3. Consider The Quadratic Equation X2 + 2x - 35 = 0. Solve By Factoring And Using The Zero-product Property.

3. Consider The Quadratic Equation X2 + 2x - 35 = 0. Solve By Factoring And Using The Zero-product Property.

Quadratic equations are fundamental in algebra, representing a wide array of real-world problems, from physics to finance. Among various methods to solve quadratic equations, factoring combined with the zero-product property is one of the most straightforward and efficient techniques when applicable. In this article, we will explore how to solve the quadratic equation $X^2 + 2x - 35 = 0$ by factoring and applying the zero-product property, along with a detailed explanation of each step, relevant concepts, and additional tips for mastering this method.

Understanding the Quadratic Equation $X^2 + 2x - 35 = 0$

Before diving into the solution process, it is important to understand the structure of the quadratic equation at hand.

Standard Form of a Quadratic Equation

A quadratic equation is generally written as:

$$ ax^2 + bx + c = 0 $$

where:
- $a \neq 0$,
- b and c are coefficients,
- x is the variable.

In our case, the equation is:

$$ X^2 + 2x - 35 = 0 $$

which aligns with the standard form with:
- $a = 1$,
- $b = 2$,
- $c = -35$.

Goals of Solving the Equation

Our primary goal is to find all values of x that satisfy the equation. These solutions are called roots or zeros of the quadratic function.

Step-By-Step Solution Using Factoring and the Zero-Product Property

The process involves rewriting the quadratic into a factored form, then applying the zero-product property to find solutions.

Step 1: Identify the coefficients

From the equation $X^2 + 2x - 35 = 0$, note:
- $a = 1$,
- $b = 2$,
- $c = -35$.

Step 2: Find two numbers that multiply to $a \times c$ and add to b

This is a standard approach for factoring quadratics when $a = 1$.

- Multiply a and c:

$$1 \times (-35) = -35$$

- Find two numbers m and n such that:

$$m \times n = -35$$
$$m + n = 2$$

By considering the factors of -35:

- 1 and -35: sum = -34
- -1 and 35: sum = 34
- 5 and -7: sum = -2
- -5 and 7: sum = 2

Notice that the pair -5 and 7 multiply to -35 and add to 2, which matches our middle coefficient.

Step 3: Rewrite the quadratic as a sum of two terms

Express the middle term $2x$ as the sum of two terms using the numbers found:

$$X^2 + 7x - 5x - 35 = 0$$

Step 4: Factor by grouping

Group terms:

$$ (X^2 + 7x) + (-5x - 35) = 0 $$

Factor out the common factors in each group:

$$ X(X + 7) - 5(X + 7) = 0 $$

Notice the common binomial factor $ (X + 7) $:

$$ (X + 7)(X - 5) = 0 $$

Applying the Zero-Product Property

The zero-product property states:

> If $ AB = 0 $, then either $ A = 0 $ or $ B = 0 $.

Using this property on our factored form:

$$ (X + 7)(X - 5) = 0 $$

We set each factor equal to zero:

1. $ X + 7 = 0 $
2. $ X - 5 = 0 $

Solving each:

- $ X + 7 = 0 \Rightarrow X = -7 $
- $ X - 5 = 0 \Rightarrow X = 5 $

Therefore, the solutions to the quadratic equation $ X^2 + 2x - 35 = 0 $ are:

$$ X = -7 \quad \text{and} \quad X = 5 $$

Verification of Solutions

It's always good practice to verify solutions by substituting them back into the original equation:

- For $ X = -7 $:

$$ (-7)^2 + 2(-7) - 35 = 49 - 14 - 35 = 0 $$

- For $ X = 5 $:

$$ 5^2 + 2(5) - 35 = 25 + 10 - 35 = 0 $$

Both solutions satisfy the original quadratic, confirming their correctness.

Additional Tips for Factoring Quadratic Equations

- When the quadratic is easily factorable (like our example), factoring is often the fastest method.
- Identify the pattern: look for two numbers that multiply to $a \times c$ and add to b.
- Use factoring by grouping if the quadratic has four terms or more.
- Always verify solutions to avoid extraneous roots, especially when dealing with more complex equations.

Limitations of Factoring and Alternative Methods

While factoring is efficient when applicable, not all quadratic equations are easily factorable. In such cases, alternative methods include:

- Quadratic formula:

$$ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} $$

- Completing the square.

These methods are more universal but may involve more computation.

Conclusion

Solving quadratic equations by factoring and using the zero-product property is a fundamental skill in algebra. By following a systematic approach—identifying coefficients, finding two numbers that satisfy the multiplication and addition conditions, rewriting the quadratic as a product of binomials, and applying the zero-product property—you can efficiently find solutions to many quadratic equations. The example of $X^2 + 2x - 35 = 0$ demonstrates this process clearly, leading to solutions $X = -7$ and $X = 5$, both verified to satisfy the original equation. Mastery of factoring not only simplifies solving quadratics but also builds a strong foundation for more advanced algebraic concepts.

Frequently Asked Questions

How do you factor the quadratic equation $x^2 + 2x - 35 = 0$?

To factor $x^2 + 2x - 35$, find two numbers that multiply to -35 and add to 2. These numbers are 7 and -5. So, the factored form is $(x + 7)(x - 5) = 0$.

What is the zero-product property and how is it used to solve quadratic equations?

The zero-product property states that if the product of two factors is zero, then at least one of the factors must be zero. It is used to solve quadratics by setting each factor equal to zero and solving for x.

What are the solutions to the equation $x^2 + 2x - 35 = 0$ by factoring?

Setting each factor equal to zero: $x + 7 = 0$ gives $x = -7$, and $x - 5 = 0$ gives $x = 5$. So, the solutions are $x = -7$ and $x = 5$.

Can you verify the solutions by plugging them back into the original equation?

Yes. For $x = -7$: $(-7)^2 + 2(-7) - 35 = 49 - 14 - 35 = 0$. For $x = 5$: $25 + 10 - 35 = 0$. Both solutions satisfy the original equation.

What steps are involved in solving $x^2 + 2x - 35 = 0$ by factoring?

First, rewrite the quadratic in standard form. Then, find two numbers that multiply to -35 and add to 2. Next, factor the quadratic as a product of binomials. Finally, set each factor equal to zero and solve for x.

Is factoring always the best method to solve quadratic equations?

Factoring is efficient when the quadratic trinomial factors easily. For other quadratics, methods like completing the square or the quadratic formula may be more appropriate.

What are common mistakes to avoid when solving quadratics by factoring?

Common mistakes include forgetting to set each factor equal to zero, misidentifying the correct pair of factors, and making algebraic errors during factoring or solving.

How can you check if your factored form is correct?

You can expand the factored form to see if it simplifies back to the original quadratic equation. If it matches, the factoring is correct.

What is the significance of the solutions $x = -7$ and $x = 5$ in

real-world applications?

These solutions can represent points where a real-world scenario modeled by the quadratic function intersects a particular value, such as maximum profit or break-even points.

How does understanding factoring and zero-product property help in solving other types of equations?

Mastering these concepts provides foundational skills for solving higher-degree polynomials, algebraic equations, and understanding functions in advanced mathematics.

Additional Resources

Consider The Quadratic Equation $X^2 + 2x - 35 = 0$: An In-Depth Exploration of Factoring and the Zero-Product Property

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Introduction

Quadratic equations are fundamental in algebra, forming the backbone of many mathematical concepts and applications across science, engineering, and mathematics education. Among the various methods to solve quadratic equations, factoring combined with the zero-product property is often the most accessible and insightful approach for students and practitioners alike. This article delves into the detailed process of solving the quadratic equation

$$X^2 + 2x - 35 = 0$$

by factoring and applying the zero-product property. We will analyze each step comprehensively, explore alternative methods for solving the same equation, and discuss the significance of these techniques in both theoretical and practical contexts.

Understanding the Quadratic Equation

Before diving into solving strategies, it is crucial to understand the structure of the quadratic equation in question:

$$X^2 + 2x - 35 = 0$$

This is a second-degree polynomial, where the highest exponent of variable X is 2. Standard form for quadratic equations is:

$$ax^2 + bx + c = 0$$

In our case:

- $a = 1$

- $b = 2$
- $c = -35$

The goal is to find the values of X that satisfy this equation—commonly called the roots or solutions.

The Role of Factoring in Solving Quadratic Equations

Factoring is a method that involves expressing the quadratic as a product of two binomials. If the quadratic can be factored neatly, the solutions can be directly inferred from the factors using the zero-product property.

Zero-Product Property: If the product of two factors is zero, then at least one of the factors must be zero:

$$(A)(B) = 0 \Rightarrow A = 0 \text{ or } B = 0$$

This property forms the basis of solving factored quadratics.

Step-by-Step Solution of $X^2 + 2x - 35 = 0$ by Factoring

Step 1: Identify coefficients and set up the problem

Given:

$$X^2 + 2x - 35 = 0$$

Here:

- $a = 1$
- $b = 2$
- $c = -35$

The coefficients suggest that the quadratic is monic (leading coefficient is 1), simplifying the factoring process.

Step 2: Find two numbers that multiply to $a \times c = 1 \times -35 = -35$ and add to $b = 2$

This is a key step:

- Find two numbers, m and n, such that:

$$
m \times n = -35
$$

$$
m + n = 2
$$

Let's analyze the factors of -35:

- 1 and -35, sum: $1 + (-35) = -34$
- -1 and 35, sum: $-1 + 35 = 34$
- 5 and -7, sum: $5 + (-7) = -2$
- -5 and 7, sum: $-5 + 7 = 2$

The pair -5 and 7 satisfy both conditions:

$$
-5 \times 7 = -35
$$

$$
-5 + 7 = 2
$$

Step 3: Rewrite the quadratic as a sum of two terms

Using these numbers, rewrite the middle term:

$$
X^2 - 5x + 7x - 35 = 0
$$

This step decomposes the quadratic into four terms, facilitating factoring by grouping.

Step 4: Factor by grouping

Group the terms:

$$
(X^2 - 5x) + (7x - 35) = 0
$$

Factor each group:

$$
X (X - 5) + 7 (X - 5) = 0
$$

Now, factor out the common binomial factor $(X - 5)$:

$$
(X - 5)(X + 7) = 0
$$

Step 5: Apply the zero-product property

Set each factor equal to zero:

$$
X - 5 = 0 \Rightarrow X = 5
$$
$$
X + 7 = 0 \Rightarrow X = -7
$$

Solutions: $X = 5$ and $X = -7$

Validating the Solutions

It is good practice to verify solutions by substituting back into the original equation:

- For $X = 5$:

$$
(5)^2 + 2(5) - 35 = 25 + 10 - 35 = 0
$$

- For $X = -7$:

$$
(-7)^2 + 2(-7) - 35 = 49 - 14 - 35 = 0
$$

Both solutions satisfy the original equation, confirming their validity.

Alternative Methods for Solving the Same Equation

While factoring is often the most straightforward approach for quadratics with easily recognizable factors, other methods exist, including:

1. Quadratic Formula

The quadratic formula:

$$
X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
$$

Applying to our equation:

$$
X = \frac{-2 \pm \sqrt{(2)^2 - 4(1)(-35)}}{2}
$$

$$
X = \frac{-2 \pm \sqrt{4 + 140}}{2}
$$

$$
X = \frac{-2 \pm \sqrt{144}}{2}
$$

$$
X = \frac{-2 \pm 12}{2}
$$

Solutions:

$$
X = \frac{-2 + 12}{2} = \frac{10}{2} = 5
$$

$$
X = \frac{-2 - 12}{2} = \frac{-14}{2} = -7
$$

Matching the solutions obtained via factoring.

2. Completing the Square

Rearranged as:

$$
X^2 + 2X = 35
$$

Add $(\frac{b}{2})^2 = 1$ to both sides:

$$
X^2 + 2X + 1 = 36
$$

Expressed as:

$$
(X + 1)^2 = 36
$$

Taking square roots:

$$
X + 1 = \pm 6
$$

Solutions:

$$
X = -1 \pm 6
$$

Which gives:

$$
X = 5 \quad \text{or} \quad X = -7
$$

Again, matching previous solutions.

Significance of Factoring and Zero-Product Property in Education and Practice

The process of solving quadratics by factoring and zero-product property is not only foundational but also enhances conceptual understanding of algebraic structures. It emphasizes:

- Recognizing factor pairs
- Understanding the relationships between coefficients and roots
- Developing problem-solving intuition

In practical applications, factoring is often the quickest method when applicable, especially in engineering contexts where quick solutions are necessary. Moreover, mastering these techniques builds the groundwork for understanding more advanced algebraic concepts, including polynomial functions, roots, and graphing.

Limitations and Considerations

While factoring is elegant and straightforward for certain quadratics, it is not always feasible—particularly when the quadratic has irrational or complex roots. In such cases, the quadratic formula or completing the square become necessary.

Furthermore, some quadratics do not factor over the rationals, and attempting to factor without consideration of the discriminant may lead to incorrect conclusions. The discriminant $D = b^2 - 4ac$ indicates the nature of roots:

- $D > 0$: Two real roots
- $D = 0$: One real root (a repeated root)
- $D < 0$: Two complex roots

In our example, $D = 144 > 0$, confirming two real solutions.

Conclusion

The quadratic equation $X^2 + 2x - 35 = 0$ illustrates the power and elegance of factoring combined with the zero-product property. Through strategic decomposition, it is possible to efficiently find solutions without resorting to more complex formulas. This approach not only simplifies problem-solving but also deepens understanding of algebraic relationships.

The solutions $X = 5$ and $X = -7$ are verified through substitution, highlighting the importance of validation in mathematical problem-solving. Recognizing when factoring is applicable—and understanding alternative methods—equips students and practitioners with versatile tools for tackling quadratic equations in various contexts.

By mastering these techniques, learners build a solid foundation for more advanced mathematical topics, fostering analytical thinking and problem-solving confidence essential in academic and professional settings.

Key Takeaways:

- Factoring involves rewriting the quadratic as a product of binomials.
- The zero-product property allows direct solutions once factored.
- Recognizing factor pairs is crucial for efficient factoring.
- Alternative methods include quadratic formula and completing the

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