A Bag Contains 14 Blue, 6 Red, 12 Green, And 8 Purple Buttons. 25 Buttons Are Removed From The Bag Randomly.

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This scenario presents an intriguing problem often encountered in probability and combinatorics. When dealing with a collection of differently colored items, understanding the likelihood of various outcomes after a random selection is fundamental. In this article, we will explore the problem of randomly removing 25 buttons from a bag containing a specific number of buttons of each color, analyze the possible distributions of colors among the removed buttons, and delve into the associated probabilistic calculations. Such problems are not only mathematically interesting but also have practical applications in quality control, inventory management, and game theory.

Understanding the Composition of the Bag

Before analyzing the removal process, it is essential to comprehend the initial composition of the buttons in the bag.

Breakdown of Buttons by Color

The bag contains four different colors of buttons:
- Blue: 14 buttons
- Red: 6 buttons
- Green: 12 buttons
- Purple: 8 buttons

The total number of buttons in the bag is:
- Total buttons = 14 (Blue) + 6 (Red) + 12 (Green) + 8 (Purple) = 40 buttons

This initial composition provides the baseline for understanding the probabilities involved in randomly removing 25 buttons.

The Process of Random Removal

How Are Buttons Removed?

The problem states that 25 buttons are removed randomly from the bag. This randomness can be modeled in different ways depending on the context:
- Without replacement: Once a button is removed, it is not returned to the bag.
- Equal probability: Each button has an equal chance of being selected at each draw.

Typically, such problems assume without replacement removal, meaning the total number of buttons decreases with each removal, and the composition of the remaining buttons changes dynamically.

Implications of Random Selection

Given the initial counts, the key question is:
- What is the probability that a certain number of buttons of each color are among the 25 removed?

This involves calculating the likelihood of specific color distributions among the removed buttons, which can be approached using combinatorial methods.

Modeling the Problem: Hypergeometric Distribution

Introduction to the Hypergeometric Distribution

The hypergeometric distribution is a discrete probability distribution that describes the probability of drawing a specific number of successes (e.g., buttons of a particular color) in a fixed number of draws from a finite population without replacement.

In this context:
- Population size (N): 40 buttons
- Number of success states in the population (K): Number of buttons of a specific color
- Number of draws (n): 25 buttons removed
- Number of observed successes (k): Number of removed buttons of that color

Applying Hypergeometric Distribution to Each Color

For each color, the probability of removing exactly k buttons of that color is given by:

$$
P(K = k) = \frac{\binom{K}{k} \binom{N - K}{n - k}}{\binom{N}{n}}
$$

Where:
- K is the total number of buttons of that color in the initial bag
- N is the total number of buttons initially (40)
- n is the total number of buttons removed (25)
- k is the number of buttons of that color removed

Calculating Probabilities for Specific Color Counts

Let's analyze the probabilities for each color, considering their initial counts.

Blue Buttons (14 initially)

- Possible values of k: from 0 up to 14 (since we can't remove more than available)

The probability that exactly k blue buttons are among the removed 25 is:

$$
P_{\text{Blue}}(k) = \frac{\binom{14}{k} \binom{26}{25 - k}}{\binom{40}{25}}
$$

Similarly, for other colors:

Red Buttons (6 initially)

$$
P_{\text{Red}}(k) = \frac{\binom{6}{k} \binom{34}{25 - k}}{\binom{40}{25}}
$$

Green Buttons (12 initially)

$$
P_{\text{Green}}(k) = \frac{\binom{12}{k} \binom{28}{25 - k}}{\binom{40}{25}}
$$

Purple Buttons (8 initially)

$$
P_{\text{Purple}}(k) = \frac{\binom{8}{k} \binom{32}{25 - k}}{\binom{40}{25}}
$$

Note:
- The sum over all possible k for each color should equal 1.
- To find the probability of a specific distribution (e.g., 5 blue, 2 red, 10 green, 8 purple), additional combinatorial calculations are needed, often involving multinomial distributions.

Expected Values and Variances

Calculating Expected Number of Removed Buttons per Color

The expected number of buttons of each color removed can be calculated using the properties of the hypergeometric distribution:

$$
E[K] = n \times \frac{K}{N}
$$

Applying this:

- Blue: $E[\text{Blue}] = 25 \times \frac{14}{40} = 25 \times 0.35 = 8.75$
- Red: $E[\text{Red}] = 25 \times \frac{6}{40} = 25 \times 0.15 = 3.75$
- Green: $E[\text{Green}] = 25 \times \frac{12}{40} = 25 \times 0.3 = 7.5$
- Purple: $E[\text{Purple}] = 25 \times \frac{8}{40} = 25 \times 0.2 = 5$

These expected values give an average perspective of how many buttons of each color are likely to be removed.

Variance of the Number of Removed Buttons

Variance provides insight into the variability of the number of buttons removed per color:

$$
Var[K] = n \times \frac{K}{N} \times \left(1 - \frac{K}{N}\right) \times \frac{N - n}{N - 1}
$$

Calculations:

- Blue:
$Var[\text{Blue}] = 25 \times 0.35 \times 0.65 \times \frac{15}{39} \approx 25 \times 0.2275 \times 0.3846 \approx 2.19$

- Red:
$Var[\text{Red}] = 25 \times 0.15 \times 0.85 \times \frac{15}{39} \approx 25 \times 0.1275 \times 0.3846 \approx 1.23$

- Green:
$Var[\text{Green}] = 25 \times 0.3 \times 0.7 \times \frac{15}{39} \approx 25 \times 0.21 \times 0.3846 \approx 2.02$

- Purple:
$Var[\text{Purple}] = 25 \times 0.2 \times 0.8 \times \frac{15}{39} \approx 25 \times 0.16 \times 0.3846 \approx 1.54$

Understanding these variances helps in assessing the likelihood of deviations from the expected counts.

Practical Applications and Extensions

Inventory Management

In real-world scenarios, understanding the probabilities of certain quantities being removed informs stock replenishment strategies and quality assurance processes.

Quality Control

Random sampling is often used to test products or components. Knowing the expected distribution of items after random sampling ensures that sampling sizes are adequate to detect defects or issues.

Game Theory and Decision Making

In games involving chance, such as drawing colored tokens or buttons, calculating these probabilities influences strategy development.

Advanced Topics: Multinomial Distributions and Joint Probabilities

While the hypergeometric distribution handles the probability for each color independently, calculating the joint probability of specific distributions across all colors involves the multinomial distribution:

$$
P(k_1, k_2, k_3, k_4) = \frac{\binom{N}{k_1, k_2, k_3, k_4}}{\binom{N}{n}}
$$

Where:
- k_1, k_2, k_3, k_4 are the counts of each color removed,
- and they sum to 25.

The multinomial coefficient:

$$
\binom{N}{k_1, k_2, k_3, k_4} = \frac{N!}{k_1! \times k_2! \times k_3! \times k_4!}
$$

This approach allows for calculating the probability of any specific combination of colors among the removed buttons.

Conclusion

The problem of removing 25 buttons randomly from a bag containing a fixed number of colored buttons exemplifies several key concepts in probability and combinatorics. By understanding the initial composition, applying hypergeometric distributions, and calculating expected

Frequently Asked Questions

What is the total number of buttons initially in the bag?

The total number of buttons is 14 (Blue) + 6 (Red) + 12 (Green) + 8 (Purple) = 40 buttons.

If 25 buttons are removed randomly, what is the probability that all removed buttons are Blue?

Since there are 14 Blue buttons out of 40, the probability that all 25 removed are Blue is <calculated>, which is extremely low because there are only 14 Blue buttons.

What is the expected number of Green buttons in the 25 removed?

The expected number of Green buttons removed is <(12/40) imes 25 = 7.5>.

After removing 25 buttons, what is the expected number of remaining Purple buttons?

Expected remaining Purple buttons = 8 - (<(8/40) imes 25>) = 8 - 5 = 3.

What is the probability that no Red buttons are removed in the 25 buttons taken out?

The probability is <calculated using hypergeometric distribution>, which accounts for choosing all 25 buttons from the 34 non-Red buttons.

If exactly 10 Green buttons are removed, how many Green buttons remain in the bag?

Initially 12 Green buttons; after removing 10, there are 12 - 10 = 2 Green buttons remaining in the bag.

What is the probability that the number of Blue buttons in the remaining buttons is at least 10?

This involves calculating the probability that at most 4 Blue buttons are removed (since 14 total Blue), making at least 10 Blue remaining.

If you randomly pick one button from the remaining 15 buttons, what is the probability it is Purple?

First, estimate how many Purple buttons remain after removal; then, divide that number by 15 to find the probability.

Are Green buttons more likely to be removed than Red buttons in the 25 buttons taken out?

Yes, because Green buttons make up a larger proportion (12/40) compared to Red buttons (6/40), so they are more likely to be removed.

What is the expected number of buttons remaining in the bag after removal?

Since 25 buttons are removed from 40, the expected remaining is 40 - 25 = 15 buttons.

Additional Resources

A Bag Contains 14 Blue, 6 Red, 12 Green, And 8 Purple Buttons. 25 Buttons Are Removed From The Bag Randomly. This scenario presents an interesting problem in probability and combinatorics, offering a rich context for exploring how to analyze outcomes when dealing with multiple categories and random sampling without replacement. Whether you're a student tackling a math problem, a teacher designing a lesson, or a data analyst examining sampling techniques, understanding the nuances of this problem can deepen your grasp of probability distributions, expected values, and contingency analysis.

Introduction to the Problem

Imagine you have a large bag filled with buttons of different colors. Specifically, the bag contains:

- 14 Blue buttons
- 6 Red buttons
- 12 Green buttons
- 8 Purple buttons

The total number of buttons in the bag is:

14 (Blue) + 6 (Red) + 12 (Green) + 8 (Purple) = 40 buttons

Now, suppose you randomly remove 25 buttons from this bag without replacement. This process leaves the remaining buttons in the bag, but the focus here is on understanding the distribution and probabilities associated with the number of each color removed.

This situation exemplifies a classic problem in hypergeometric probability — determining the likelihood of drawing certain counts of each category in a sample drawn without replacement.

Fundamental Concepts and Terminology

Before delving into calculations, it's essential to understand some core concepts:

1. Population and Sample
- Population: The total set of items, here, 40 buttons.
- Sample: The subset selected, here, 25 buttons removed.

2. Categories/Classes
- The different types within the population:
- Blue (14)
- Red (6)
- Green (12)
- Purple (8)

3. Sampling Without Replacement
- Once a button is removed, it isn't replaced, meaning the probabilities change dynamically as sampling proceeds.
- This is different from sampling with replacement, where the probabilities stay constant.

4. Hypergeometric Distribution
- Used to calculate probabilities of a certain number of successes in draws without replacement.

- The probability of drawing exactly k items of a certain type from the population.

Analyzing the Distribution of Colors in the Sample

1. Basic Probabilistic Model

When removing 25 buttons randomly from the bag, the counts of each color in the sample are random variables. For each color C, the number of buttons X_C in the sample follows a hypergeometric distribution:

$$
P(X_C = k) = \frac{\binom{N_C}{k} \binom{N - N_C}{n - k}}{\binom{N}{n}}
$$

Where:
- N = total population size (40)
- N_C = number of items of color C
- n = sample size (25)
- k = number of items of color C in the sample

Step-by-Step Breakdown: Calculating Expected Values and Variances

1. Expected Number of Each Color in the Sample

The expected value $E[X_C]$ for each color:

$$
E[X_C] = n \times \frac{N_C}{N}
$$

This means:

- Blue:

$$
E[X_{Blue}] = 25 \times \frac{14}{40} = 25 \times 0.35 = 8.75
$$

- Red:

$$
E[X_{Red}] = 25 \times \frac{6}{40} = 25 \times 0.15 = 3.75
$$

- Green:

$$

E[X_{Green}] = 25 \times \frac{12}{40} = 25 \times 0.30 = 7.5
\]

- Purple:

\[
E[X_{Purple}] = 25 \times \frac{8}{40} = 25 \times 0.20 = 5
\]

Interpretation: On average, in a random sample of 25 buttons, you expect to find approximately 8.75 Blue, 3.75 Red, 7.5 Green, and 5 Purple buttons.

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2. Variance and Covariance Between Colors

Variance measures the spread of the number of each color, and covariance indicates how the counts of two colors vary together.

- Variance of \(X_C \):

\[
Var(X_C) = n \times \frac{N_C}{N} \times \left(1 - \frac{N_C}{N}\right) \times \frac{N - n}{N - 1}
\]

Calculating variances:

- Blue:

\[
Var[X_{Blue}] = 25 \times 0.35 \times 0.65 \times \frac{15}{39} \approx 25 \times 0.2275 \times 0.3846 \approx 25 \times 0.0875 \approx 2.19
\]

- Red:

\[
Var[X_{Red}] = 25 \times 0.15 \times 0.85 \times \frac{15}{39} \approx 25 \times 0.1275 \times 0.3846 \approx 25 \times 0.049 \approx 1.22
\]

- Green:

\[
Var[X_{Green}] = 25 \times 0.30 \times 0.70 \times \frac{15}{39} \approx 25 \times 0.21 \times 0.3846 \approx 25 \times 0.081 \approx 2.03
\]

- Purple:

\[

$$
Var[X_{Purple}] = 25 \times 0.20 \times 0.80 \times \frac{15}{39} \approx 25 \times 0.16 \times 0.3846 \approx 25 \times 0.0615 \approx 1.54
$$

- Covariance between colors C_i and C_j:

$$
Cov(X_{C_i}, X_{C_j}) = - n \times \frac{N_{C_i}}{N} \times \frac{N_{C_j}}{N} \times \frac{N - n}{N - 1}
$$

The negative covariance indicates that selecting more of one color reduces the chance of selecting more of another, due to the fixed total of 25 buttons.

Practical Applications and Extensions

1. Probabilities of Specific Outcomes

Suppose you're interested in the probability that exactly 10 Blue, 4 Red, 8 Green, and 3 Purple buttons are in the sample.

While calculating the exact probability involves multinomial hypergeometric calculations, in practice, such probabilities are obtained by multiplying the individual hypergeometric probabilities for each color, considering the constraints.

2. Expected Counts and Variability

Understanding the expected counts helps in planning and quality control scenarios where certain color proportions are desired or need to be maintained.

3. Estimating Population Composition

Given the sample counts, one could estimate the proportions of each color in the population, especially if multiple samples are drawn. This is akin to survey sampling techniques.

Considerations and Limitations

1. Dependence of Counts
- Counts of different colors are not independent; they are negatively correlated because the total sample size is fixed.

2. Variability
- While expected values provide a central point, actual outcomes can vary significantly due to randomness.

3. Assumptions
- The model assumes truly random sampling and that each button has an equal chance of being

selected.

Summary and Takeaways

- Understanding the distribution of colors in a sample drawn from a mixed population involves hypergeometric probability.
- Expected counts of each color in the sample are proportional to their representation in the population.
- Variances and covariances provide insights into the variability and relationships between counts of different colors.
- These concepts are fundamental in many fields, from quality control to ecological sampling, offering tools for prediction and analysis.

Final Thoughts

The problem of analyzing a bag containing various colored buttons and drawing a subset at random is a classic illustration of hypergeometric distribution principles. It underscores the importance of understanding how sampling without replacement influences the probability and expected outcomes of categorical data. Whether for classroom exercises, statistical modeling, or real-world decision-making, mastering these concepts enhances your ability to interpret and analyze complex sampling scenarios effectively.

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