A Triangle Is Defined By The Points A(8,5,7) , B(3,6,6), And C(4,k,9). The Area Of The Triangle Is (8920.5).

A Triangle Is Defined By The Points A(8,5,7) , B(3,6,6), And C(4,k,9). The Area Of The Triangle Is (8920.5).

Understanding the properties of triangles in three-dimensional space is fundamental in fields such as geometry, physics, engineering, and computer graphics. In this article, we will explore how to analyze a triangle defined by three points in 3D space—specifically, points A, B, and C—and how to determine the unknown coordinate \(k \) based on the given area. This comprehensive guide covers the mathematical concepts involved, including vector operations, the formula for the area of a triangle in 3D, and practical steps to compute the missing coordinate.

Introduction to Triangles in 3D Space

A triangle in three-dimensional space is formed by three non-collinear points. Unlike in a 2D plane, where the area can be easily computed using simple base and height, calculating the area of a triangle in 3D involves vector operations. The key to understanding such triangles lies in vector algebra—particularly, the cross product.

Coordinates of the Given Points

Let's first restate the points involved:

- Point A: (8, 5, 7)
- Point B: (3, 6, 6)
- Point C: (4, k, 9)

Here, \(k \) is an unknown coordinate that we need to determine based on the given area of the triangle.

Understanding the Area of a Triangle in 3D Space

The area \(S \) of a triangle formed by points \(A \), \(B \), and \(C \) in 3D space can be calculated using vector operations as follows:

\[
S = \frac{1}{2} \| \vec{AB} \times \vec{AC} \|
\]

where:

- \(\vec{AB} \) is the vector from point A to point B
- \(\vec{AC} \) is the vector from point A to point C
- \(\times \) denotes the cross product
- \(\| \cdot \| \) is the magnitude (length) of the resulting vector

The magnitude of the cross product gives twice the area of the triangle, so dividing by 2 yields the area.

Step-by-Step Calculation Process

To find \(k \), follow these steps:

1. Compute Vectors \(\vec{AB} \) and \(\vec{AC} \)

\[
\vec{AB} = B - A = (3 - 8, 6 - 5, 6 - 7) = (-5, 1, -1)
\]

\[
\vec{AC} = C - A = (4 - 8, k - 5, 9 - 7) = (-4, k - 5, 2)
\]

2. Calculate the Cross Product \(\vec{AB} \times \vec{AC} \)

Using the determinant method:

\[
\vec{AB} \times \vec{AC} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-5 & 1 & -1 \\

-4 & k - 5 & 2
\end{vmatrix}
\]

Expanding this determinant:

\[
\mathbf{i} \left(1 \times 2 - (-1) \times (k - 5) \right) -
\mathbf{j} \left(-5 \times 2 - (-1) \times (-4) \right) +
\mathbf{k} \left(-5 \times (k - 5) - 1 \times (-4) \right)
\]

Simplify each component:

- \(\mathbf{i} \) component:

\[
2 - (-1)(k - 5) = 2 + (k - 5) = (k - 3)
\]

- \(\mathbf{j} \) component:

\[
- (-10 - 4) = - (-14) = 14
\]

- \(\mathbf{k} \) component:

\[
-5(k - 5) + 4 = -5k + 25 + 4 = -5k + 29
\]

So, the cross product vector is:

\[
\vec{AB} \times \vec{AC} = (k - 3, 14, -5k + 29)
\]

---

3. Compute the magnitude of the cross product

\[
\| \vec{AB} \times \vec{AC} \| = \sqrt{ (k - 3)^2 + 14^2 + (-5k + 29)^2 }
\]

Calculating each term:

\[

(k - 3)^2 = k^2 - 6k + 9
\]
\[
14^2 = 196
\]
\[
(-5k + 29)^2 = 25k^2 - 290k + 841
\]

Adding all together:

\[
k^2 - 6k + 9 + 196 + 25k^2 - 290k + 841 = (k^2 + 25k^2) + (-6k - 290k) + (9 + 196 + 841)
\]

Simplify:

\[
26k^2 - 296k + 1046
\]

Therefore,

\[
\| \vec{AB} \times \vec{AC} \| = \sqrt{26k^2 - 296k + 1046}
\]

Determining \(k \) Using the Given Area

The area \(S \) is given as 8920.5. Recall:

\[
S = \frac{1}{2} \| \vec{AB} \times \vec{AC} \|
\]

Thus,

\[
8920.5 = \frac{1}{2} \times \sqrt{26k^2 - 296k + 1046}
\]

Multiply both sides by 2:

\[
17841 = \sqrt{26k^2 - 296k + 1046}
\]

Square both sides:

$$
(17841)^2 = 26k^2 - 296k + 1046
$$

Calculate 17841^2:

$$
17841^2 = 318,099,681
$$

Now, set up the quadratic equation:

$$
26k^2 - 296k + 1046 = 318,099,681
$$

Subtract 318,099,681 from both sides:

$$
26k^2 - 296k + 1046 - 318,099,681 = 0
$$

Simplify:

$$
26k^2 - 296k - 318,098,635 = 0
$$

Divide the entire equation by 2 to simplify:

$$
13k^2 - 148k - 159,049,317.5 = 0
$$

Solving for k

The quadratic equation:

$$
13k^2 - 148k - 159,049,317.5 = 0
$$

Using the quadratic formula:

\[

k = \frac{148 \pm \sqrt{(-148)^2 - 4 \times 13 \times (-159,049,317.5)}}{2 \times 13}
\]

Compute discriminant \(D \):

\[
D = 148^2 - 4 \times 13 \times (-159,049,317.5)
\]

Calculate:

\[
148^2 = 21,904
\]

\[
4 \times 13 = 52
\]

\[
52 \times (-159,049,317.5) = -8,259,763,267
\]

Since it's multiplied by a negative, the subtraction becomes addition:

\[
D = 21,904 + 8,259,763,267 = 8,259,785,171
\]

Calculate the square root:

\[
\sqrt{8,259,785,171} \approx 90,882.68
\]

Now, compute \(k \):

\[
k = \frac{148 \pm 90,882.68}{26}
\]

Two solutions:

1.

\[
k = \frac{148 + 90,882.68}{26} \approx \frac{90,930.68}{26} \approx 3,504.25
\]

2.

$$
k = \frac{148 - 90,882.68}{26} \approx \frac{-90,734.68}{26} \approx -3,495.19
$$

Interpreting the Results

The two possible values for k:

- $k \approx 3,504$

Frequently Asked Questions

How can we find the value of k in the triangle with points A(8,5,7), B(3,6,6), and C(4,k,9) given its area is 8920.5?

To find k, we can use the formula for the area of a triangle in 3D space: Area = (1/2) |AB × AC|. First, find vectors AB and AC, then compute their cross product magnitude, set the area equal to 8920.5, and solve for k.

What is the step-by-step process to determine the value of k for the given triangle's area?

1. Calculate vectors AB = B - A and AC = C - A.
2. Find the cross product AB × AC.
3. Compute the magnitude of this cross product.
4. Set (1/2) times this magnitude equal to 8920.5.
5. Solve the resulting equation for k.

Can the area of 8920.5 be correct for a triangle with points in 3D space like A(8,5,7), B(3,6,6), and C(4,k,9)?

Given the coordinates, an area of 8920.5 is unusually large and likely unrealistic for these points unless the points are very far apart. It's important to verify calculations, but mathematically, it is possible if the points are scaled accordingly.

What formulas are used to calculate the area of a triangle in three-dimensional space?

The area of a triangle in 3D space can be found using the formula: Area = (1/2) |AB × AC|, where AB and AC are vectors from one vertex to the other two, and × denotes the cross product.

How does the value of k affect the shape and size of the triangle formed by points A, B, and C?

The coordinate k determines the position of point C along the y-axis. Changing k alters vector AC, which impacts the cross product magnitude and thus the area, influencing the size and shape of the triangle.

Is it necessary to verify the given area before solving for k in the triangle with specified points?

Yes, verifying the area calculation ensures consistency and correctness before solving for k, especially since the given area value is quite large and may require confirmation or correction based on actual point coordinates.

Additional Resources

A Triangle Is Defined By The Points A(8,5,7), B(3,6,6), And C(4,k,9). The Area Of The Triangle Is 8920.5

Understanding the geometry of a triangle in three-dimensional space involves analyzing the positions of its vertices and applying vector operations to determine properties such as area. In this guide, we delve into the problem of a triangle defined by the points A(8,5,7), B(3,6,6), and C(4,k,9), with a known area of 8920.5. We will explore how to find the unknown coordinate k, interpret the problem, and understand the underlying mathematics step-by-step.

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1. Introduction to the Problem

Given three points in 3D space:
- $A(8, 5, 7)$
- $B(3, 6, 6)$
- $C(4, k, 9)$

the area of the triangle formed by these points is 8920.5.

The goal is to determine the value of k that satisfies this area condition.

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2. Mathematical Foundations

2.1 Coordinates and Vectors in 3D Space

Points in 3D space are represented as ordered triples (x, y, z). To analyze the area of a triangle, we typically consider vectors formed from the vertices:
- $\vec{AB} = \vec{B} - \vec{A}$
- $\vec{AC} = \vec{C} - \vec{A}$

The area of a triangle in 3D space can be found using the cross product of two of its side vectors:
$$
\text{Area} = \frac{1}{2} \left| \vec{AB} \times \vec{AC} \right|
$$
where $\left| \cdot \right|$ denotes the magnitude of the vector.

2.2 Cross Product and Its Magnitude

For vectors $\vec{u} = (u_1, u_2, u_3)$ and $\vec{v} = (v_1, v_2, v_3)$:
$$
\vec{u} \times \vec{v} = (u_2 v_3 - u_3 v_2,\ u_3 v_1 - u_1 v_3,\ u_1 v_2 - u_2 v_1)
$$
and the magnitude is:
$$
|\vec{u} \times \vec{v}| = \sqrt{(u_2 v_3 - u_3 v_2)^2 + (u_3 v_1 - u_1 v_3)^2 + (u_1 v_2 - u_2 v_1)^2}
$$

---

3. Computing the Vectors $\vec{AB}$ and $\vec{AC}$

3.1 Vector $\vec{AB}$

$$
\vec{AB} = (3 - 8, 6 - 5, 6 - 7) = (-5, 1, -1)
$$

3.2 Vector $\vec{AC}$

$$
\vec{AC} = (4 - 8, k - 5, 9 - 7) = (-4, k - 5, 2)
$$

---

4. Calculating the Cross Product $\vec{AB} \times \vec{AC}$

Using the vectors:

$$
\vec{AB} = (-5, 1, -1)
$$
$$
\vec{AC} = (-4, k - 5, 2)
$$

The cross product components:

$$
\begin{aligned}

(\vec{AB} \times \vec{AC})_x &= (1)(2) - (-1)(k - 5) = 2 + (k - 5) = k - 3 \\
(\vec{AB} \times \vec{AC})_y &= -\left((-5)(2) - (-1)(-4)\right) = -(-10 - 4) = -(-14) = 14 \\
(\vec{AB} \times \vec{AC})_z &= (-5)(k - 5) - (1)(-4) = -5(k - 5) + 4 = -5k + 25 + 4 = -5k + 29
\end{aligned}
\]

Thus,

\[
\vec{AB} \times \vec{AC} = (k - 3,\ 14,\ -5k + 29)
\]

---

5. Finding the Magnitude of the Cross Product

The magnitude is:

\[
|\vec{AB} \times \vec{AC}| = \sqrt{(k - 3)^2 + 14^2 + (-5k + 29)^2}
\]

Expanding:

\[
\begin{aligned}
(k - 3)^2 &= k^2 - 6k + 9 \\
14^2 &= 196 \\
(-5k + 29)^2 &= 25k^2 - 290k + 841
\end{aligned}
\]

Adding these:

\[
k^2 - 6k + 9 + 196 + 25k^2 - 290k + 841 = (k^2 + 25k^2) + (-6k - 290k) + (9 + 196 + 841)
\]
\[
= 26k^2 - 296k + 1046
\]

Therefore,

\[
|\vec{AB} \times \vec{AC}| = \sqrt{26k^2 - 296k + 1046}
\]

---

6. Relating the Cross Product Magnitude to the Triangle Area

Given that the area of the triangle is 8920.5:

$$
\text{Area} = \frac{1}{2} |\vec{AB} \times \vec{AC}| \Rightarrow |\vec{AB} \times \vec{AC}| = 2 \times 8920.5 = 17841
$$

Set the magnitude equal to 17841:

$$
\sqrt{26k^2 - 296k + 1046} = 17841
$$

Squaring both sides:

$$
26k^2 - 296k + 1046 = (17841)^2
$$

Calculate $(17841)^2$:

$$
17841^2 = 318,135,681
$$

---

7. Solving the Quadratic Equation for k

$$
26k^2 - 296k + 1046 = 318,135,681
$$

Bring all to one side:

$$
26k^2 - 296k + 1046 - 318,135,681 = 0
$$

Simplify:

$$
26k^2 - 296k - 318,134,635 = 0
$$

Divide through by 2 to simplify:

$$
13k^2 - 148k - 159,067,317.5 = 0
$$

This is a quadratic in k:

$$
13k^2 - 148k - 159,067,317.5 = 0
$$

Using the quadratic formula:

$$
k = \frac{148 \pm \sqrt{(-148)^2 - 4 \times 13 \times (-159,067,317.5)}}{2 \times 13}
$$

Calculate discriminant:

$$
D = 148^2 - 4 \times 13 \times (-159,067,317.5)
$$

$$
148^2 = 21,904
$$

$$
4 \times 13 = 52
$$

$$
D = 21,904 + 52 \times 159,067,317.5
$$

Compute:

$$
52 \times 159,067,317.5 = 52 \times 159,067,317.5
$$

Approximate:

$$
159,067,317.5 \times 50 = 7,953,365,875
$$

$$
159,067,317.5 \times 2 = 318,134,635
$$

Adding:

$$
7,953,365,875 + 318,134,635 = 8,271,500,510
$$

So,

$$
D = 21,904 + 8,271,500,510 \approx 8,271,522,414
$$

Now, compute the square root:

$$
\sqrt{8,271,522,414} \approx 91,008.44
$$

Finally, calculate k:

$$
k = \frac{148 \pm 91,008.44}{26}
$$

Two solutions:

- Solution 1:

$$
k = \frac{148 + 91,008.44}{26} \approx \frac{91,156.44}{26} \approx 3,514.47
$$

- Solution

A Triangle Is Defined By The Points A 8 5 7 B 3 6 6 And C 4 K 9 The Area Of The Triangle Is 8920 5

A Triangle Is Defined By The Points A 8 5 7 B 3 6 6 And C 4 K 9 The Area Of The Triangle Is 8920 5

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