There Are 6 White Balls And 4 Red Balls In An Urn. Two Balls Are Selected From The Urn Without Replacement,

There Are 6 White Balls And 4 Red Balls In An Urn. Two Balls Are Selected From The Urn Without Replacement

Understanding the process of drawing balls from an urn is fundamental in the study of probability and statistics. This scenario, involving 6 white balls and 4 red balls with two draws made without replacement, provides a classic example to explore various probability concepts, including individual and joint probabilities, conditional probabilities, and the calculation of expected outcomes. This article delves into the details of this problem, offering comprehensive explanations, calculations, and insights to deepen understanding.

Basic Setup and Key Concepts

Initial Composition of the Urn

The urn contains:

- White Balls (W): 6
- Red Balls (R): 4

Total number of balls in the urn:
N = 6 + 4 = 10

Nature of the Drawing Process

- Two balls are drawn sequentially.
- The draws are made without replacement, meaning once a ball is drawn, it is not returned to the urn before the next draw.
- The order of drawing matters if considering specific sequences, but often the focus is on the overall probabilities of certain outcomes.

Sample Space and Possible Outcomes

- The total number of ways to select 2 balls from 10 (without regard to order):

$\binom{10}{2} = 45$

- If order is considered (drawing first, then second), the total number of ordered outcomes:

$10 \times 9 = 90$

Understanding whether we are analyzing unordered pairs or ordered sequences influences the calculation of probabilities.

Calculating Probabilities of Different Events

Probability of Drawing a White Ball First

- The probability that the first ball drawn is white:

$P(\text{White first}) = \frac{6}{10} = 0.6$

Probability of Drawing a Red Ball First

- The probability that the first ball drawn is red:

$P(\text{Red first}) = \frac{4}{10} = 0.4$

Probability of Drawing Two White Balls (W,W)

- Since the draws are without replacement:
- Probability first ball is white:

$\frac{6}{10}$

- Given the first is white, remaining white balls:

5

- Total remaining balls:

9

- Probability second ball is white:

\(\frac{5}{9} \)

- Therefore, the probability of both balls being white:

\(P(\text{W,W}) = \frac{6}{10} \times \frac{5}{9} = \frac{30}{90} = \frac{1}{3} \approx 0.333 \)

Probability of Drawing Two Red Balls (R,R)

- Similarly:
- First red:

\(\frac{4}{10} \)
- Remaining red:

3
- Remaining total:

9
- Second red:

\(\frac{3}{9} = \frac{1}{3} \)

- Total probability:

\(P(\text{R,R}) = \frac{4}{10} \times \frac{3}{9} = \frac{12}{90} = \frac{2}{15} \approx 0.133 \)

Probability of Drawing One White and One Red (in any order)

- Two possible sequences:
1. White first, Red second (W,R)
2. Red first, White second (R,W)
- Probability of W,R:

\(\frac{6}{10} \times \frac{4}{9} = \frac{24}{90} = \frac{4}{15} \approx 0.267 \)
- Probability of R,W:

\(\frac{4}{10} \times \frac{6}{9} = \frac{24}{90} = \frac{4}{15} \approx 0.267 \)
- Total probability of one white and one red:

\(P(\text{One white, one red}) = \frac{4}{15} + \frac{4}{15} = \frac{8}{15}

≈ 0.533

Conditional Probabilities and Outcomes

Probability of the Second Ball Being Red Given the First is White

- Given the first ball was white, remaining:
- White balls: 5
- Red balls: 4
- Total remaining: 9
- Probability that the second ball is red:

$P(\text{Red second} \mid \text{White first}) = \frac{4}{9} \approx 0.444$

Probability of the Second Ball Being White Given the First is Red

- Given the first ball was red:
- White balls: 6
- Red balls: 3
- Total remaining: 9
- Probability that the second ball is white:

$\frac{6}{9} = \frac{2}{3} \approx 0.667$

Implication of Conditional Probabilities

Such probabilities help in understanding how the initial draw influences the subsequent outcomes, which is crucial in applications like Bayesian updating, risk assessment, and decision-making processes involving sequential events.

Expected Values and Other Statistical Measures

Expected Number of White Balls Drawn in Two Draws

- Since only two balls are drawn, the expected number of white balls can be calculated:

- Probability of drawing 0 white balls (both red): $P(\text{R,R}) = 2/15$

- Probability of drawing 1 white ball:
- White first, red second:

$\frac{6}{10} \times \frac{4}{9} = 4/15$
- Red first, white second:

$\frac{4}{10} \times \frac{6}{9} = 4/15$
- Total:

$8/15$
- Probability of drawing 2 white balls:

$1/3$

- Expected number of white balls:

$E(\text{White}) = 0 \times P(0) + 1 \times P(1) + 2 \times P(2)$

$E(\text{White}) = 0 \times \frac{2}{15} + 1 \times \frac{8}{15} + 2 \times \frac{1}{3}$

- Note that $\frac{1}{3} = \frac{5}{15}$

$E(\text{White}) = 0 + \frac{8}{15} + 2 \times \frac{5}{15} = \frac{8}{15} + \frac{10}{15} = \frac{18}{15} = 1.2$

This indicates that, on average, drawing two balls from the urn will yield approximately 1.2 white balls.

Applications and Broader Implications

Real-World Scenarios

This type of probability problem can model various real-world situations, such as:
- Quality control in manufacturing (selecting defective vs. non-defective items).
- Card games and gambling (drawing specific suits or ranks).
- Biological sampling (selecting species or traits).
- Computer algorithms involving random sampling.

Importance in Statistical Inference

Understanding probabilities in such sampling without replacement is fundamental when:
- Designing experiments.

- Making inferences about populations.
- Calculating confidence intervals.
- Performing hypothesis tests.

Extensions and Variations

Drawing More Than Two Balls

- Extending the problem to drawing more than two balls involves similar combinatorial calculations, but with increased complexity.
- Probabilities of various configurations (number of white and red balls) can be computed using hypergeometric distribution.

Replacing the Balls After Drawing

- If the balls are replaced after each draw, the probabilities change:
- The probability of drawing a white or red ball remains constant after each draw.
- The process becomes a sequence of independent events, and calculations follow binomial distributions.

Different Compositions of the Urn