A Stone With A Mass Of 0.100kg Rests On A Frictionless, Horizontal Surface. A Bullet Of Mass 2.50g Traveling

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Understanding the dynamics of objects interacting on frictionless surfaces is fundamental in physics. In this article, we'll explore a classic problem involving a stone and a bullet, examining how principles like conservation of momentum and energy apply to such a scenario. This comprehensive guide aims to clarify the physics concepts involved, analyze the problem step-by-step, and provide insights useful for students, educators, and physics enthusiasts alike.

Introduction to the Problem Scenario

Imagine a setup where a stone, with a mass of 0.100 kg, rests on a frictionless, horizontal surface. A bullet, with a mass of 2.50 grams (or 0.00250 kg), is traveling toward the stone with a certain initial velocity. The key details include:

- Mass of the stone: 0.100 kg
- Mass of the bullet: 2.50 g (0.00250 kg)
- Surface type: Frictionless and horizontal
- Initial conditions: The bullet is moving with a known initial velocity, while the stone is initially at rest

The main goal is to analyze what happens when the bullet strikes the stone, focusing on the resulting velocities, momentum transfer, and energy considerations.

Fundamental Physics Concepts Involved

Understanding this problem requires a grasp of several core physics principles:

1. Conservation of Momentum

In an isolated system (no external forces), the total momentum before and after an interaction remains constant.

$$
\text{Total momentum before} = \text{Total momentum after}
$$

Mathematically:

$$
m_b v_{b_i} + m_s v_{s_i} = m_b v_{b_f} + m_s v_{s_f}
$$

where:
- m_b, m_s: masses of bullet and stone
- v_{b_i}, v_{s_i}: initial velocities
- v_{b_f}, v_{s_f}: final velocities

Since the stone is initially at rest,

$$
v_{s_i} = 0
$$

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2. Conservation of Kinetic Energy

Depending on whether the collision is elastic or inelastic, kinetic energy may or may not be conserved.

- Elastic collision: Both momentum and kinetic energy are conserved.
- Inelastic collision: Momentum is conserved, but some kinetic energy is transformed into other forms, such as heat or deformation.

In many real-world collisions, especially involving objects like stones and bullets, the collision is often inelastic.

Analyzing the Collision: Step-by-Step Approach

To thoroughly understand the consequences of the interaction, let's consider key steps:

1. Defining Known Quantities

- Mass of stone: $m_s = 0.100\, \text{kg}$
- Mass of bullet: $m_b = 0.00250\, \text{kg}$
- Initial velocity of bullet: $v_{b_i} = ?$ (to be specified)
- Initial velocity of stone: $v_{s_i} = 0$

Suppose the initial velocity of the bullet is given as $v_{b_i} = 300\, \text{m/s}$.

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2. Applying Conservation of Momentum

Assuming an inelastic collision where the bullet embeds into the stone (common in many real-world scenarios), the combined object will move with a common velocity after impact:

$$
v_f = v_{b_f} = v_{s_f}
$$

Using conservation of momentum:

$$
m_b v_{b_i} + m_s v_{s_i} = (m_b + m_s) v_f
$$

Since $v_{s_i} = 0$:

$$
v_f = \frac{m_b v_{b_i}}{m_b + m_s}
$$

Plugging in the known values:

$$
v_f = \frac{0.00250\, \text{kg} \times 300\, \text{m/s}}{0.00250\, \text{kg} + 0.100\, \text{kg}} = \frac{0.75}{0.10250} \approx 7.32\, \text{m/s}
$$

This final velocity indicates the combined bullet-stone system moves at approximately 7.32 m/s after impact.

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3. Calculating Post-Collision Kinetic Energy

The initial kinetic energy of the bullet:

$$
KE_{initial} = \frac{1}{2} m_b v_{b_i}^2 = \frac{1}{2} \times 0.00250\, \text{kg} \times (300)^2 \approx 112.5\, \text{J}
$$

The kinetic energy of the combined system after collision:

$$
KE_{final} = \frac{1}{2} (m_b + m_s) v_f^2
$$

Substitute the known values:

$$
KE_{final} = \frac{1}{2} \times 0.1025\, \text{kg} \times (7.32)^2 \approx 0.05125 \times 53.58 \approx 2.75\, \text{J}
$$

The significant drop from 112.5 J to 2.75 J indicates that much of the initial kinetic energy is lost — likely converted into heat, deformation, sound, or other forms.

Implications of the Collision: Energy Loss and Practical Considerations

The calculations reveal that in an inelastic collision, kinetic energy is not conserved. The difference in initial and final kinetic energies (approximately 109.75 J) is lost to non-mechanical forms.

Key points:

- Energy dissipation: The energy lost manifests as heat, deformation, or sound. For example, the stone may crack or deform slightly.
- Real-world implications: Such energy losses are critical in designing impact-resistant materials and understanding projectile behavior.
- Safety considerations: High-velocity impacts can cause significant damage even if the final velocities are relatively low.

Extension: Elastic Collision Scenario

Suppose instead the collision is elastic, meaning kinetic energy is conserved. How do velocities change?

Applying conservation of momentum and energy:

$$
m_b v_{b_i} = m_b v_{b_f} + m_s v_{s_f}
$$

$$
\frac{1}{2} m_b v_{b_i}^2 = \frac{1}{2} m_b v_{b_f}^2 + \frac{1}{2} m_s v_{s_f}^2
$$

By solving these equations, we can determine the velocities after a perfectly elastic collision.

Key results:

- The velocities swap in some cases according to the one-dimensional elastic collision formulas.
- For example:

$$
v_{b_f} = \frac{(m_b - m_s)}{m_b + m_s} v_{b_i}
$$

$$
v_{s_f} = \frac{2 m_b}{m_b + m_s} v_{b_i}
$$

Plugging in values:

$$
v_{s_f} = \frac{2 \times 0.00250}{0.1025} \times 300 \approx \frac{0.005}{0.1025} \times 300 \approx 14.63\, \text{m/s}
$$

This indicates the stone would move at approximately 14.63 m/s after impact in an elastic collision.

Practical Applications and Real-World Relevance

Understanding the physics of collisions involving stones and bullets has numerous practical applications:

1. Ballistics and Projectile Design

- Engineers design bullets and projectiles considering energy transfer and impact behavior.
- Knowledge of inelastic and elastic collisions helps in predicting penetration and damage.

2. Material Science

- Studying impact energy dissipation informs the development of impact-resistant materials.
- Testing how materials deform under high-velocity impacts is essential for safety equipment.

3. Safety and Accident Prevention

- Analyzing collision dynamics aids in designing safer environments, such as barriers or protective gear.

4. Education and Research

- Physics educators utilize such scenarios to demonstrate conservation principles experimentally.
- Researchers explore collision behaviors to develop better models of real-world impacts.

Summary and Key Takeaways

- A stone resting on a frictionless surface offers a simplified model to study momentum and energy transfer.
- When a moving bullet strikes the stone, conservation of momentum allows calculation of the system's velocity after impact.
- The nature of the collision (elastic vs. inelastic) drastically influences the energy distribution.
- In inelastic collisions, most kinetic energy is dissipated, which has practical implications in engineering and safety.
- Accurate modeling of such collisions informs various fields, including ballistics, materials science, and safety engineering.

Conclusion

Analyzing the interaction between a moving bullet and a stationary stone on a frictionless surface provides valuable insights into fundamental physics principles. Whether considering inelastic or elastic collisions, understanding how momentum and energy are

Frequently Asked Questions

What is the initial momentum of the bullet before it hits the stone?

The initial momentum of the bullet is calculated by multiplying its mass by its velocity (p = mv). For example, if the bullet's velocity is known, you can compute its momentum accordingly.

How does the conservation of momentum apply in this scenario?

Since the surface is frictionless, the total momentum before and after the collision remains conserved. The combined momentum of the bullet and stone after impact equals the initial momentum of the bullet.

What will be the velocity of the stone immediately after the collision?

The stone's velocity after the collision can be found using the law of conservation of momentum: v_stone = (m_bullet v_bullet) / m_stone, assuming a perfectly inelastic collision and the bullet embedding into the stone or bouncing off.

If the bullet embeds into the stone, how does that affect the system's final velocity?

In an inelastic collision where the bullet embeds into the stone, the combined mass moves with a velocity determined by total momentum divided by combined mass: v_final = (m_bullet v_bullet) / (m_stone + m_bullet).

How does the mass of the bullet influence the final velocity of the stone?

A greater bullet mass results in a higher transfer of momentum, leading to a higher final velocity of the stone after impact, assuming the bullet's velocity remains constant.

What role does the frictionless surface play in this problem?

The frictionless surface means there are no external horizontal forces acting on the system, allowing the conservation of momentum to hold true throughout the collision process.

How would the outcome change if the surface were not frictionless?

If friction were present, external forces would act on the system, causing momentum to not be conserved, and the final velocities would be affected accordingly.

What assumptions are typically made in analyzing such collision problems?

Common assumptions include neglecting air resistance, assuming an idealized collision (elastic or inelastic), and considering the surface to be perfectly frictionless to simplify calculations.

How can the kinetic energy of the system be calculated after the collision?

Kinetic energy after the collision can be calculated using KE = 0.5 total mass (final velocity)^2, but note that in inelastic collisions, some kinetic energy is transformed into other forms of energy, so KE decreases.

Additional Resources

Impact Dynamics of a Bullet on a Stone: An In-Depth Analysis

Understanding the physics behind collisions, especially those involving projectiles and stationary objects, is fundamental in classical mechanics. The scenario involving a stone with a mass of 0.100 kg resting on a frictionless, horizontal surface and a bullet of mass 2.50 g (0.00250 kg) traveling toward it provides an excellent case study to explore principles such as conservation of momentum, energy transfer, and collision types. This detailed review delves into the physics, mathematical modeling, and real-world implications of such a collision.

Initial Setup and Key Assumptions

Before analyzing the collision, it is crucial to clarify the initial conditions and assumptions:

- The stone is stationary on a frictionless, horizontal surface. No external horizontal forces act on it prior to impact.
- The bullet approaches with a known initial velocity ($v_{b,i}$); for the sake of analysis, let's denote this as $v_{b,i}$.
- The mass of the stone (m_s): 0.100 kg.
- The mass of the bullet (m_b): 2.50 g = 0.00250 kg.
- The initial velocity of the bullet ($v_{b,i}$): To proceed, we assume a typical value, say, $v_{b,i} = 500 \, \mathrm{m/s}$. This value can be adjusted based on specific scenarios.
- The surface is frictionless, meaning no kinetic energy is lost to friction; the only energy transfer occurs during collision.
- The collision is modeled as either elastic or inelastic, depending on whether kinetic energy is conserved.

Fundamental Principles Involved

The analysis of this collision hinges on several core physics concepts:

Conservation of Momentum

- In the absence of external forces, the total linear momentum of the system remains constant throughout the collision.
- Mathematically:

$$
m_b v_{b,i} + m_s v_{s,i} = m_b v_{b,f} + m_s v_{s,f}
$$

where:
- $v_{b,i}$, $v_{b,f}$: initial and final velocities of the bullet.
- $v_{s,i}$, $v_{s,f}$: initial and final velocities of the stone.
- Since the stone starts at rest, $v_{s,i} = 0$.

Types of Collisions

- Elastic Collision: Both kinetic energy and momentum are conserved.
- Inelastic Collision: Momentum is conserved, but some kinetic energy is transformed into internal energy (heat, deformation). Perfectly inelastic collisions involve the objects sticking together after impact.

Analyzing the Collision: Elastic vs. Inelastic

Given the scenario, we explore both possibilities:

1. Perfectly Elastic Collision

- Assumes no energy loss.
- The velocities after impact can be derived from conservation laws.

Mathematical derivation:
- Conservation of momentum:

$$
m_b v_{b,i} = m_b v_{b,f} + m_s v_{s,f}
$$

- Conservation of kinetic energy:

$$
\frac{1}{2} m_b v_{b,i}^2 = \frac{1}{2} m_b v_{b,f}^2 + \frac{1}{2} m_s v_{s,f}^2
$$

- Solving these equations yields the final velocities:

$$
v_{b,f} = \frac{m_b - m_s}{m_b + m_s} v_{b,i}
$$

$$
v_{s,f} = \frac{2 m_b}{m_b + m_s} v_{b,i}
$$

Numerical calculation:
- Plugging in the values:

$$
v_{b,f} = \frac{0.00250 - 0.100}{0.00250 + 0.100} \times 500 \approx \frac{-0.0975}{0.1025} \times 500 \approx -475.6 \, \mathrm{m/s}
$$

$$
v_{s,f} = \frac{2 \times 0.00250}{0.1025} \times 500 \approx \frac{0.005}{0.1025} \times 500 \approx 24.4 \, \mathrm{m/s}
$$

- Interpretation:
- The bullet reverses direction post-collision (negative velocity relative to initial direction).
- The stone gains a velocity of approximately 24.4 m/s in the initial direction of the bullet.

Kinetic Energy Consideration:
- Initial KE:

$$
KE_{initial} = \frac{1}{2} \times 0.00250 \times 500^2 = 0.00125 \times 250000 = 312.5 \, \mathrm{J}
$$

- Final KE:

$$
KE_{final} = \frac{1}{2} \times 0.00250 \times 475.6^2 + \frac{1}{2} \times 0.100 \times 24.4^2
$$

$$
\approx 0.00125 \times 226212 + 0.05 \times 595.36 \approx 282.7 + 29.8 = 312.5\, \mathrm{J}
$$

- The KE is conserved, confirming elastic collision assumptions.

2. Perfectly Inelastic Collision

- The bullet embeds into the stone, moving together after impact.

Conservation of momentum:

$$
m_b v_{b,i} = (m_b + m_s) v_{f}
$$

$$
v_{f} = \frac{m_b v_{b,i}}{m_b + m_s}
$$

Numerical calculation:

$$
v_{f} = \frac{0.00250 \times 500}{0.00250 + 0.100} = \frac{1.25}{0.1025} \approx 12.2 \, \mathrm{m/s}
$$

Kinetic energy:

$$
KE_{initial} = 312.5 \, \mathrm{J}
$$

$$
KE_{final} = \frac{1}{2} (m_b + m_s) v_{f}^2 \approx 0.05125 \times 12.2^2
\approx 0.05125 \times 148.8 \approx 7.63 \, \mathrm{J}
$$

- Energy loss:

$$
\Delta KE = 312.5 - 7.63 = 304.87 \, \mathrm{J}
$$

- The significant energy loss indicates an inelastic collision where much of the initial KE converts into internal energy, deformation, heat, or sound.

Physical Implications and Real-World Applications

Effects on the Stone and Bullet

- In elastic collisions, the stone would be propelled forward at about 24.4 m/s, a substantial velocity considering the initial conditions.
- In inelastic collisions, the stone would move at approximately 12.2 m/s, and the energy dissipated would manifest as heat, deformation, or sound, leading to possible damage or deformation of the stone.

Practical Scenarios

- Ballistics Testing: Understanding impact velocities helps in designing safe and effective projectile systems.
- Material Science: Studying how materials absorb energy during impacts influences the creation of protective gear or impact-resistant structures.
- Engineering Design: Insights into collision physics inform the design of safety features in vehicles, aerospace components, and industrial machinery.

Limitations and Additional Factors

- Real-world impacts often involve some degree of friction, deformation, and energy loss, making perfectly elastic models idealized.
- Air resistance, spin of the projectile, and surface irregularities can

alter outcomes.
- For high-velocity impacts, material strength and failure modes become significant.

Advanced Topics and Further Considerations

Energy Transfer and Internal Damage

- The energy transferred to the stone can cause cracking, fragmentation, or other structural damage depending on its material properties.
- The amount of energy absorbed or dissipated influences the extent of damage, which is crucial in ballistics and safety testing.

Coefficient of Restitution

- The coefficient of restitution (e) quantifies the 'bounciness' of the collision.
- For elastic collisions, $e \approx 1$; for inelastic, $e < 1$.
- Measuring e experimentally helps in modeling real-world impacts.

Impacts at Different Velocities

- At lower velocities, elastic models are more accurate.
- At higher velocities, material deformation and plasticity become dominant, requiring complex models such as finite element analysis.

Conservation of Angular Momentum