The Volume Of The Solid Generated By Revolving The Regron Enclosed By The Trangle With Vertices (4,1),(4,6)

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Introduction

Revolutions of regions about axes are a fundamental concept in integral calculus, particularly in the calculation of volumes of solids of revolution. When a region bounded by certain curves or lines is revolved around an axis, it generates a three-dimensional solid. Determining the volume of such a solid involves applying methods like the disk/washer method or the shell method, which rely on setting up appropriate integrals.

In this article, we explore the problem of finding the volume of the solid generated by revolving a specific region—bounded by a triangle with vertices at (4, 1) and (4, 6)—about a particular axis. Although the problem statement specifies only two vertices, we interpret it as referring to a triangle with vertices at (4, 1), (4, 6), and another point that completes the triangle, possibly the x-axis or another line. Our goal is to understand, define, and compute this volume in detail.

Understanding the Region and Its Boundaries

Identifying the Vertices and the Region

The problem mentions a triangle with vertices at (4, 1) and (4, 6). To define a triangle, a third vertex must be specified. Common interpretations could include:

- The third vertex at (x, 0), forming a triangle with the base on the x-axis.
- The third vertex at some other point on the plane, such as (x, y) with different coordinates.

For the purpose of this analysis, we'll assume the triangle is bounded by:

- The vertical line x = 4, connecting (4, 1) and (4, 6).
- The line segment connecting these points along the y-axis or another line,

depending on the context.

Possible interpretations:

- Scenario 1: The triangle is bounded by x = 4, the x-axis (y=0), and the line connecting (4, 1) and (4, 6). But since both points share the same x-coordinate, this would be a line segment vertical at x=4, which is not sufficient for a triangle boundary.

- Scenario 2: The triangle is bounded between x = 4 and some other x-value, with the vertices at (4, 1), (4, 6), and another point, say (x0, y0). Alternatively, perhaps the problem refers to the triangle with vertices at (4, 1), (4, 6), and (x, 1), which would be a right triangle.

Given the ambiguity, we will interpret the problem as referring to a region enclosed between the vertical line x=4 and the x-axis between y=1 and y=6, forming a vertical strip. To proceed, we will analyze the region bounded by:

- The vertical line x = 4,
- The horizontal lines y=1 and y=6,
- The x-axis (y=0), and
- The line x=0 (the y-axis).

This forms a rectangular region between x=0 and x=4, and y=1 and y=6, which can be revolved around a specified axis.

Alternatively, for clarity, suppose the triangle has vertices at:

- (4, 1),
- (4, 6),
- (x, 1), where x<4.

This forms a triangle with a base along y=1 between (x, 1) and (4, 1), and a vertical side at x=4 from y=1 to y=6.

For the purpose of this detailed analysis, we'll assume the region is bounded by:

- The vertical line x=4,
- The horizontal line y=1,
- The horizontal line y=6,
- The line connecting (x=0, y=1) to (x=4, y=1),
- And the line connecting (0, 1) to (0, 6), as well as (4, 6).

Thus, the region is a rectangle between x=0 and x=4, and y=1 and y=6.

Defining the Region Precisely

To proceed, we'll define the region R as:

- Bounded on the left by the line x=0,
- Bounded on the right by the line x=4,
- Bounded below by y=1,
- Bounded above by y=6.

This rectangle is then revolved about an axis—say, the x-axis or y-axis—to generate a solid.

Choosing the Axis of Revolution

The problem statement does not specify the axis about which the region is revolved. Common axes include:

- The x-axis (y=0)
- The y-axis (x=0)
- Other vertical or horizontal lines

Given the vertices are at x=4, it is natural to consider revolving around the x-axis for simplicity.

Revolution About the x-Axis

In this case, the region between x=0 and x=4, from y=1 to y=6, is revolved around y=0, creating a three-dimensional solid.

Calculating the Volume of the Solid

Methodology Overview

To find the volume of the solid generated by revolving the region around the x-axis, we employ the disk method.

Disk method principle:
- The volume of the solid is obtained by integrating the cross-sectional areas perpendicular to the axis of revolution.
- For a horizontal strip at height y, the radius of the disk is the x-value at that y, and the area is π(radius)^2.
- Alternatively, if the region is between x=0 and x=4, and the y-values are between 1 and 6, then the volume can be integrated with respect to y.

In our case, since the region is a rectangle, the cross-section at a given y is a line segment from x=0 to x=4 (constant), so the radius is 4, and the cross-sectional area is $\pi(4)^2 = 16\pi$.

But this would imply the entire rectangle is revolved, which results in a cylindrical shell of fixed radius.

Alternatively, for a more general case, suppose the boundary between x and y is given by functions, and we want to consider the actual varying shape.

Setting Up the Integral

Assuming the region is bounded by:

- $x=0$,
- $x=4$,
- $y=1$,
- $y=6$.

and revolved about the x-axis, the volume is calculated as:

$$V = \int_{y=1}^{6} A(y)\, dy,$$

where $A(y)$ is the cross-sectional area at height y.

Since the cross-section at y is a line segment from $x=0$ to $x=4$, the area of the cross-section is a rectangle with width 4 and height negligible (since it's a thin slice).

Alternatively, using the shell method:

- For shell method, consider vertical shells at x, with height from $y=1$ to $y=6$.

- The volume is:

$$V = \int_{x=0}^{4} 2\pi\, x\, \text{height}(x)\, dx,$$

where $\text{height}(x) = 6 - 1 = 5$ (constant).

Thus,

$$V = 2\pi \int_{0}^{4} x\, 5\, dx = 10\pi \int_{0}^{4} x\, dx = 10\pi \left[x^2/2\right]_{0}^{4} = 10\pi (16/2) = 10\pi\, 8 = 80\pi.$$

Therefore, the volume obtained by revolving the rectangle between $x=0$ and $x=4$, $y=1$ and $y=6$, about the y-axis is 80π cubic units.

Generalizing for Different Boundaries and Revolving Axes

Revolving the Region About the y-Axis

- When revolving around y=0, the shell method applies with respect to x.
- The volume is:

V = 2π ∫_{x=a}^{b} x height(x) dx.

- Here, with the rectangle between x=0 and x=4, y=1 to y=6, the volume is 80π, as shown.

Revolving the Region About a Vertical Line x = c

- The shell method can be adapted for axes other than x=0.
- The radius becomes |x - c|.

Conclusion and Summary

The calculation of the volume of the solid generated by revolving a region enclosed by a triangle with specific vertices depends heavily on the region's precise boundaries and the axis of revolution.

In the interpreted scenario where the region is a rectangle between x=0 and x=4, y=1 and y=6, revolving around the y-axis yields a volume of 80π cubic units.

Key points:

- Determining the exact region is crucial for setting up the integral correctly.
- The shell method is often more straightforward for regions bounded between vertical lines and revolved around the y-axis.
- The disk/washer method is suitable when revolving around the x-axis or horizontal lines

Frequently Asked Questions

How do I set up the integral to find the volume of the solid generated by revolving the region bounded by the triangle with vertices (4,1) and (4,6)?

First, identify the line segment connecting the vertices; since both have x=4, the region is a vertical line segment from y=1 to y=6. To generate a solid by revolving this region around the x-axis, you need to express y as a function of x or vice versa, and then set up the integral accordingly. Typically, if revolving around the x-axis, you would integrate with respect to y, using the radius as the x-value, to find the volume using the washer method.

What is the formula for calculating the volume of the solid formed by revolving the given triangle about the x-axis?

Since the region is a vertical line segment at x=4 between y=1 and y=6, revolving around the x-axis creates a cylindrical shell with radius y and height equal to the difference in x-values. But because the region is a line at x=4, the volume is essentially a cylinder with radius 4 and height 5 (from y=1 to y=6). The volume is then V = π (radius)^2 height = π 4^2 5 = 80π units^3.

Can I use the disk or washer method to find the volume in this scenario?

In this specific case, since the region is a vertical line segment at x=4, revolving around the x-axis creates a circular cross-section with radius equal to the x-coordinate. The volume can be computed as a cylinder with radius 4 and height 5, so a direct application of the cylinder formula is appropriate. For more complex regions, the disk or washer methods are used, but here, a simple cylinder formula suffices.

What are the implications of the vertices (4,1) and (4,6) on the shape of the generated solid?

The vertices indicate the region is a vertical line segment at x=4 between y=1 and y=6. When revolved about the x-axis, this creates a circular solid with a fixed radius of 4 (the x-coordinate), extending along the y-interval from 1 to 6. The resulting solid is a cylinder with height 5 and radius 4.

If the region was bounded by a line y=x+1 from x=4 to x=4, how would the volume calculation change?

If the region was bounded by y=x+1 between x=4 and x=4, which is a single point, then the region would have zero width, resulting in zero volume. However, if the region extended over an interval, you'd set up an integral with respect to x, revolving around the x-axis, integrating the area of disks with radius y=x+1, from x=4 to x=some endpoint.

What is the significance of the vertices (4,1) and (4,6) in understanding the boundaries of the region?

These vertices define a vertical line segment at x=4, with y-values from 1 to 6. They establish the region's position in the coordinate plane, indicating that the region is a line segment at a fixed x-value, which simplifies the process of calculating the volume of the solid generated by revolving this segment around an axis.

Additional Resources

The Volume of the Solid Generated by Revolving the Region Enclosed by the Triangle with Vertices (4, 1) and (4, 6)

Understanding how to compute the volume of a solid generated by revolving a region around an axis is a fundamental problem in integral calculus, especially within the context of volume by disks, washers, and shells. In this detailed guide, we will explore the volume of the solid generated by revolving the region enclosed by the triangle with vertices (4, 1) and (4, 6). This problem exemplifies the practical application of the disk and shell methods, illustrating how to set up integral expressions and interpret the geometric aspects involved.

Introduction to the Problem

Imagine a triangle in the coordinate plane with vertices at specific points, notably at (4, 1) and (4, 6). To visualize this, consider the following:

- The points (4, 1) and (4, 6) are both on the line $x = 4$.
- The third vertex can be inferred based on the problem context; often, in such problems, the third point is at (x, y) with y-values between 1 and 6, and potentially other constraints.

But since only two vertices are specified, it suggests that the region may be bounded between these two points and some line or boundary in the coordinate plane.

The key is to clarify the specific region enclosed. For the purpose of this analysis, let's assume the region enclosed is the triangle with vertices at:

- $A(4, 1)$
- $B(4, 6)$
- $C(a, 1)$, where a is some value $a < 4$.

However, given that the vertices are only (4, 1) and (4, 6), and the problem mentions 'the region enclosed by the triangle,' we need to specify the third point or boundary to fully define the triangle.

Clarifying the Region

Suppose the third point is at $(a, 1)$, with $a < 4$. The triangle would then be formed by the points:

- $(a, 1)$
- $(4, 1)$
- $(4, 6)$

This triangle is right-angled at $(4, 1)$ and is bounded by:

- The vertical line segment from $(4, 1)$ to $(4, 6)$
- The horizontal line segment from $(a, 1)$ to $(4, 1)$
- The hypotenuse connecting $(a, 1)$ to $(4, 6)$

This is a common setup in volume problems involving revolutions, especially when revolving around the x-axis or y-axis.

Step 1: Defining the Region

Geometric Description

- The region is a triangle with vertices at:
- $(a, 1)$
- $(4, 1)$
- $(4, 6)$

- The base runs along $y = 1$ from $x = a$ to $x = 4$.
- The height extends vertically from $y=1$ to $y=6$.

Assumptions

- The problem involves revolving this triangular region around a specific axis — most likely the x-axis or y-axis.
- The exact axis of revolution determines the method (disk/washer or shell).

For clarity, let's assume the region is revolved about the x-axis.

Step 2: Sketching and Visualizing the Region

Visualizing the region helps in selecting the proper method:

- Draw the triangle with vertices at $(a, 1)$, $(4, 1)$, and $(4, 6)$.
- The hypotenuse is the line connecting $(a, 1)$ and $(4, 6)$.
- The line equation for the hypotenuse is:

$$
\text{Line passing through } (a, 1) \text{ and } (4, 6): \\
\text{slope} = m = \frac{6 - 1}{4 - a} = \frac{5}{4 - a} \\
\Rightarrow y - 1 = m(x - a) \\
\Rightarrow y = 1 + \frac{5}{4 - a}(x - a)
$$

This line forms the upper boundary of the region.

Step 3: Choosing the Method of Integration

Disk/Washer Method

- Suitable if rotating around the x-axis and the region is expressed as a function of x.
- The volume element is a disk or washer with radius depending on y.

Shell Method

- Useful if rotating around the y-axis or when the region is better expressed as x in terms of y.
- The volume element is a shell with radius and height depending on the variables.

Given our assumptions:

- The region is bounded between $y=1$ and $y=6$.
- The boundary between $x=a$ and $x=4$ is linear.

We will proceed with the shell method for the rotation around the x-axis:

- The shells are formed by revolving vertical slices at different y values.

Step 4: Setting Up the Integral Using the Shell Method

Shell Method Principles

- For rotation about the x-axis, the shell radius is y, and the shell height is the x-extent at each y.
- The volume element:

$$
dV = 2 \pi \times \text{radius} \times \text{height} \times dy
$$

- The limits are from $y=1$ to $y=6$.

Express x in terms of y

From the line equation:

$$
y = 1 + \frac{5}{4 - a}(x - a) \\
\Rightarrow y - 1 = \frac{5}{4 - a}(x - a) \\
\Rightarrow x - a = \frac{4 - a}{5}(y - 1) \\
\Rightarrow x = a + \frac{4 - a}{5}(y - 1)
$$

- For each y, the left boundary is at $x = a + \frac{4 - a}{5}(y - 1)$.

- The right boundary is at $x = 4$.

Shell height

- When revolving about the x-axis, the shells are formed by slicing vertically from $x = a + \frac{4 - a}{5}(y - 1)$ to $x=4$.
- The radius of the shell at height y is simply y.

- The length of the shell (circumference) is $2 \pi y$.

- The height of each shell is the difference in x:

$$
\text{height} = 4 - \left(a + \frac{4 - a}{5}(y - 1) \right) = 4 - a - \frac{4 - a}{5}(y - 1)
$$

Volume integral

$$
V = \int_{y=1}^{6} 2 \pi y \times \left[4 - a - \frac{4 - a}{5}(y - 1) \right] dy
$$

Step 5: Simplifying the Integral

Expressed explicitly:

$$
V = 2 \pi \int_{1}^{6} y \left[4 - a - \frac{4 - a}{5}(y - 1) \right] dy
$$

Distribute y:

$$

V = 2 \pi \int_{1}^{6} \left[y(4 - a) - y \frac{4 - a}{5}(y - 1) \right] dy
\]

Rewrite:

\[
V = 2 \pi \left[(4 - a) \int_{1}^{6} y \, dy - \frac{4 - a}{5} \int_{1}^{6} y(y - 1) dy \right]
\]

Compute each integral separately.

Step 6: Computing the Integrals

Integral 1:

\[
\int_{1}^{6} y \, dy = \left[\frac{y^2}{2} \right]_1^6 = \frac{36}{2} - \frac{1}{2} = 18 - 0.5 = 17.5
\]

Integral 2:

\[
\int_{1}^{6} y(y - 1) dy = \int_{1}^{6} (y^2 - y) dy
\]

Calculate:

\[
\left[\frac{y^3}{3} - \frac{y^2}{2} \right]_1^6
\]

At \(y=6 \):

\[
\frac{216}{3} - \frac{36}{2} = 72 - 18 = 54

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