

11. One Number Is Twelve More Than Another. The Sum Of The Two Numbers Is The Same As Fourtimes The Smaller.

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This statement introduces a classic algebraic problem that involves understanding relationships between two numbers and translating those relationships into equations. Such problems are common in mathematics, especially in algebra, and they serve as excellent exercises for developing problem-solving skills. In this article, we will explore how to approach this problem step-by-step, derive the necessary equations, and find the solution. Additionally, we will provide insights into similar problems and tips for solving them efficiently.

Understanding the Problem

Before jumping into equations and calculations, it's vital to understand what the problem is asking. Let's break down the statement:

- One number is twelve more than another: This indicates a relationship between two numbers, say x and y , where one is larger by 12.
- The sum of the two numbers is the same as four times the smaller: The total of x and y equates to four times the smaller number.

By analyzing these clues, we can set up equations that represent these relationships.

Defining Variables

The first step in solving algebraic problems is to assign variables to the unknowns. Here, we are dealing with two numbers, so:

- Let x = the smaller number.
- Let y = the larger number.

Based on the problem statement, we have:

- y is twelve more than x :

$$y = x + 12$$

- The sum of the two numbers equals four times the smaller:

$$x + y = 4x$$

Our goal is to find the values of x and y that satisfy these equations.

Formulating the Equations

With the variables defined, the next step is to write the equations explicitly:

1. Relationship between the two numbers:

$$y = x + 12$$

2. Sum of the numbers equals four times the smaller:

$$x + y = 4x$$

Substituting the first into the second allows us to solve for x .

Solving the Equations Step-by-Step

Let's proceed with solving for x and y .

Step 1: Substitute the expression for y into the sum equation

$$x + (x + 12) = 4x$$

Step 2: Simplify the equation

$$x + x + 12 = 4x$$

$$\begin{aligned} & \backslash[\\ & 2x + 12 = 4x \\ & \backslash] \end{aligned}$$

Step 3: Isolate x

Subtract 2x from both sides:

$$\begin{aligned} & \backslash[\\ & 12 = 4x - 2x \\ & \backslash] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & 12 = 2x \\ & \backslash] \end{aligned}$$

Step 4: Solve for x

$$\begin{aligned} & \backslash[\\ & x = \frac{12}{2} = 6 \\ & \backslash] \end{aligned}$$

Step 5: Find y

Recall that $y = x + 12$:

$$\begin{aligned} & \backslash[\\ & y = 6 + 12 = 18 \\ & \backslash] \end{aligned}$$

Final Answer and Verification

The two numbers are:

- Smaller number (x): 6
- Larger number (y): 18

Let's verify the solution:

- Is y twelve more than x?

$$\begin{aligned} & \backslash[\\ & 18 = 6 + 12 \quad \checkmark \\ & \backslash] \end{aligned}$$

- Is the sum of the numbers equal to four times the smaller?

$$\begin{aligned} & \backslash[\\ & 6 + 18 = 24 \quad \text{and} \quad 4 \times 6 = 24 \quad \checkmark \\ & \backslash] \end{aligned}$$

Both conditions are satisfied, confirming the correctness of the solution.

Understanding the Key Concepts

This problem illustrates several important concepts in algebra:

1. Translating Word Problems into Equations

Identifying the relationships described in words and converting them into algebraic expressions is fundamental.

2. Substitution Method

Using one equation to replace a variable in another simplifies the problem, making it easier to solve.

3. Verification of Solutions

Always check your solutions against the original conditions to ensure accuracy.

Additional Practice Problems

To deepen understanding, consider practicing with similar problems:

- One number is three less than twice another. The sum of the two numbers is 20. Find the numbers.
- The difference between two numbers is 8. The larger number is twice the smaller. Find both numbers.
- Three more than a number is equal to twice another number. Their sum is 30. Find the numbers.

Tips for Solving Similar Problems

- Clearly define variables: Assign meaningful letters to unknowns.
- Translate relationships carefully: Pay close attention to words like "more than," "less than," "twice," etc.
- Write equations systematically: Use the relationships described to formulate equations.
- Check units and conditions: Make sure the solutions satisfy all parts of the problem.
- Verify your answer: Substitute back into original conditions to confirm.

Conclusion

The problem "11. One Number Is Twelve More Than Another. The Sum Of The Two Numbers Is The Same As Fourtimes The Smaller." demonstrates how algebra can be used to solve real-world-like problems involving relationships between numbers. By carefully defining variables, translating the problem into equations, and solving systematically, we can find the unknown numbers efficiently and accurately. Mastering such techniques not only enhances problem-solving skills but also prepares you for more complex algebraic challenges. Practice regularly with similar problems to build confidence and proficiency in algebra.

Frequently Asked Questions

What is the general approach to solve the problem where one number is twelve more than another, and their sum equals four times the smaller number?

Set variables for the two numbers, express the larger in terms of the smaller, then form equations based on the given conditions and solve for the variables.

How can I represent the two numbers algebraically in the problem where one is twelve more than the other?

Let the smaller number be x ; then, the larger number is $x + 12$.

What equation do I set up if the sum of the two

numbers equals four times the smaller number?

The equation is $x + (x + 12) = 4x$.

How do I solve for the smaller number in this type of problem?

Combine like terms in the equation and isolate x , for example: $2x + 12 = 4x$, then solve for x .

What is the solution for the smaller number in this problem?

The smaller number is 6, obtained by solving the equation $2x + 12 = 4x$, which simplifies to $x = 6$.

What is the larger number in this problem after solving for the smaller?

The larger number is 18, since it is 12 more than the smaller number 6.

Are there any common mistakes to avoid when solving this problem?

Yes, common mistakes include mixing up the variables, incorrectly setting up the equation, or algebraic errors while solving for x .

Can you verify the solution by plugging the numbers back into the original conditions?

Yes, substitute the smaller number (6) and larger number (18) into the conditions: $6 + 18 = 24$ and $4 \times 6 = 24$, which confirms the solution is correct.

How can this problem be extended to more complex scenarios?

It can be extended by adding more numbers with similar relationships or incorporating additional conditions, then setting up systems of equations to solve for multiple variables.

Additional Resources

One Number Is Twelve More Than Another. The Sum Of The Two Numbers Is The Same As Four Times The Smaller.

This intriguing algebraic problem presents a compelling opportunity to explore the relationships between two numbers through the lens of simple equations. It combines basic arithmetic concepts with logical reasoning, making it an excellent example for students and enthusiasts alike to sharpen their problem-solving skills. By dissecting the problem carefully, understanding the underlying structure, and applying systematic methods, one can arrive at the solution while appreciating the elegance of algebraic relationships.

Understanding the Problem Statement

The problem provides two key pieces of information:

1. One number is twelve more than another.
2. The sum of the two numbers is the same as four times the smaller number.

These statements set the stage for formulating algebraic expressions and equations. To effectively analyze and solve the problem, it's essential to clarify what the terms "one number" and "another" refer to and how they relate.

Breaking Down the Key Statements

- The phrase "one number" and "another" typically implies two distinct numbers, which we can denote as variables: say, x and y .
- The phrase "one number is twelve more than another" suggests a relationship such as:

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\[
\text{Number 1} = \text{Number 2} + 12
\]
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or vice versa. To proceed systematically, we need to decide on which variable represents the smaller number.

- The second statement "the sum of the two numbers is the same as four times the smaller" indicates that the sum $(x + y)$ equals $(4 \times \text{smaller number})$.

Setting Up Variables and Equations

To analyze the problem, let's assign:

- s = the smaller number
- l = the larger number

Based on the first statement:

$$l = s + 12$$

From the second statement:

$$s + l = 4s$$

Since $(l = s + 12)$, substituting into the second equation:

$$s + (s + 12) = 4s$$

Simplify:

$$2s + 12 = 4s$$

Rearranged:

$$4s - 2s = 12$$

$$2s = 12$$

$$s = 6$$

Now, find (l) :

$$l = s + 12 = 6 + 12 = 18$$

Solution:

- Smaller number $(s = 6)$
- Larger number $(l = 18)$

Verification of the Solution

It's crucial to verify whether the solution satisfies both conditions:

1. The larger number is twelve more than the smaller:

$$\begin{aligned} &[\\ 18 &= 6 + 12 \quad \checkmark \\ &] \end{aligned}$$

2. The sum of the two numbers equals four times the smaller:

$$\begin{aligned} &[\\ 6 + 18 &= 24 \\ &] \\ &[\\ 4 \times 6 &= 24 \quad \checkmark \\ &] \end{aligned}$$

Both conditions are satisfied, confirming the solution's correctness.

Alternative Approaches and Generalizations

While the above method is straightforward, exploring alternative approaches can deepen understanding:

Using Different Variable Assignments

Suppose we initially assign:

- (x) as the smaller number
- (y) as the larger number

Given:

$$\begin{aligned} &[\\ y &= x + 12 \\ &] \end{aligned}$$

$$\begin{aligned} & \backslash[\\ & x + y = 4x \\ & \backslash] \end{aligned}$$

Substituting:

$$\begin{aligned} & \backslash[\\ & x + (x + 12) = 4x \\ & \backslash] \\ & \backslash[\\ & 2x + 12 = 4x \\ & \backslash] \\ & \backslash[\\ & 4x - 2x = 12 \\ & \backslash] \\ & \backslash[\\ & 2x = 12 \\ & \backslash] \\ & \backslash[\\ & x = 6 \\ & \backslash] \end{aligned}$$

And then:

$$\begin{aligned} & \backslash[\\ & y = 6 + 12 = 18 \\ & \backslash] \end{aligned}$$

This approach is essentially the same but underscores the flexibility of variable assignment.

Generalizing the Problem

What if the number "twelve" were replaced with another number, say $\backslash(k\backslash)$? Then, the problem becomes:

- One number is $\backslash(k\backslash)$ more than the other.
- The sum equals four times the smaller.

Expressed as:

$$\begin{aligned} & \backslash[\\ & \text{\text{Larger}} = \text{\text{Smaller}} + k \\ & \backslash] \\ & \backslash[\\ & \text{\text{Smaller}} + \text{\text{Larger}} = 4 \times \text{\text{Smaller}} \\ & \backslash] \end{aligned}$$

Substituting:

$$\begin{aligned} & s + (s + k) = 4s \\ & 2s + k = 4s \\ & 4s - 2s = k \\ & 2s = k \\ & s = \frac{k}{2} \end{aligned}$$

And the larger:

$$l = s + k = \frac{k}{2} + k = \frac{3k}{2}$$

This generalization reveals a pattern:

- The smaller number is half of (k) .
- The larger is $(1.5 \times k)$.

This demonstrates the problem's scalability and how changing the constant affects the solutions.

Features and Insights of the Problem

Understanding this problem offers several features and insights:

- Linear relationships: The problem exemplifies how two variables can be related through simple linear equations.
- Variable assignment flexibility: Choosing which variable represents the smaller number simplifies the algebra.
- Parameterization: Introducing a parameter (k) allows exploration of a family of solutions, illustrating the beauty of algebra.
- Logical consistency: The verification step ensures that solutions satisfy all initial conditions, reinforcing rigorous mathematical reasoning.

Pros and Cons of This Type of Problem

Pros:

- Educational value: Enhances understanding of algebraic relationships and solving equations.
- Problem-solving skills: Develops logical reasoning and systematic approach abilities.
- Flexibility: Can be adapted to various similar problems with different constants.
- Conceptual clarity: Demonstrates the importance of defining variables accurately.

Cons:

- Potential for misinterpretation: Ambiguous language can lead to multiple interpretations.
- Limited complexity: Simple problems may not challenge advanced learners.
- Over-reliance on algebra: Might discourage exploration of alternative methods like graphical solutions or inequalities.

Conclusion and Final Thoughts

The problem "One number is twelve more than another. The sum of the two numbers is the same as four times the smaller" exemplifies fundamental algebraic concepts in a clear, approachable manner. By systematically defining variables, formulating equations, and verifying solutions, learners can develop confidence in their problem-solving abilities. Moreover, the problem's structure lends itself to generalizations, fostering deeper insights into linear relationships. Whether viewed as a straightforward exercise or a stepping stone towards more complex problems, it underscores the elegance and utility of algebra in understanding relationships between quantities.

In summary:

- The solution demonstrates the power of simple algebraic equations.
- The approach emphasizes clarity, verification, and logical reasoning.
- The problem encourages exploration beyond fixed constants, introducing parameterization.
- Its features make it an excellent educational tool for developing core mathematical skills.

By mastering such problems, students build a strong foundation that enables tackling more sophisticated mathematical challenges in the future.

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