15. For A Population With = 80 And = 8, Find The X Value That Corresponds To Each Of The Following Z-scores.

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Understanding how to find specific X values corresponding to given Z-scores is a fundamental concept in statistics, especially when working with normal distributions. This process is crucial for data analysis, hypothesis testing, and confidence interval estimation. In this article, we will explore the method to determine X values given a population mean and standard deviation, and how to interpret Z-scores in various contexts. We will also provide detailed examples and step-by-step calculations to enhance your comprehension of this essential statistical skill.

What Are Z-scores and Why Are They Important?

Definition of Z-scores

A Z-score indicates how many standard deviations an individual data point (X) is from the population mean (μ). It is a standardized score that allows for comparison between data points from different normal distributions.

Mathematically, the Z-score is calculated as:
$$ Z = \frac{X - \mu}{\sigma} $$
where:
- X = data point
- μ = population mean
- σ = population standard deviation

Why Use Z-scores?

Z-scores are vital because they:
- Standardize different data points for comparison
- Help identify outliers
- Facilitate probability calculations using standard normal distribution tables
- Enable the determination of X values corresponding to specific Z-scores

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Given Data and Objective

Before diving into calculations, let's clarify the provided data and what we aim to find.

- Population mean (μ) = 80
- Population standard deviation (σ) = 8

Our goal is to find the X values that correspond to various Z-scores.

How to Find X Values From Z-scores

The fundamental formula connecting X and Z-score is:
$$X = Z \times \sigma + \mu$$

Given μ and σ, for any Z-score, the corresponding X value can be calculated directly.

Key steps:
1. Identify the Z-score(s) of interest.
2. Plug the Z-score into the formula:
$$X = Z \times \sigma + \mu$$
3. Compute the X value.

Examples: Calculating X Values for Different Z-scores

Let's examine some common Z-scores and determine their corresponding X values.

Example 1: Z = 0

When Z = 0, the data point is exactly at the mean.

$$X = 0 \times 8 + 80 = 80$$

Interpretation: The X value is 80, which is the population mean.

Example 2: Z = 1

This indicates a value one standard deviation above the mean.

$$X = 1 \times 8 + 80 = 88$$

Interpretation: The data point is 88 when Z=1.

Example 3: Z = -1

This indicates a value one standard deviation below the mean.

$$X = -1 \times 8 + 80 = 72$$

Interpretation: The data point is 72 when $Z=-1$.

Example 4: Z = 2

Two standard deviations above the mean.

$$ X = 2 \times 8 + 80 = 96 $$

Example 5: Z = −2

Two standard deviations below the mean.

$$ X = -2 \times 8 + 80 = 64 $$

More Z-scores and Corresponding X Values

To extend your understanding, here is a list of common Z-scores and their corresponding X values for the given population parameters:

- $Z = 1.96$ (approximate value for 97.5th percentile):
  - $X = 1.96 \times 8 + 80 \approx 15.68 + 80 = 95.68$
- $Z = -1.96$:
  - $X = -1.96 \times 8 + 80 \approx -15.68 + 80 = 64.32$
- $Z = 2.33$ (approximate for 98th percentile):
  - $X = 2.33 \times 8 + 80 \approx 18.64 + 80 = 98.64$
- $Z = -2.33$:
  - $X = -2.33 \times 8 + 80 \approx -18.64 + 80 = 61.36$

These calculations are crucial in statistical inference, such as constructing confidence intervals or conducting hypothesis tests.

Practical Applications of Finding X Values from Z-scores

Understanding how to determine X values from Z-scores has numerous practical applications in various fields:

1. Confidence Intervals

- Estimating the range within which a population parameter lies with a certain confidence level.
- Using Z-scores corresponding to desired confidence levels (e.g., 95%, 99%).

2. Hypothesis Testing

- Comparing sample data to population parameters.
- Calculating critical values for test statistics.

3. Quality Control

- Identifying outliers or unusual observations.
- Ensuring products meet quality standards based on statistical thresholds.

4. Data Standardization

- Converting raw scores into standardized scores for comparison across different datasets.

Tips for Accurate Calculations

When calculating X values from Z-scores, keep these tips in mind:

1. Always ensure your Z-scores are accurate, especially when using statistical tables or software.
2. Use consistent units; standard deviations should match the context of the data.
3. Double-check calculations to avoid errors, particularly with negative Z-scores.
4. Use software tools like Excel, R, or statistical calculators for complex computations.

Conclusion

Mastering the process of finding X values corresponding to Z-scores is an essential skill in statistics. Given a population mean of 80 and a standard deviation of 8, you can determine the specific data points associated with various Z-scores using a straightforward formula. This knowledge enables statisticians and data analysts to interpret data points within the context of the normal distribution, perform hypothesis testing, construct confidence intervals, and make informed decisions based on statistical evidence.

By practicing the calculation methods outlined in this article and understanding their applications, you will enhance your statistical literacy and analytical capabilities. Whether working in research, quality control, or data analysis, knowing how to convert Z-scores to X values is a fundamental step toward accurate and meaningful statistical interpretation.

Keywords for SEO Optimization:

- Z-scores and X-values
- Calculate X from Z-score
- Normal distribution calculations
- Population mean and standard deviation
- Statistical inference
- Confidence intervals from Z-scores
- Hypothesis testing with Z-scores
- Standard normal distribution table
- How to find X given Z
- Statistical analysis tools

Frequently Asked Questions

What is the formula to find the X value given a Z-score, population mean, and standard deviation?

The formula is $X = \mu + Z \sigma$, where μ is the population mean, σ is the population standard deviation, and Z is the Z-score.

How do you calculate the X value for a Z-score of 1.5 with a mean of 80 and standard deviation of 8?

$X = 80 + 1.5\ 8 = 80 + 12 = 92$.

What is the X value corresponding to a Z-score of -2 with $\mu = 80$ and $\sigma = 8$?

$X = 80 + (-2)\ 8 = 80 - 16 = 64$.

If the Z-score is 0, what is the corresponding X

value for $\mu = 80$ and $\sigma = 8$?

$X = 80 + 0 8 = 80$.

How would you find the X value for a Z-score of 2.5 with a population mean of 80 and standard deviation of 8?

$X = 80 + 2.5 8 = 80 + 20 = 100$.

What are the steps to find X values for different Z-scores in a population with mean 80 and standard deviation 8?

First, identify the Z-score, then multiply it by the standard deviation (8), and finally add the population mean (80) to find the X value: $X = \mu + Z \sigma$.

Additional Resources

15. For a Population With N = 80 and μ = 8, Find The X Value That Corresponds To Each Of The Following Z-scores

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Introduction

In the realm of statistics, understanding the relationship between a population's parameters—namely, the mean (μ), the standard deviation (σ), and the individual data points (X)—is fundamental. The concept of Z-scores serves as a vital tool for standardizing data points and understanding their position relative to the population mean. This article delves deeply into the process of calculating the specific data point (X value) corresponding to given Z-scores for a population with a known size (N=80) and mean (μ=8). Through methodical exploration, we aim to clarify the procedure, interpret the results, and contextualize their significance within statistical analysis.

Understanding the Context and Key Parameters

Before diving into calculations, it is essential to establish a clear understanding of the given parameters:

- Population Size (N): 80
- Population Mean (μ): 8
- Standard Deviation (σ): Not explicitly given; however, understanding how to proceed depends on whether the population standard deviation is known or estimated.

In typical problems involving Z-scores, the focus is on the standard deviation (σ) of the population. If this value is not directly provided, it is often derived from sample data or assumed based on context. Since the problem statement does not specify σ, we will examine scenarios assuming known σ and explore how the calculation adapts if σ is unknown, involving sample standard deviation (s).

Deciphering the Z-Score Formula

The Z-score for an individual data point X is given by:

$$ Z = \frac{X - \mu}{\sigma} $$

Rearranged to find X:

$$ X = Z \times \sigma + \mu $$

This formula underscores that knowing σ is essential to determine X for a given Z-score.

Scenario 1: Known Population Standard Deviation

Suppose the population standard deviation (σ) is known. This is common in classical statistical problems where the population parameters are available.

Step 1: Identify the Z-score

For each Z-score, the corresponding X value can be computed directly using the formula above.

Step 2: Compute X

$$ X = Z \times \sigma + 8 $$

The key is plugging in the specific Z-score and the known σ value into this formula.

Scenario 2: Unknown Population Standard Deviation

If σ is unknown, the standard practice involves using the sample standard deviation (s) as an estimate, and the calculations are typically performed within the framework of confidence intervals or hypothesis testing. For the purpose of this discussion, we will assume σ is known for clarity.

Calculating X Values for Specific Z-scores

Suppose the Z-scores to be considered are standard (e.g., Z = -2, -1, 0, 1, 2). We will demonstrate how to find X values corresponding to these Z-scores assuming a known σ.

Example:

- For Z = 1.5
- Assume σ = 2 (just as an illustrative example)

Calculation:

$$ X = 1.5 \times 2 + 8 = 3 + 8 = 11 $$

Similarly, for Z = −1.5:

$$ X = -1.5 \times 2 + 8 = -3 + 8 = 5 $$

Deeper Analysis: Impact of Population Size (N=80)

While the population size (N=80) might seem auxiliary in this context, it plays a vital role in understanding the sampling distribution, especially when dealing with sample means or proportions.

Sampling Distribution of the Sample Mean:

- The mean of the sampling distribution remains $\mu = 8$.
- The standard deviation of the sampling distribution (standard error) is:

$$ \text{Standard Error} = \frac{\sigma}{\sqrt{N}} $$

Thus, the population size influences the variability of sample means, not individual data points.

Example:

Assuming $\sigma = 2$:

$$ \text{Standard Error} = \frac{2}{\sqrt{80}} \approx \frac{2}{8.944} \approx 0.2236 $$

This is crucial when translating Z-scores related to sample means rather than individual data points.

Practical Applications and Implications

Understanding how to find X values for given Z-scores in a population with specific parameters has direct implications in various fields:

- Quality Control: Identifying the range of acceptable measurements based on standard deviations.
- Psychology and Education: Standardizing test scores within a population.
- Healthcare: Determining patient measurements that are within or outside normal ranges.

This process aids in making data-driven decisions, assessing anomalies, and evaluating the extremity of specific observations.

Limitations and Considerations

While the computations seem straightforward, several caveats should be addressed:

1. Assumption of Normality: Z-scores assume the data follows a normal distribution. Deviations from normality can affect the accuracy of X

estimates.

2. Known σ: The calculations assume that the population standard deviation is known, which is often unrealistic. When σ is unknown, alternative approaches involving sample standard deviations and t-scores are necessary.

3. Sample Size Effects: Larger N reduces the standard error, making sample means more representative of the population.

4. Applicability to Small Samples: For N=80, the sample size is generally considered sufficient for the Central Limit Theorem to hold, but caution is advised in smaller samples.

Summary of Calculation Steps

1. Identify the Z-score for the data point.
2. Determine the population standard deviation (σ).
3. Apply the formula: $X = Z \times \sigma + \mu$.
4. Compute X for each Z-score of interest.

Concluding Remarks

The process of translating Z-scores into raw data points (X values) in a population with known parameters is fundamental in statistical analysis. It enables practitioners to interpret standardized scores within the context of the original data distribution. By understanding the influence of population size, standard deviation, and mean, statisticians can accurately map standardized measures back to tangible data points, facilitating informed decision-making across diverse disciplines.

In practical applications, always verify assumptions—particularly regarding the normality of data and knowledge of population parameters—to ensure the validity of the calculations. The methodology outlined here provides a clear framework for such conversions, supporting robust statistical analysis and interpretation.

Note: For specific Z-scores provided in an actual problem, substitute the given values into the formula accordingly. If additional data (like the actual Z-scores) are available, the same principles apply to derive the corresponding X values.

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