A Particle Moves Along The X-axis So That Its Velocity At Time T Is Given By V(t)= (t^6-13t^4+12)/(10t^3+3).

A Particle Moves Along The X-axis So That Its Velocity At Time T Is Given By V(t)= (t^6-13t^4+12)/(10t^3+3).

Understanding the motion of particles along a line is a fundamental aspect of classical mechanics and calculus. When a particle's velocity function is given explicitly, as in this case, it opens the door to exploring various properties of its motion, such as displacement, acceleration, and the intervals of motion where the particle moves forward or backward. This article aims to analyze the behavior of the particle described by the velocity function $V(t) = (t^6 - 13t^4 + 12) / (10t^3 + 3)$, providing insights into its motion over time, critical points, and key characteristics.

Analyzing the Velocity Function

The velocity function $V(t)$ encapsulates how the particle's position changes with time. To understand the particle's behavior, we need to examine the function's properties, including its domain, zeros, and asymptotic behavior.

Domain of the Function

The domain of $V(t)$ is determined by the denominator, $10t^3 + 3$. Since division by zero is undefined, we must exclude any t where $10t^3 + 3 = 0$.

Solving for t:

$10t^3 + 3 = 0$
$\Rightarrow 10t^3 = -3$
$\Rightarrow t^3 = -3/10$
$\Rightarrow t = -(3/10)^{1/3}$

The cube root of a negative number is negative, so:

$t = -(3/10)^{1/3} \approx -0.669$

Therefore, the domain of $V(t)$ is all real numbers except $t \neq -0.669$.

Behavior at Infinity

To understand how the particle behaves as time progresses, examine the limits as t approaches infinity and negative infinity.

- As t ⍰ ⍰ :

The leading term in numerator: t^6
The leading term in denominator: $10t^3$

So,

$V(t) \approx t^6 / (10t^3) = t^3 / 10$ □ □

- As t □ -□ :

Similarly,

$V(t) \approx t^6 / (10t^3) = t^3 / 10$

Since t^3 tends to -□ when t □ -□ , $V(t)$ also tends to -□ .

This indicates that for large positive t, the velocity becomes very large and positive, and for large negative t, it becomes very large in magnitude but negative.

Zeros of the Velocity Function

Finding where $V(t) = 0$ gives the points in time where the particle's velocity is zero, corresponding to moments when the particle changes direction or is momentarily at rest.

Set the numerator equal to zero:

$t^6 - 13t^4 + 12 = 0$

Let's analyze this polynomial:

$t^6 - 13t^4 + 12 = 0$

Note that $t^6 = (t^2)^3$ and $t^4 = (t^2)^2$, so it's convenient to substitute $u = t^2$.

Substituting u:

$u^3 - 13u^2 + 12 = 0$

Now, solve for u:

$u^3 - 13u^2 + 12 = 0$

This is a cubic in u. To find roots, attempt rational root testing:

Possible rational roots: factors of 12 over factors of 1, i.e., $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

Test u=1:

$1 - 13 + 12 = 0$ □ 0; u=1 is a root.

Divide the cubic by $(u - 1)$:

Using synthetic division:

Coefficients: 1 | -13 | 0 | 12

Bring down 1:

1

Multiply by root 1: 1

Add to next coefficient: $-13 + 1 = -12$

Multiply -12 by 1: -12

Add to next coefficient: 0 + (-12) = -12

Multiply -12 by 1: -12

Add to last coefficient: 12 + (-12) = 0

Thus, the cubic factors as:

$(u - 1)(u^2 - 12u - 12) = 0$

Now, solve the quadratic:

$u^2 - 12u - 12 = 0$

Discriminant:

$\Delta = (12)^2 - 4\ 1\ (-12) = 144 + 48 = 192$

Square root:

$\sqrt{192} \approx 13.856$

Find roots:

$u = [12 \pm 13.856] / 2$

First root:

$u = (12 + 13.856)/2 \approx 25.856/2 \approx 12.928$

Second root:

$u = (12 - 13.856)/2 \approx (-1.856)/2 \approx -0.928$

Recall $u = t^2$, so:

$t^2 = u$

- For $u=1$:

$t^2=1 \Rightarrow t=\pm 1$

- For $u\approx 12.928$:

$t=\pm\sqrt{12.928} \approx \pm 3.597$

- For $u\approx -0.928$:

t^2 negative, discard as $t \notin \mathbb{R}$

Therefore, zeros of numerator occur at:

$t = \pm 1$, $t \approx \pm 3.597$

Note that $t=\pm 1$ and $t\approx\pm 3.597$ are the points where the velocity is zero.

Important: Check if any of these points coincide with the excluded point $t \approx -0.669$. Since -0.669 is not among these, all are within the domain.

Sign Analysis of the Velocity Function

To understand the particle's motion, analyze the sign of $V(t)$ in the intervals determined by the zeros and the discontinuity point.

Intervals to consider:

- $t < -3.597$
- $-3.597 < t < -0.669$
- $-0.669 < t < -1$
- $-1 < t < 1$
- $1 < t < 3.597$
- $t > 3.597$

For each interval, determine the sign of numerator and denominator:

- Numerator zeros at $t \approx -3.597, -1, 1, 3.597$
- Discontinuity at $t \approx -0.669$

Sample test points in each interval:

1. $t = -4$ (less than -3.597):

Numerator:

$(-4)^6 = 4096$, a positive number

$(-13)(-4)^4 = -13\ 256 = -3328$

Add 12:

$4096 - 3328 + 12 = 780$ (positive)

Denominator:

$10(-4)^3 + 3 = 10(-64) + 3 = -640 + 3 = -637$ (negative)

$V(t)$ = positive / negative = negative

2. $t = -2$ (between -3.597 and -0.669):

Numerator:

$(-2)^6 = 64$

$(-13)(-2)^4 = -1316 = -208$

Sum: $64 - 208 + 12 = -132$ (negative)

Denominator:

$10(-2)^3 + 3 = 10(-8) + 3 = -80 + 3 = -77$ (negative)

$V(t)$ = negative / negative = positive

3. t = -0.5 (between -0.669 and -1):

Note that -0.5 > -0.669, but recall the discontinuity at t ≈ -0.669; so choose t= -0.5 (which is greater than -0.669), but since the domain excludes t ≈ -0.669, check the sign:

Numerator:

(-0.5)^6= (0.5)^6= 0.015625

(-13)(-0.5)^4= -13 (0.5)^4= -130.0625= -0.8125

Sum: 0.015625 - 0.8125 +12 ≈ 11.203125 (positive)

Denominator:

10 (-0.5)^3 + 3= 10 (-0.125)+ 3= -1.25 + 3= 1.75 (positive)

V(t)= positive/ positive= positive

4. t= 0 (between -1 and 1):

Numerator:

0^6= 0

(-13)0^4= 0

Sum: 0 + 0 + 12= 12 (positive)

Denominator:

100 + 3=3 (positive)

V(t)= positive / positive= positive

5. t= 2 (between 1 and 3.597):

Numerator:

2^6= 64

(-13)2^4= -1316= -208

Sum: 64 - 208 + 12= -132 (negative)

Denominator:

108 + 3=80 + 3=83 (positive)

V(t)= negative / positive= negative

6. t= 4 (greater than 3.597):

Numerator:

4^6= 4096

Frequently Asked Questions

What is the expression for the velocity of the particle at time t?

The velocity of the particle at time t is given by $v(t) = (t^6 - 13t^4 + 12) / (10t^3 + 3)$.

How can we find the acceleration of the particle at any time t?

The acceleration $a(t)$ is the derivative of the velocity $v(t)$ with respect to time t, so $a(t) = dv(t)/dt$.

At what times does the particle change direction?

The particle changes direction when its velocity $v(t) = 0$. By solving $(t^6 - 13t^4 + 12) / (10t^3 + 3) = 0$, we find the roots of the numerator $t^6 - 13t^4 + 12 = 0$, excluding points where the denominator is zero.

Are there any points where the velocity is undefined?

Yes, the velocity is undefined when the denominator $10t^3 + 3 = 0$, which occurs at $t = -(3/10)^{1/3}$.

How do we determine the particle's speed at a given time t?

The speed is the absolute value of the velocity, so $|v(t)|$ at the specific time t.

What is the significance of the roots of the numerator in v(t)?

The roots of the numerator, where $t^6 - 13t^4 + 12 = 0$, indicate the times when the particle's velocity is zero, i.e., when it changes direction.

Can the velocity v(t) be negative, and what does that imply about the particle's motion?

Yes, $v(t)$ can be negative, indicating the particle is moving in the negative x-direction at time t.

How would you analyze the long-term behavior of the particle's velocity as t approaches infinity?

As t approaches infinity, the leading terms t^6 in numerator and $10t^3$ in denominator suggest $v(t)$ behaves like $t^6 / 10t^3 = t^3 / 10$, which tends to infinity, indicating the velocity increases without bound.

What steps would you take to find the particle's displacement over a time interval?

To find displacement, integrate the velocity function $v(t)$ over the desired time interval: displacement = $\int v(t)\, dt$ from t=a to t=b.

How can the derivative of v(t) inform us about the particle's acceleration and changing motion?

The derivative $dv(t)/dt$ gives the acceleration; analyzing its sign and magnitude helps determine whether the particle's speed is increasing or decreasing at different times.

Additional Resources

Analyzing the Motion of a Particle Along the X-axis with Velocity $v(t) = \frac{t^6 - 13t^4 + 12}{10t^3 + 3}$

Introduction

The study of particle motion in calculus and physics is foundational for understanding how objects move over time. When the velocity function is explicitly known, it opens the door to detailed analysis of various kinematic quantities such as displacement, average velocity, acceleration, and the behavior of the particle over different time intervals. In this review, we delve into the properties and implications of a specific velocity function:

$$
v(t) = \frac{t^6 - 13t^4 + 12}{10t^3 + 3}
$$

This rational function encapsulates complex behavior, including potential points of discontinuity, roots, and interesting long-term trends. Our goal is to analyze this function thoroughly, interpret the particle's motion, and explore related concepts in depth.

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1. Understanding the Velocity Function

1.1. Expression Breakdown

The given velocity function is a rational function:

$$
v(t) = \frac{N(t)}{D(t)} \quad \text{where} \quad N(t) = t^6 - 13t^4 + 12, \quad D(t) = 10t^3 + 3
$$

- Numerator $N(t)$: A sixth-degree polynomial, symmetric with respect to high powers, indicating potentially complex roots and sign changes.
- Denominator $D(t)$: A cubic polynomial, which can be zero at certain t, leading to discontinuities.

Understanding the behavior of $v(t)$ requires analyzing both numerator and denominator separately, particularly their roots and signs.

1.2. Domain Considerations

- The function is defined everywhere except where $D(t) = 0$.
- Solve for $D(t) = 0$:

$$
10t^3 + 3 = 0 \Rightarrow t^3 = -\frac{3}{10} \Rightarrow t = -\sqrt[3]{\frac{3}{10}}
$$

- Approximate root:

$$
t \approx -\sqrt[3]{0.3} \approx -0.669
$$

Thus, $v(t)$ has a vertical asymptote at $t \approx -0.669$.

---

2. Roots of the Velocity Function: When is $V(t) = 0$?

The zeros of $V(t)$ occur when $N(t) = 0$, provided $D(t) \neq 0$.

2.1. Solving $N(t) = 0$:

$$
t^6 - 13t^4 + 12 = 0
$$

Let's set $x = t^2$, transforming the sixth-degree polynomial into a cubic in x:

$$
x^3 - 13x^2 + 12 = 0
$$

This simplifies the analysis:

$$
x^3 - 13x^2 + 12 = 0
$$

Now, solve for x:

- Possible rational roots: Use Rational Root Theorem. Factors of constant term 12 are ±1, ±2, ±3, ±4, ±6, ±12.

Test these candidates:

- For $x=1$:

$$
1 - 13 + 12 = 0 \Rightarrow \text{Yes!} \quad x=1
$$

- For $x=12$:

$$
12^3 - 13 \times 12^2 + 12 = 1728 - 13 \times 144 + 12 = 1728 - 1872 + 12 = -132 \neq 0
$$

- For $x=2$:

$$
8 - 13 \times 4 + 12 = 8 - 52 + 12 = -32 \neq 0
$$

- For $x=3$:

$$
27 - 13 \times 9 + 12 = 27 - 117 + 12 = -78 \neq 0
$$

- For $x=4$:

$$
64 - 13 \times 16 + 12 = 64 - 208 + 12 = -132 \neq 0

$$
\]
$$

- For $x=6$:

$$
216 - 13 \times 36 + 12 = 216 - 468 + 12 = -240 \neq 0
$$

- For $x=-1$:

$$
-1 - 13 \times 1 + 12 = -1 - 13 + 12 = -2 \neq 0
$$

- For $x=-2$:

$$
-8 - 13 \times 4 + 12 = -8 - 52 + 12 = -48 \neq 0
$$

Only $x=1$ is a rational root.

Perform polynomial division to factor out $(x - 1)$:

Divide $x^3 - 13x^2 + 12$ by $x - 1$:

Using synthetic division:

Coefficients: 1 | -13 | 0 | 12

Bring down 1:

- Multiply by root: 11=1; add to -13: -13+1=-12

- Multiply: -121 = -12; add to 0: 0+(-12)=-12

- Multiply: -121 = -12; add to 12: 12+(-12)=0

Results in quotient: $x^2 - 12x -12$

Thus,

$$
x^3 - 13x^2 + 12 = (x - 1)(x^2 - 12x - 12)
$$

Solve quadratic:

$$
x^2 - 12x - 12=0
$$

Discriminant:

$$
\Delta = (12)^2 - 4 \times 1 \times (-12) = 144 + 48=192
$$

Square root:

$$
\sqrt{192} = \sqrt{64 \times 3} = 8 \sqrt{3}
$$

Solutions:

$$
x = \frac{12 \pm 8 \sqrt{3}}{2} = 6 \pm 4 \sqrt{3}
$$

Approximate:

$$
\sqrt{3} \approx 1.732
$$

- $x = 6 + 4 \times 1.732 \approx 6 + 6.928 = 12.928$

- $x = 6 - 6.928 \approx -0.928$

Recall that $x = t^2$, so:

- For $x = 1$:

$$
t^2=1 \Rightarrow t= \pm 1
$$

- For $x \approx 12.928$:

$$
t= \pm \sqrt{12.928} \approx \pm 3.6
$$

- For $x \approx -0.928$:

$$
t^2 \approx -0.928 \Rightarrow \text{no real solution}
$$

Summary of roots where numerator $N(t) = 0$:

$$
t = -3.6, \quad -1, \quad 1, \quad 3.6
$$

Corresponding to approximate values.

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3. Sign Analysis and Particle Motion

Understanding where $V(t)$ is positive or negative helps interpret the particle's direction:

- Positive velocity: particle moves in the positive x-direction.
- Negative velocity: particle moves in the negative x-direction.

Using the roots:

- The numerator $N(t)$ changes sign at $t \approx -3.6, -1, 1, 3.6$.
- The denominator $D(t)$ changes sign at $t \approx -0.669$.

Construct sign charts to analyze these intervals.

---

4. Behavior of $v(t)$ in Different Intervals

Interval $t < -3.6$:

- $N(t)$: Negative (since leading term t^6 is positive and dominant, but check at a test point, e.g., $t=-4$):

$$
N(-4) = 4096 - 13 \times 256 + 12 = 4096 - 3328 + 12 = 780
$$

Positive.

- $D(t)$:

$$
10(-4)^3 + 3 = 10(-64) + 3 = -640 + 3 = -637
$$

Negative.

- $v(t)$:

$$
\text{positive numerator} / \text{negative denominator} \Rightarrow v(t) < 0
$$

So, for $t < -3.6$, the velocity is negative; the particle moves in the negative direction.

Interval $-3.6 < t < -0.669$:

- At $t=-2$:

$$
N(-2) = 64 - 13 \times 16 + 12 = 64 -
$$

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