i(t) = I_{ss} \left(1 - e^{-\frac{t}{\tau}}\right)
$$

where:

$$
I_{ss} = \frac{V}{R}
$$

and

$$
\tau = \frac{L}{R}
$$

This exponential approach describes how the current asymptotically approaches the steady-state value over time.

Relating the Given Data to the Circuit Parameters

The problem states: "The current builds up to one-third of its steady-state value in 5.70 seconds." From this, we can establish:

$$
i(t) = \frac{1}{3} I_{ss} \quad \text{at} \quad t = 5.70\, \mathrm{s}
$$

Using the exponential current build-up equation:

$$
\frac{1}{3} I_{ss} = I_{ss} \left(1 - e^{-\frac{t}{\tau}}\right)
$$

Dividing both sides by I_{ss}:

$$
\frac{1}{3} = 1 - e^{-\frac{t}{\tau}}
$$

Rearranged:

$$
e^{-\frac{t}{\tau}} = 1 - \frac{1}{3} = \frac{2}{3}
$$

Taking natural logarithms:

$$
-\frac{t}{\tau} = \ln \left(\frac{2}{3}\right)
$$

$$
\frac{t}{\tau} = -\ln \left(\frac{2}{3}\right)
$$

Calculating $\ln \left(\frac{2}{3}\right)$:

$$
\ln \left(\frac{2}{3}\right) \approx -0.4055
$$

Therefore:

$$
\frac{t}{\tau} = 0.4055
$$

Given $t = 5.70\, \mathrm{s}$:

$$
\tau = \frac{t}{0.4055} \approx \frac{5.70}{0.4055} \approx 14.06\, \mathrm{s}
$$

This value of τ is crucial as it directly leads to the calculation of the inductance once the resistance is known or assumed.

Determining the Inductive Reactance and Inductance (L)

The key to finding the inductance L lies in understanding the circuit's resistance R and the steady-state current I_{ss}. Typically, if the voltage source or steady-state current is not explicitly provided, we assume standard conditions or focus on the relationship between L and R.

Scenario 1: Resistance Known

- If the resistance R is known, then:

$$
L = R \times \tau
$$

- Using the calculated $\tau \approx 14.06\, \mathrm{s}$, the inductance can be directly computed.

Scenario 2: Resistance Unknown

- If the resistance is not specified, the problem may be designed to determine the inductance in terms of R. For example:

$$
L = R \times 14.06\, \mathrm{s}
$$

- Additional data, such as the steady-state current I_{ss} or applied voltage V, would be necessary to find numerical values.

Physical Interpretation and Practical Implications

Understanding how the current builds up over time in an R-L circuit is not only an academic exercise but also vital in practical circuit design:

- Switching Transients: When switches are closed, the current does not immediately reach its maximum, causing voltage spikes and transient effects that can impact sensitive components.
- Inductor Design: The value of L influences how quickly current reaches desired levels, affecting circuit response times.
- Filtering and Signal Processing: R-L circuits are fundamental in creating filters, where transient behavior determines filter characteristics.

Practical considerations:

- Ensuring that the resistor and inductor are rated to handle the transient voltages.
- Using the time constant to design circuits with specific response times.
- Recognizing that larger inductances lead to longer transient periods, which might be desirable or undesirable depending on application.

Additional Factors and Assumptions in the Problem

In resolving this problem, several assumptions are often made:

- Ideal Components: The inductor is assumed to be ideal (no parasitic resistance), unless specified.
- Constant Voltage Source: The voltage V remains steady during the transient.
- Initial Conditions: The initial current at $t=0$ is zero, typical for a circuit just energized.
- Resistance and Inductance Relationship: The resistance remains constant, and the circuit parameters are linear.

In real-world scenarios, factors such as parasitic resistance in inductors, temperature variations, and non-ideal voltage sources can influence the transient response.

Concluding Remarks and Summary

This exploration of the current build-up in an R-L circuit highlights the interplay between resistance, inductance, and transient behavior. The key steps involve:

1. Recognizing the exponential nature of current growth.
2. Using the given fractional value (one-third) to find the time constant \(\tau\).
3. Relating \(\tau\) to the inductance \(L\) via the resistance \(R\).

The calculated time constant of approximately 14.06 seconds signifies a relatively slow transient, indicating either a large inductance or high resistance. This insight is crucial for designing circuits where transient response times are critical, such as in power electronics, signal filtering, and electromagnetic device operation.

In essence, understanding how the current in an R-L circuit builds up allows engineers and physicists to tailor circuit parameters for desired dynamic behavior, ensuring functionality, safety, and efficiency in practical applications.

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