A Box Contains Cards Numbered 1 - 10. Two Cards Are Randomly Picked With Replacement.What Is The Probability

A Box Contains Cards Numbered 1 - 10. Two Cards Are Randomly Picked With Replacement. What Is The Probability

Understanding probability is an essential aspect of mathematics that helps us quantify the likelihood of events occurring. One common scenario involves drawing items from a set and analyzing the chances of specific outcomes. In this article, we will explore the probability associated with drawing cards from a box containing cards numbered 1 through 10, where two cards are chosen randomly with replacement. By the end, you will have a comprehensive understanding of how to calculate these probabilities and interpret their significance.

Introduction to the Scenario

Understanding the Setup

The problem involves a box containing 10 distinct cards, each numbered from 1 to 10. The process of drawing involves two key points:

- **Random selection:** Each card has an equal chance of being selected in each draw.

- **With replacement:** After a card is drawn, it is put back into the box before the next draw. This means the probability remains unchanged for each draw.

What Is Being Asked?

The primary question is: What is the probability of a specific event occurring when two cards are randomly drawn with replacement? For example:

- What is the probability both cards show the same number?
- What is the probability that the sum of the two numbers is greater than 10?
- What is the probability that at least one card is a number greater than 5?

While the problem as stated is broad, we will focus on the general methodology to compute such probabilities, including common types of events and their calculations.

Fundamentals of Probability in This Context

Basic Probability Principles

Probability measures the likelihood of an event happening, expressed as a value between 0 and 1, or as a percentage between 0% and 100%. The fundamental formula is:

$$
\text{Probability of event } A = \frac{\text{Number of favorable outcomes}}{\text{Total number of possible outcomes}}
$$

In cases involving multiple steps or events, the rules of probability, such as multiplication for independent events, come into play.

Independent Events and Replacement

Since the cards are drawn with replacement, each draw is independent; the outcome of one draw does not influence the next. Therefore:

- The probability of an event in the first draw remains the same in the second draw.
- The joint probability of multiple independent events is the product of their individual probabilities.

This simplifies calculation considerably because each draw can be analyzed separately, then combined as needed.

Calculating Probabilities in the "Cards Numbered 1 - 10" Scenario

Sample Space and Outcomes

- Total possible outcomes for two draws with replacement:

$$
10 \times 10 = 100
$$

- Each outcome can be represented as an ordered pair (x, y), where x is the number drawn first, and y is the second.

Common Events and Their Probabilities

Let’s examine some specific events and how to calculate their probabilities.

1. **Both cards show the same number:**

- Favorable outcomes: $(1,1), (2,2), ..., (10,10)$
- Number of favorable outcomes: 10
- Probability:

$$
P(\text{both same}) = \frac{10}{100} = 0.10
$$

2. **Both cards show different numbers:**

- Total outcomes: 100
- Favorable outcomes: 100 - 10 = 90
- Probability:

$$
P(\text{both different}) = \frac{90}{100} = 0.90
$$

3. **Sum of the two numbers exceeds 10:**

- To find this, count outcomes where $x + y > 10$.

Calculating the favorable outcomes:

- For each possible first card x, count the number of second cards y such that $x + y > 10$.

First card x	Second card y satisfying $x + y > 10$	Number of outcomes
1	$y > 9$ (since $1 + y > 10$) ▯ $y=10$	1
2	$y > 8$ ▯ $y=9,10$	2
3	$y > 7$ ▯ $y=8,9,10$	3
4	$y > 6$ ▯ $y=7,8,9,10$	4
5	$y > 5$ ▯ $y=6,7,8,9,10$	5
6	$y > 4$ ▯ $y=5,6,7,8,9,10$	6
7	$y > 3$ ▯ $y=4,5,6,7,8,9,10$	7
8	$y > 2$ ▯ $y=3,4,5,6,7,8,9,10$	8
9	$y > 1$ ▯ $y=2,3,4,5,6,7,8,9,10$	9
10	$y > 0$ ▯ $y=1,2,3,4,5,6,7,8,9,10$	10

Sum the counts:

$$
1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55
$$

- Total favorable outcomes: 55
- Probability:

$$
P(\text{sum} > 10) = \frac{55}{100} = 0.55
$$

Probability of At Least One Card Greater Than 5

- Event: At least one card is greater than 5.
- Complement event: Neither card is greater than 5, i.e., both cards are from 1 to 5.

Calculations:

- Probability that a single card is from 1 to 5:

$$
P(\text{card} \leq 5) = \frac{5}{10} = 0.5
$$

- Probability both cards are from 1 to 5:

$$
P(\text{both} \leq 5) = 0.5 \times 0.5 = 0.25
$$

- Therefore, probability that at least one card is greater than 5:

$$
P(\text{at least one} > 5) = 1 - P(\text{both} \leq 5) = 1 - 0.25 = 0.75
$$

Generalized Approach to Computing Probabilities

Step-by-Step Methodology

To analyze similar problems, follow these steps:

1. **Identify the sample space:** Determine the total number of possible outcomes, especially considering whether the process involves replacement or not.

2. **Define the event:** Clearly specify what constitutes a favorable outcome.

3. **Count favorable outcomes:** Enumerate or calculate how many outcomes satisfy the event's conditions.

4. **Compute the probability:** Divide the number of favorable outcomes by the total outcomes.

Using Probabilities of Independent Events

When dealing with multiple independent events, the probability of their joint occurrence is the product of individual probabilities. For example:

- Probability that both cards are even numbers:

$$
P(\text{first even}) \times P(\text{second even}) = \left(\frac{5}{10}\right) \times \left(\frac{5}{10}\right) = 0.25
$$

Real-World Applications and Examples

Game Design and Probability

Understanding these probability calculations aids in designing fair games and predicting outcomes. For instance, in card games or lottery systems, knowing the odds helps players and organizers assess risk.

Educational Tools

This type of problem is frequently used in educational settings to teach students about combinatorics, probability rules, and problem-solving strategies.

Decision Making and Risk Assessment

Businesses and analysts use probability calculations like these to evaluate risks, forecast outcomes, and make informed decisions.

Conclusion

Calculating probabilities in scenarios involving drawing cards with replacement involves understanding the fundamental principles of probability, the nature of independent events, and systematic counting of outcomes. In the specific case of a box containing cards numbered 1 to 10, the total number of outcomes for two draws is 100, and the probabilities of various events, such as both cards being the same or the sum exceeding a certain number, can be precisely calculated using combin

Frequently Asked Questions

What is the probability of drawing a specific card (e.g., card number 5) twice with replacement from a box containing cards numbered 1 to 10?

The probability of drawing card number 5 twice with replacement is (1/10) (1/10) = 1/100.

What is the probability that the two cards drawn (with replacement) have different numbers?

The probability that the two cards have different numbers is 1 - (1/10) (1/10) = 1 - 1/100 = 99/100.

How do you calculate the probability of drawing two even-numbered cards in succession with replacement?

First, find the probability of drawing an even card (2, 4, 6, 8, 10), which is 5/10 = 1/2. Then, for two draws: (1/2) (1/2) = 1/4.

What is the probability that the sum of the two cards drawn is greater than 10?

Possible pairs where the sum exceeds 10 include (6,5), (6,6), (6,7), etc. Calculating all such pairs, the probability is 21/100.

If two cards are drawn with replacement, what is the probability that both cards are prime numbers?

Prime numbers between 1 and 10 are 2, 3, 5, 7. Probability of drawing a prime number each time is 4/10 = 2/5. For two draws: (2/5) (2/5) = 4/25.

Additional Resources

A Box Contains Cards Numbered 1 - 10. Two Cards Are Randomly Picked With Replacement. What Is The Probability?

Introduction

In the realm of probability theory, simple experiments often serve as foundational examples that illuminate the fundamental principles governing randomness and chance. Among these, the scenario of drawing cards from a finite set stands out for its clarity and instructive value. Specifically, consider a box containing cards numbered from 1 to 10. When two cards are drawn with replacement—that is, after each draw, the card is returned to the box—what is the probability of various outcomes? This question, deceptively straightforward, opens the door to exploring key concepts such as probability space, independence, and the calculation of compound probabilities.

In this article, we undertake a comprehensive investigation into this problem, presenting a detailed analysis of the probability calculations involved, exploring different events of interest, and contextualizing these findings within the broader framework of probability theory. We aim to not only answer the core question but also to provide a thorough understanding suitable for readers seeking a rigorous yet accessible discussion.

Understanding the Basic Setup

The Composition of the Sample Space

The experiment involves a box containing 10 cards, each uniquely numbered from 1 to 10. When two cards are drawn sequentially with replacement, the sample space—the set of all possible outcomes—is constructed from the ordered pairs of these numbers.

- Total number of outcomes: Since each draw is independent and the card is replaced, each draw has 10 possible outcomes. Therefore, the total number of ordered pairs (first draw, second draw) is:

$$
10 \times 10 = 100
$$

- Sample space notation:

$$
S = \{ (i, j) \,|\, i, j \in \{1, 2, \dots, 10\} \}
$$

Each element in this set represents a possible outcome, such as (3, 7) or (10, 10).

Assumption of Equal Probabilities

Since each card is equally likely to be drawn and the process is with replacement, the probability of drawing any particular card in a single draw is:

$$
P(\text{drawing card } i) = \frac{1}{10}
$$

for each $i \in \{1, 2, \dots, 10\}$.

Probabilistic Principles at Play

Independence of Draws

Because each card is replaced after drawing, the two draws are independent events. This independence means:

$$
P(\text{outcome of second draw} \,|\, \text{outcome of first draw}) = P(\text{second draw})
$$

This property simplifies the calculation of joint probabilities, allowing us to multiply probabilities of individual events to find the probability of compound events.

Probability of Specific Outcomes

Given the uniformity and independence:

- The probability of drawing any particular first card:

$$
P(\text{first card} = i) = \frac{1}{10}
$$

- Similarly, the probability of drawing any particular second card:

$$
P(\text{second card} = j) = \frac{1}{10}
$$

- Therefore, the probability of a specific ordered pair (i, j):

$$
P(i, j) = P(\text{first card} = i) \times P(\text{second card} = j) = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100}
$$

Calculating Probabilities for Various Events

To explore the problem thoroughly, we consider different events of interest, including the probability of drawing specific numbers, matching numbers, or satisfying particular conditions.

1. Probability of Drawing a Specific Ordered Pair

Since all outcomes are equally likely:

$$
P(\text{any specific pair } (i, j)) = \frac{1}{100}
$$

for all $i, j \in \{1, \ldots, 10\}$.

---

2. Probability that Both Cards Are the Same Number

This event, called "matching," occurs when the first and second draws are equal:

$$
\text{Event } A: \text{first card} = \text{second card}
$$

- Number of favorable outcomes: For each number i, the pair (i, i) is favorable.

- Count of favorable outcomes:

$$
10
$$

- Probability:

$$
P(A) = \sum_{i=1}^{10} P(i, i) = 10 \times \frac{1}{100} = \frac{10}{100} = \frac{1}{10}
$$

This aligns with the intuitive expectation that, with replacement, there's a 10% chance that both chosen cards are identical.

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3. Probability of Drawing a Number Greater Than 5

Suppose we are interested in the probability that the first draw is a number greater than 5:

- Favorable outcomes for the first draw:

$$
\{6, 7, 8, 9, 10\}
$$

- Probability that the first card is greater than 5:

$$
P(\text{first} > 5) = \frac{5}{10} = \frac{1}{2}
$$

Similarly, for the second draw:

$$
P(\text{second} > 5) = \frac{1}{2}
$$

If we want the probability that both cards are greater than 5:

$$
P(\text{first} > 5 \text{ and second} > 5) = P(\text{first} > 5) \times P(\text{second} > 5) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}
$$

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4. Probability of the Sum of the Two Cards Exceeding 10

An interesting composite event involves the sum of the numbers on the two cards:

- Total outcomes: 100
- Favorable outcomes: pairs where $i + j > 10$

Calculating this directly:

- For each $i \in \{1, 2, \dots, 10\}$, count how many j satisfy $i + j > 10$:

First Card i	j Values with $i + j > 10$	Count of j values
1	None ($j > 9$)	0
2	None ($j > 8$)	0
3	$j > 7$	2 (8, 9, 10)
4	$j > 6$	3 (7, 8, 9, 10)
5	$j > 5$	5 (6, 7, 8, 9, 10)
6	$j > 4$	6 (5, 6, 7, 8, 9, 10)
7	$j > 3$	7 (4, 5, 6, 7, 8, 9, 10)
8	$j > 2$	8 (3, 4, 5, 6, 7, 8, 9, 10)
9	$j > 1$	9 (2, 3, 4, 5, 6, 7, 8, 9, 10)
10	$j > 0$	10 (1 through 10)

- Total favorable outcomes:

$$
0 + 0 + 2 + 3 + 5 + 6 + 7 + 8 + 9 + 10 = 50
$$

- Probability:

$$
P(i + j > 10) = \frac{50}{100} = \frac{1}{2}
$$

This example demonstrates how combinatorial counting can be combined with probability calculations.

Broader Interpretations and Applications

Implications of Replacement in Random Sampling

The process of drawing with replacement is fundamental in many statistical applications, including sampling theory, Monte Carlo simulations, and randomized algorithms. It ensures independence between draws and simplifies probability calculations, as seen in the above examples.

Extensions to Non-Uniform Probabilities

While this analysis assumes uniform probabilities, real-world scenarios often involve biased or weighted sampling. For instance, if some cards are more likely to be drawn, the probability calculations must adapt accordingly. The core principles, however, remain similar, relying on defining the probability space accurately and applying

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