

A Random Sample Of 980 Adults Found That 65% Said They Believe In Ghosts. Construct A 95% Confidence Interval

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Understanding statistical concepts such as confidence intervals is essential for interpreting survey data accurately. In this article, we will explore how to construct a 95% confidence interval based on a survey where a random sample of 980 adults revealed that 65% believe in ghosts. We will break down the process, explain key terms, and discuss the significance of the findings in a clear, organized manner.

Introduction to Confidence Intervals

Confidence intervals are a fundamental concept in statistics that provide a range of values within which we expect a population parameter to lie, with a certain degree of confidence. They are particularly useful in survey research, where it is rarely feasible to survey an entire population.

What Is a Confidence Interval?

A confidence interval offers an estimated range of values that likely contains the true proportion or mean of a population, based on a sample statistic. For example, a 95% confidence interval suggests that if we were to take many samples and compute a confidence interval from each, approximately 95% of these intervals would contain the true population parameter.

Why Use Confidence Intervals?

- They provide a measure of estimation uncertainty.
- They help in making informed decisions based on sample data.
- They are more informative than a simple point estimate (like 65%).

Understanding the Survey Data

The survey data in question states:

- Sample size (n) = 980 adults
- Number of adults who believe in ghosts (x) = 65% of 980
- Sample proportion ($\hat{p}$) = 0.65

Let's analyze these components step by step.

Calculating the Number of Supporters

To find the actual number of adults who believe in ghosts:

- $x = 0.65 \times 980 = 637$ adults (rounded to the nearest whole number).

Sample Proportion ($\hat{p}$)

- $\hat{p} = x / n = 637 / 980 \approx 0.65$

This means approximately 65% of the sample believes in ghosts.

Constructing the 95% Confidence Interval

The process involves several steps, which we will detail below.

Step 1: Identify the Sample Proportion

- $\hat{p} = 0.65$

Step 2: Determine the Standard Error (SE)

The standard error measures the variability of the sample proportion and is calculated as:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Applying the numbers:

$$SE = \sqrt{\frac{0.65 \times (1 - 0.65)}{980}} = \sqrt{\frac{0.65 \times 0.35}{980}}$$

Calculating step-by-step:

- $0.65 \times 0.35 = 0.2275$

- Divide by 980: $0.2275 / 980 \approx 0.000232$

- Take the square root: $\sqrt{0.000232} \approx 0.01523$

So, the standard error:

$$SE \approx 0.01523$$

Step 3: Find the Z-Score for 95% Confidence Level

For a 95% confidence level, the Z-score corresponds to the critical value:

- $Z \approx 1.96$

This value is derived from the standard normal distribution, capturing the middle 95% of the distribution.

Step 4: Calculate the Margin of Error (ME)

The margin of error quantifies the maximum expected difference between the sample estimate and the true population parameter:

$$ME = Z \times SE$$

Calculating:

$$ME = 1.96 \times 0.01523 \approx 0.02984$$

Step 5: Determine the Confidence Interval

Finally, we compute the interval:

- Lower bound: $\hat{p} - ME = 0.65 - 0.02984 \approx 0.6202$
- Upper bound: $\hat{p} + ME = 0.65 + 0.02984 \approx 0.6798$

Expressed as percentages:

- Lower bound: approximately 62.02%
- Upper bound: approximately 67.98%

Therefore, the 95% confidence interval for the proportion of adults who believe in ghosts is approximately 62.02% to 67.98%.

Interpreting the Confidence Interval

This interval suggests that we can be 95% confident that the true proportion of all adults in the population who believe in ghosts falls between approximately 62% and 68%. This is a crucial insight because it accounts for sampling variability and provides a range rather than a single estimate.

Implications of the Findings

- The data indicates that belief in ghosts is relatively common among adults.
- The confidence interval provides a precise estimate, reducing uncertainty.
- Policymakers, researchers, and marketers can use this information to understand cultural or societal beliefs.

Factors Affecting the Confidence Interval

Several factors influence the width and accuracy of a confidence interval:

1. **Sample Size:** Larger samples reduce the standard error, resulting in a narrower confidence interval.
2. **Sample Variability:** More variability in responses increases the standard error, widening the interval.
3. **Confidence Level:** Higher confidence levels (e.g., 99%) increase the Z-score, leading to wider

intervals.

In this case, the sample size of 980 is substantial, providing a relatively precise estimate.

Limitations and Considerations

While confidence intervals are powerful tools, they are based on certain assumptions and limitations:

- **Random Sampling:** The sample must be randomly selected to ensure representativeness.
- **Normality:** The approximation relies on the sample size being sufficiently large; generally, $n \times \hat{p}$ and $n \times (1 - \hat{p})$ should both be ≥ 10 .
- **Survey Bias:** Non-response bias or inaccurate responses can affect the validity.

It is essential to consider these factors when interpreting the results.

Practical Applications of Confidence Intervals

Understanding confidence intervals extends beyond academic exercises. Here are some practical scenarios:

1. **Marketing Strategies:** Companies targeting demographics with belief in supernatural phenomena can tailor campaigns based on belief prevalence.
2. **Public Policy:** Policymakers can gauge public opinion to inform educational or cultural programs.
3. **Further Research:** Researchers can compare different populations or regions to understand variations in beliefs.

Summary and Conclusion

In conclusion, constructing a 95% confidence interval based on the survey data reveals that approximately between 62% and 68% of adults believe in ghosts. This range provides a statistically sound estimate of the true proportion in the population, accounting for sampling variability. Such insights are valuable for researchers, marketers, policymakers, and anyone interested in understanding public beliefs and trends.

To recap, the steps involved in constructing the confidence interval are:

- Calculate the sample proportion ($\hat{p}$).
- Compute the standard error (SE).
- Determine the z-score for the desired confidence level.
- Calculate the margin of error (ME).
- Derive the confidence interval bounds.

Recognizing the importance of sample size, confidence level, and potential biases ensures accurate interpretation and application of these statistical results.

By mastering these concepts, you can confidently analyze survey data and derive meaningful insights that inform decision-making across various fields.

Frequently Asked Questions

What is the main purpose of constructing a 95% confidence interval in this context?

To estimate the range within which the true proportion of all adults who believe in ghosts likely falls, with 95% confidence.

What is the sample proportion of adults who believe in ghosts in this study?

The sample proportion is 65%, or 0.65.

How do you calculate the standard error for this confidence interval?

The standard error (SE) is calculated using the formula $SE = \sqrt{\hat{p}(1 - \hat{p}) / n}$, where $\hat{p}$ is the sample proportion and n is the sample size.

What is the formula for constructing a 95% confidence interval for a population proportion?

The 95% confidence interval is $\hat{p} \pm Z SE$, where Z is the Z-value corresponding to 95% confidence (approximately 1.96).

Calculate the standard error for this sample.

$$SE = \sqrt{0.65 (1 - 0.65) / 980} \approx 0.0134.$$

What are the lower and upper bounds of the 95% confidence interval for the proportion?

Lower bound $\approx 0.65 - 1.96 (0.0134) \approx 0.623$, and upper bound $\approx 0.65 + 1.96 (0.0134) \approx 0.677$.

What does the 95% confidence interval (approximately 62.3% to 67.7%) tell us about the beliefs of all adults?

We are 95% confident that between approximately 62.3% and 67.7% of all adults believe in ghosts.

Why is it important to specify the confidence level (e.g., 95%) when constructing such intervals?

Because it indicates the probability that the interval captures the true population parameter over many repeated samples.

Can the confidence interval be interpreted as the probability that the true proportion lies within this range?

No, the true proportion is fixed; the confidence interval either contains it or not. The 95% confidence level reflects the long-term success rate of the method used to generate intervals.

What assumptions are necessary for constructing this confidence interval accurately?

Assumptions include that the sample is random, the sample size is large enough for the normal approximation ($n\hat{p}$ and $n(1 - \hat{p})$ both ≥ 10), and the data are independent.

Additional Resources

A Random Sample Of 980 Adults Found That 65% Said They Believe In Ghosts. Construct A 95% Confidence Interval

In today's exploration of statistical inference, understanding how to construct a confidence interval based on sample data is an essential skill. A random sample of 980 adults found that 65% said they believe in ghosts, which provides a valuable snapshot into public belief systems. But how confident can we be that this percentage reflects the broader population? In this guide, we will walk through the process of constructing a 95% confidence interval for the true proportion of adults who believe in ghosts, offering insight into what this interval tells us about the wider community.

Understanding the Basics: What Is a Confidence Interval?

Before diving into calculations, it's important to clarify what a confidence interval (CI) represents. A confidence interval provides a range of values within which we expect the true population parameter—here, the proportion of all adults who believe in ghosts—to fall, with a certain level of confidence.

- In our case, a 95% confidence interval means that if we were to take many samples and build a confidence interval from each, approximately 95% of those intervals would contain the true proportion.

- Why is this useful? Because it accounts for sampling variability, giving us a measure of uncertainty around our estimate.

Step 1: Gather the Sample Data

From the problem statement, we have:

- Sample size, $n = 980$
- Number of adults who believe in ghosts, $x = 0.65 \times 980 = 637$ (since 65% of 980)

Calculating the actual number:

$$637 = 0.65 \times 980$$

Step 2: Calculate the Sample Proportion

The sample proportion $\hat{p}$ (pronounced "p-hat") is:

$$\hat{p} = \frac{x}{n} = \frac{637}{980} \approx 0.65$$

This confirms the reported percentage, which simplifies our calculations.

Step 3: Determine the Margin of Error (ME)

The margin of error quantifies the range of plausible values for the population proportion, considering sampling variability. For a proportion, the formula for the margin of error at a given confidence level is:

$$ME = Z^* \times \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}}$$

Where:

- Z^* = critical value corresponding to the confidence level (for 95%, $Z^* \approx 1.96$)
- $\hat{p}$ = sample proportion
- n = sample size

Step 4: Find the Critical Z-Value

For a 95% confidence level, the critical Z-value is:

$$\hat{Z} = 1.96$$

This value comes from the standard normal distribution, capturing the middle 95% of the distribution.

Step 5: Calculate the Standard Error (SE)

The standard error (SE) measures the variability of the sample proportion:

$$SE = \sqrt{\frac{\hat{p}(1 - \hat{p})}{n}} = \sqrt{\frac{0.65 \times (1 - 0.65)}{980}}$$

Calculations:

$$\begin{aligned} &-(1 - 0.65 = 0.35) \\ &-(0.65 \times 0.35 = 0.2275) \end{aligned}$$

Thus:

$$SE = \sqrt{\frac{0.2275}{980}} \approx \sqrt{0.00023214} \approx 0.01524$$

Step 6: Compute the Margin of Error

Now, multiply the Z-value by the standard error:

$$ME = 1.96 \times 0.01524 \approx 0.02987$$

Rounded to four decimal places, the margin of error is approximately 0.0299.

Step 7: Construct the Confidence Interval

Finally, the 95% confidence interval is:

$$\hat{p} \pm ME$$

Calculations:

- Lower bound:

$$\sqrt{0.65 - 0.0299} \approx 0.6201$$

- Upper bound:

$$\sqrt{0.65 + 0.0299} \approx 0.6799$$

Expressed as percentages, the confidence interval is approximately 62.0% to 68.0%.

Final Confidence Interval:

The 95% confidence interval for the proportion of adults who believe in ghosts is approximately 62.0% to 68.0%.

This means we are 95% confident that between 62.0% and 68.0% of the entire adult population believe in ghosts, based on this sample data.

Interpreting the Results

What does this interval tell us?

- The sample proportion (65%) suggests a majority of adults believe in ghosts.
- The confidence interval narrows down the true proportion, accounting for sampling variability.
- Since the entire interval is above 60%, it indicates a significant portion of the population holds this belief.
- The interval does not include very low or very high percentages, giving us a reasonable estimate with some margin of error.

Practical implications:

- For researchers: This interval provides a basis for further studies or for designing surveys.
- For policymakers: Understanding such beliefs can inform cultural or educational initiatives.
- For the general public: It offers insight into how widespread beliefs in supernatural phenomena might be.

Additional Considerations

Assumptions Behind the Confidence Interval

Constructing this confidence interval relies on some key assumptions:

1. Random Sampling: The sample of 980 adults was randomly selected, ensuring representativeness.
2. Independence: Each person's response is independent of others.
3. Sample Size Adequacy: The sample size is large enough for the normal approximation to be valid, which generally holds if:

$$\begin{aligned} & n \hat{p} \geq 10 \quad \text{and} \quad n (1 - \hat{p}) \geq 10 \end{aligned}$$

Let's check:

- $n \hat{p} = 980 \times 0.65 = 637 \geq 10$
- $n (1 - \hat{p}) = 980 \times 0.35 = 343 \geq 10$

Both conditions are satisfied, confirming the appropriateness of the normal approximation.

Limitations

- The confidence interval provides an estimate based on the sample; actual beliefs may vary.
- If the sampling method was biased or non-random, the interval's validity could be compromised.
- The interval does not account for potential measurement errors or response biases.

Summary

Constructing a 95% confidence interval for a population proportion involves several straightforward steps:

1. Gather sample data and calculate the sample proportion.
2. Find the critical Z-value corresponding to the confidence level.
3. Calculate the standard error.
4. Determine the margin of error.
5. Build the interval by adding and subtracting the margin from the sample proportion.

In our case, with a sample of 980 adults and a 65% belief rate, the 95% confidence interval is approximately 62.0% to 68.0%. This interval offers a statistically sound estimate of the true proportion of adults who believe in ghosts, providing valuable insights into public beliefs and attitudes.

By understanding and applying these statistical methods, researchers and analysts can effectively interpret survey data, communicate findings accurately, and make informed decisions based on sample evidence.

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