(a) Does The Plane F(s, T) = (3-2) 7+ (s-2-3r)j +2sk Contain The Point (7,4,0)? ____(b) Find The Z-component

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Introduction

In the realm of vector calculus and three-dimensional geometry, understanding how planes are defined and whether specific points lie on them is fundamental. This article explores a particular plane described by the vector function:

$$ F(s, T) = (3-2) \, 7 + (s - 2 - 3r) \, j + 2s \, k $$

and investigates whether the point $(7, 4, 0)$ lies on this plane. Additionally, we will delve into calculating the Z-component of the vector function, which often holds significance in understanding the spatial orientation of the plane. This comprehensive guide aims to clarify these concepts step-by-step, ensuring clarity for students, educators, and enthusiasts alike.

Understanding the Plane Equation

What Is a Plane in Vector Form?

A plane in three-dimensional space can often be represented parametrically using vectors. Typically, a parametric equation of a plane involves a point and two direction vectors:

$$
\vec{r}(s, t) = \vec{r}_0 + s \vec{a} + t \vec{b}
$$

where:

- $\vec{r}_0$ is a position vector to a known point on the plane.
- $\vec{a}$ and $\vec{b}$ are vectors defining the directions along the plane.
- s and t are scalar parameters.

Decoding the Given Vector Function

The provided function is:

$$
F(s, T) = (3-2) \, 7 + (s - 2 - 3r) \, j + 2s \, k

\]

However, this expression appears to have some inconsistencies or typographical ambiguities. To interpret it correctly, let's analyze each component:

- The first term: \((3-2) \, 7\) simplifies to \(1 \times 7 = 7\).
- The second term involves \((s - 2 - 3r) \, j\), which seems to involve a variable \(r\). Since the function is in terms of \(s\) and \(T\), perhaps \(r\) is a typo or meant to be \(T\).
- The third term: \(2s \, k\).

Assuming the intended function is:

\[
F(s, T) = 7 + (s - 2 - 3T) \, \hat{j} + 2s \, \hat{k}
\]

then the vector form can be written as:

\[
F(s, T) = \underbrace{7 \, \hat{i}}_{x\text{-component}} + \underbrace{(s - 2 - 3T)}_{y\text{-component}} \, \hat{j} + \underbrace{2s}_{z\text{-component}} \, \hat{k}
\]

This is a parametric vector function representing points on a surface or a family of planes parameterized by \(s\) and \(T\).

Part (a): Does the Plane Contain the Point (7, 4, 0)?

Step 1: Clarify the Equation of the Plane

Based on the assumptions above, the parametric point on the plane is:

\[
(x, y, z) = \left(7,\; s - 2 - 3T,\; 2s \right)
\]

The goal is to determine whether the point \((7, 4, 0)\) satisfies these equations for some values of \(s\) and \(T\).

Step 2: Set Up Equations for the Point

To check if \((7, 4, 0)\) lies on the plane, equate each component:

1. \(x\)-component:

\[
7 = 7
\]

which holds true, so the x-component is satisfied regardless of s or T.

2. y-component:

$$
4 = s - 2 - 3T
$$

3. z-component:

$$
0 = 2s
$$

Step 3: Solve for s and T

From the z-component:

$$
0 = 2s \Rightarrow s = 0
$$

Substitute $s = 0$ into the y-component:

$$
4 = 0 - 2 - 3T \Rightarrow 4 = -2 - 3T
$$

Solve for T:

$$
4 + 2 = -3T \Rightarrow 6 = -3T \Rightarrow T = -2
$$

Step 4: Verify the Solution

- With $s = 0$ and $T = -2$, check the point:

$$
x = 7, \quad y = 0 - 2 - 3(-2) = -2 + 6 = 4, \quad z = 2 \times 0 = 0
$$

which matches the point $(7, 4, 0)$.

Thus, yes, the point $(7, 4, 0)$ lies on the plane described by the parametric equations.

Part (b): Find the Z-Component of the Vector Function

Step 1: Recall the Z-Component

From the earlier interpretation, the vector function's components are:

$$
x = 7
$$
$$
y = s - 2 - 3T
$$
$$
z = 2s
$$

The Z-component is explicitly:

$$
z = 2s
$$

Step 2: Express the Z-Component in Terms of Parameters

The Z-component depends solely on s. To analyze or compute it, consider specific values of s, or express it as a function:

$$
\boxed{
z(s, T) = 2s
}
$$

Step 3: Applications of the Z-Component

Understanding the Z-component helps in:

- Visualizing the height or elevation of points on the plane.
- Determining the plane's orientation relative to the XY-plane.
- Calculating the slope or inclination of the plane in the Z-direction.

Step 4: Example Calculation

Suppose $s = 3$, then:

$$
z = 2 \times 3 = 6
$$

indicating the point on the plane at $s=3$ has a Z-coordinate of 6.

Additional Insights & Key Points

Key Points About the Plane and Point Containment

- The plane's parametric equations are constructed based on variables s and T.
- To determine if a point lies on the plane, substitute the point's coordinates into the parametric equations and solve for the parameters.
- Consistent solutions for parameters indicate the point is on the plane.

Significance of the Z-Component

- The Z-component reveals the vertical position of points on the plane.
- Analyzing the Z-component helps in understanding the plane's inclination and orientation in three-dimensional space.
- It is crucial in applications like physics (e.g., projectile motion), engineering, and computer graphics.

Summary

- Part (a): The point $(7, 4, 0)$ lies on the plane defined by $F(s, T) = 7 + (s - 2 - 3T)j + 2s k$. This was verified by solving the parametric equations for s and T, resulting in specific values $s=0$ and $T=-2$.

- Part (b): The Z-component of the vector function is $z = 2s$. It directly relates to the parameter s and provides insight into the vertical position of points on the plane.

Understanding these concepts enhances comprehension of three-dimensional geometry, vector functions, and their applications across various scientific and engineering disciplines.

Final Notes

For those delving into advanced vector calculus or 3D geometry, mastering parametric equations and their applications in determining point containment and component analysis is essential. Practice with various vector functions and points will strengthen spatial reasoning and problem-solving skills in multidimensional spaces.

Keywords: plane equation, vector functions, point containment, Z-component, 3D geometry, parametric equations, vector calculus, spatial analysis

Frequently Asked Questions

Does the plane defined by F(s, T) = (3-2)7 + (s - 2 - 3r)j + 2sk

contain the point (7, 4, 0)?

First, rewrite the plane equation properly and interpret the components. Since the given F(s, T) appears to be a parametric form, check if the point (7, 4, 0) satisfies the parametric equations by substituting and solving for parameters. If the parameters exist, the point lies on the plane; otherwise, it does not.

How do I determine if a point lies on a given parametric plane equation?

Substitute the coordinates of the point into the parametric equations of the plane and see if there are consistent parameter values that satisfy all equations simultaneously.

What is the significance of the Z-component in a plane equation?

The Z-component indicates the position of a point along the vertical axis in 3D space. Finding the Z-component helps locate points relative to the plane and understand the orientation of the plane.

How can I find the Z-component of a point given a parametric plane equation?

Identify the parametric expression for the Z-coordinate in the plane's equations and substitute the known parameters or values to compute the Z-component.

What is the general form of a plane equation in 3D space?

A common form is Ax + By + Cz + D = 0, where (A, B, C) is the normal vector to the plane, and (x, y, z) are the coordinates of points on the plane.

How do I determine if a point (7, 4, 0) is on a plane given in parametric form?

Express the parametric equations of the plane, then solve for parameters s and T using the point's coordinates. If solutions exist that satisfy all equations, the point is on the plane.

Can the given function F(s, T) represent a plane?

Yes, if F(s, T) is linear in s and T and defines a set of points satisfying a linear equation, it can represent a plane in 3D space.

What does the notation F(s, T) = (3-2)7 + (s - 2 - 3r)j + 2sk imply?

It appears to be a parametric function involving parameters s and T (or r), with components along the x, y, and z axes. Clarifying the exact form is essential to analyze the plane.

How do I interpret the component (3-2)7 in the plane equation?

It likely simplifies to (-2) 7 = -14, indicating the x-component might be a constant or part of the parametric expression. Confirm the expression's structure for accurate interpretation.

What steps should I follow to find whether a specific point lies on a parametric plane?

1. Write down the parametric equations. 2. Substitute the point's coordinates into these equations. 3. Solve for the parameters. 4. If solutions exist, the point is on the plane; if not, it isn't.

Additional Resources

(a) Does The Plane F(s, T) = (3-2) 7+ (s-2-3r)j + 2sk Contain The Point (7,4,0)? ____(b) Find The Z-component

---

Introduction

In the realm of vector calculus and three-dimensional geometry, understanding whether a particular point lies on a given plane is fundamental. This question not only tests one's grasp of spatial reasoning but also the application of algebraic techniques to geometric problems. Here, we examine a specific vector function:

$F(s, T) = (3 - 2)\cdot 7 + (s - 2 - 3r)\cdot j + 2s\cdot k$

and analyze whether it contains the point (7, 4, 0). Subsequently, we will determine the Z-component of the vector, which is crucial for understanding the depth or height associated with the point in the context of the plane's orientation.

Part (a): Does the Plane F(s, T) Contain the Point (7, 4, 0)?

Understanding the Vector Function F(s, T)

Before addressing whether the point lies on the plane, it is essential to interpret the given function accurately.

The function is:

$F(s, T) = (3 - 2)\cdot 7 + (s - 2 - 3r)\cdot j + 2s\cdot k$

At first glance, the expression appears to be a combination of scalar multiples and vector components. To clarify, let's dissect it:

- The first term: $(3 - 2)\cdot 7$ — a scalar multiplication, resulting in $1\cdot 7 = 7$. However, it's not explicitly associated with a vector component unless it is intended as a scalar multiple of a basis vector.
- The second term: $(s - 2 - 3r)\cdot j$ — indicating a component along the y-axis, scaled by the expression $(s - 2 - 3r)$.
- The third term: $2s\cdot k$ — indicating a component along the z-axis, scaled by $2s$.

However, there's some ambiguity. Usually, a vector function in three dimensions is expressed as:

$$F(s, T) = x(s, T)\cdot i + y(s, T)\cdot j + z(s, T)\cdot k$$

Given the notation, it's likely that the original expression intends:

$$F(s, T) = [\ (3 - 2)\cdot 7\ ]\cdot i + (s - 2 - 3r)\cdot j + 2s\cdot k$$

which simplifies to:

$$F(s, T) = 7\cdot i + (s - 2 - 3r)\cdot j + 2s\cdot k$$

Note: The variable r might be a typo or an additional parameter. Since the point in question is (7, 4, 0), and the function depends on s and T, perhaps r is a typo for T or another parameter.

Assumption: For clarity, assume the function is:

$$F(s, T) = 7\cdot i + (s - 2 - 3T)\cdot j + 2s\cdot k$$

This aligns with standard parametric forms, where s and T are parameters controlling the position on the plane.

Expressing the Plane in Component Form

Using the assumption, the parametric equations become:

- $x(s, T) = 7$
- $y(s, T) = s - 2 - 3T$
- $z(s) = 2s$

Note that the x component is constant, independent of s and T, which indicates the plane is parallel to the yz-plane at $x = 7$.

Checking if (7, 4, 0) Lies on the Plane

Given that:

- $x = 7$
- $y = 4$
- $z = 0$

and the parametric equations:

- $x = 7$ (constant, matches the point's x-coordinate)
- $y = s - 2 - 3T$
- $z = 2s$

To determine if the point lies on the plane, find s and T such that the equations are satisfied:

1. From $x = 7$, no issue; the point's x-coordinate matches the plane's x-value.

2. For $z = 0$:

$z = 2s = 0 \rightarrow s = 0$

3. Substitute $s = 0$ into the y equation:

$4 = 0 - 2 - 3T \rightarrow 4 = -2 - 3T$

Solving for T:

$-3T = 4 + 2 \rightarrow -3T = 6 \rightarrow T = -2$

Since $T = -2$ is a real number, and $s = 0$ is consistent with the z component, the point (7, 4, 0) lies on the plane described by the parametric equations, provided $T = -2$.

Summary of Part (a):

- The point (7, 4, 0) is contained within the plane represented by the function F(s, T).
- The specific parameters $s = 0$ and $T = -2$ satisfy the parametric equations and confirm the point's inclusion.

Part (b): Find the Z-Component

Understanding the Z-Component in Context

The Z-component in a parametric plane equation corresponds to the z value derived from the parameters s and T.

From the assumed parametric form:

$z(s) = 2s$

Given $s = 0$ (from the previous calculation), the z-component associated with this point is:

$z = 2 \cdot 0 = 0$

To generalize, if we consider the plane or any point on it, the z-component depends linearly on s:

- When s varies, z varies proportionally as 2s.
- The maximum or minimum z depends on the range of s.

Significance of the Z-Component

- The z-coordinate provides information about the height or depth of the point relative to the xy-plane.
- Since z = 2s, adjusting s moves the point along the vertical direction.

Practical Implication

Suppose you want to find the z-value for a specific point on the plane with known s:

- For s = 1, z = 2(1) = 2
- For s = -1, z = -2

This direct relation simplifies calculations and helps visualize the plane's orientation in 3D space.

Final Remarks

Understanding the relationships in parametric equations is key to solving spatial problems. The initial query about whether the point (7, 4, 0) lies on the plane described by F(s, T) reveals that, with appropriate parameters, it indeed does. Moreover, recognizing that z = 2s offers a straightforward way to determine the vertical component for any point on the plane.

In summary:

- The point (7, 4, 0) is on the plane when s = 0 and T = -2.
- The Z-component of this point is 0, directly derived from z = 2s with s = 0.

This analysis showcases the power of parametric equations in 3D geometry, enabling precise pinpointing of points and understanding of spatial relationships. Whether for academic purposes or practical applications like engineering and computer graphics, mastering these concepts is invaluable.

A Does The Plane F S T 3 2 7 S 2 3r J 2sk Contain The Point 7 4 0 B Find The Z Component

A Does The Plane F S T 3 2 7 S 2 3r J 2sk Contain The Point 7 4 0 B Find The Z Component

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