An Object Has A Position Given By R = [2.0 M + (5.00 M/s)t] I^ + [3.0 M (3.00 M/s2)t2] J^ , Where Quantities

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Understanding the position of an object in physics is fundamental to analyzing its motion. The given position vector

$$ R = [2.0\, \text{M} + (5.00\, \text{M/s})t]\, \hat{\imath} + [3.0\, \text{M} \times (3.00\, \text{M/s}^2)t^2]\, \hat{\jmath} $$

provides a comprehensive way to describe the object's location at any given time t. This article explores the various quantities involved, how to interpret this position vector, and the underlying physics concepts such as velocity, acceleration, and trajectory that can be derived from this expression.

Deciphering the Position Vector R

The position vector R details the object’s position in a two-dimensional plane, combining the components along the x-axis ($\hat{\imath}$) and y-axis ($\hat{\jmath}$).

Component Breakdown

The vector can be split into two main parts:

1. **X-component:** $R_x(t) = 2.0\, \text{M} + (5.00\, \text{M/s}) t$
2. **Y-component:** $R_y(t) = 3.0\, \text{M} \times (3.00\, \text{M/s}^2) t^2$

This indicates:

- The x-position starts at 2.0 meters when $t = 0$ and increases linearly with time at a rate of 5.00 meters per second.
- The y-position starts at zero (since $t = 0$, $R_y = 0$), but accelerates quadratically over time, following a parabolic trajectory.

Interpreting the Quantities Involved

Understanding the physical meaning of each term and quantity is essential for analyzing the object's motion.

Initial Position Components

- Initial x-position (R_{x0}): 2.0 meters
- Initial y-position (R_{y0}): 0 meters (since $t = 0$, $R_y = 0$)

Velocity Components

- The x-velocity component is constant at 5.00 m/s, indicative of uniform motion in the x-direction.
- The y-velocity component is time-dependent because of acceleration, as it involves a quadratic term.

Quantities and Their Physical Significance

- **Position (R)**: The location of the object at a specific time t.
- **Time (t)**: The independent variable determining the position.
- **Velocity (v)**: The rate of change of position with respect to time, which can be obtained by differentiating R.
- **Acceleration (a)**: The rate of change of velocity, especially relevant in the y-direction where quadratic dependence indicates acceleration.

Calculating Velocity from the Position Vector

Velocity is a crucial quantity in kinematics, describing how fast and in what direction an object moves.

Deriving Velocity Components

To find the velocity components, differentiate each component of R with respect to time:

$$
v_x(t) = \frac{d R_x}{dt} = \frac{d}{dt} [2.0\, \text{M} + 5.00\, \text{M/s} \times t] = 5.00\, \text{M/s}
$$

$$
v_y(t) = \frac{d R_y}{dt} = \frac{d}{dt} [3.0\, \text{M} \times 3.00\, \text{M/s}^2 \times t^2] = 3.0\, \text{M} \times 3.00\, \text{M/s}^2 \times 2t = 18.0\, \text{M}^2/\text{s}^2 \times t
$$

Note that:

- The x-velocity remains constant at 5.00 m/s.
- The y-velocity varies linearly with time, indicating acceleration in the y-direction.

Summary of Velocity Components

- $v_x(t) = 5.00\, \text{M/s}$
- $v_y(t) = 18.0\, \text{M}^2/\text{s}^2 \times t$

Implications

- The object moves uniformly along the x-axis.
- The y-motion involves acceleration, with the velocity increasing linearly over time, typical of uniformly accelerated motion.

Calculating Acceleration

Acceleration describes how the velocity changes over time.

Deriving Acceleration Components

Differentiate the velocity components:

$$
a_x(t) = \frac{d v_x}{dt} = 0
$$

$$
a_y(t) = \frac{d v_y}{dt} = 18.0\, \text{M}^2/\text{s}^2
$$

This indicates:

- No acceleration in the x-direction ($a_x = 0$), consistent with constant velocity.

- Constant acceleration in the y-direction ($a_y = 18.0\, \text{M}^2/\text{s}^2$).

Physical Significance

- The object experiences uniform acceleration vertically, with magnitude 18.0 m/s^2.
- The acceleration causes the quadratic increase in the y-position over time.

Analyzing the Trajectory of the Object

The path traced by the object can be visualized by plotting $R_x(t)$ versus $R_y(t)$.

Equation of the Trajectory

Express R_y in terms of R_x:

$$
R_x(t) = 2.0\, \text{M} + 5.00\, \text{M/s} \times t
$$
$$
t = \frac{R_x - 2.0\, \text{M}}{5.00\, \text{M/s}}
$$

Substitute into $R_y(t)$:

$$
R_y = 3.0\, \text{M} \times 3.00\, \text{M/s}^2 \times \left(\frac{R_x - 2.0\, \text{M}}{5.00\, \text{M/s}} \right)^2
$$

Simplify:

$$
R_y = 3.0\, \text{M} \times 3.00\, \text{M/s}^2 \times \frac{(R_x - 2.0\, \text{M})^2}{(5.00\, \text{M/s})^2}
$$

$$
R_y = \frac{3.0 \times 3.00}{25.00} \times (R_x - 2.0)^2\, \text{M}
$$

$$
R_y = \frac{9.00}{25.00} \times (R_x - 2.0)^2\, \text{M} \approx 0.36 \times (R_x - 2.0)^2\, \text{M}
$$

This quadratic relation describes a parabolic trajectory, characteristic of projectile motion under constant acceleration.

Visual Interpretation

- The object starts at $R = (2.0\, \text{M}, 0)$.
- As R_x increases linearly, R_y increases quadratically, forming a parabola.

Key Physical Quantities and Their Calculations at Specific Times

Knowing the position, velocity, and acceleration at specific moments can provide deeper insights into the motion.

At $t = 0\, \text{s}$

- $R_x = 2.0\, \text{M}$
- $R_y = 0\, \text{M}$
- $v_x = 5.00\, \text{M/s}$
- $v_y = 0\, \text{M/s}$
- $a_x = 0\, \text{M/s}^2$
- $a_y = 18.0\, \text{M/s}^2$

At $t = 2\, \text{s}$

- R_x

Frequently Asked Questions

What are the components of the position vector R for the object?

The position vector R has components along the I and J directions: R = [2.0 M + (5.00 M/s) t] I^ and [3.0 M + (3.00 M/s^2) t^2] J^.

How do you interpret the time-dependent terms in the position vector?

The terms involving t represent how the position changes over time: the I component changes linearly with time, while the J component changes quadratically, indicating accelerated motion.

What is the initial position of the object at t = 0?

At t = 0, the position is R = 2.0 M I^ + 3.0 M J^, since the terms involving t are zero.

How can you find the velocity of the object at any time t?

Velocity is the derivative of the position vector with respect to time: v = dR/dt. Differentiating each component gives v = (5.00 M/s) I^ + (2 3.00 M/s^2 t) J^.

What is the acceleration of the object based on the given position vector?

Acceleration is the derivative of velocity with respect to time. Since the I component is constant, its acceleration is zero, while the J component's acceleration is 2 3.00 M/s^2 = 6.00 M/s^2 in the J direction.

At what time t does the object reach a position where the I component is 12 meters?

Set the I component equal to 12 M: 2.0 M + (5.00 M/s) t = 12 M. Solving for t gives t = (12 M - 2.0 M) / 5.00 M/s = 10 M / 5.00 M/s = 2.0 seconds.

What is the magnitude of the position vector R at any time t?

The magnitude is |R| = sqrt([2.0 + 5.00 t]^2 + [3.0 + 3.00 t^2]^2).

How does the quadratic term in the J component affect the object's motion?

The quadratic term indicates that the object experiences acceleration in the J direction, causing the position to change more rapidly as time increases, resulting in non-uniform, accelerated motion.

Additional Resources

Understanding how an object moves through space is fundamental in physics, especially when analyzing dynamics and kinematics. When an object has a position given by R = [2.0 M + (5.00 M/s)t] I^ + [3.0 M (3.00 M/s^2)t^2] J^, we're presented with a detailed vector expression that encapsulates how the object’s location changes over time. This type of problem is a classic example of kinematic analysis in two dimensions, combining linear motion with acceleration. In this guide, we'll break down the components of this position vector, interpret what it signifies, and explore how to analyze such motion step-by-step.

Understanding the Position Vector

The position vector R(t) describes the location of an object in a two-dimensional plane over time. It is expressed as:

R(t) = [2.0 M + (5.00 M/s)t] I^ + [3.0 M (3.00 M/s²)t²] J^

Let's dissect this expression:

- R(t): Represents the position vector as a function of time.
- I^ and J^: Unit vectors in the x- and y-directions, respectively.
- Constants and variables:
- The initial x-position is 2.0 meters.
- The x-component velocity is 5.00 m/s, which implies constant velocity in the x-direction.
- The y-component involves an acceleration term, with a quadratic dependence on time, indicating uniformly accelerated motion in the y-direction.

Breaking Down the Components

X-Component: Constant Velocity Motion

The x-component of the position vector is:

x(t) = 2.0 M + (5.00 M/s) t

- Initial position (at t=0): 2.0 meters.
- Velocity: 5.00 m/s, constant, indicating uniform linear motion in the x-direction.
- Interpretation: The object starts at x = 2.0 meters and moves along the x-axis at a steady rate.

Y-Component: Accelerated Motion

The y-component is:

$y(t) = 3.0\ M\ (3.00\ M/s^2)\ t^2$

Simplified as:

$y(t) = (3.0\ M)\ (3.00\ M/s^2)\ t^2 = (9.00\ M^2/s^2)\ t^2$

Note:

- The presence of a t^2 term indicates uniformly accelerated motion in the y-direction.

- The initial y-position at t=0 is zero (since no constant term is added).
- The acceleration in the y-direction (a_y) can be inferred from the quadratic coefficient.

Analyzing Motion Step-by-Step

To fully understand the movement of the object, we analyze position, velocity, and acceleration components separately.

1. Position as a Function of Time

- x(t): Linear in time, moving at constant velocity.
- y(t): Quadratic in time, accelerating in the y-direction.

At any time t:

- $x(t) = 2.0 + 5.00\, t$ (meters)
- $y(t) = 9.00\, t^2$ (meters)

---

2. Velocity Components

Velocity is the first derivative of position with respect to time.

- $V(t) = dR/dt = V_x\, I\hat{} + V_y\, J\hat{}$

Calculating:

- $V_x = d/dt\, [x(t)] = 5.00$ m/s

- $V_y = d/dt\, [y(t)] = d/dt\, [9.00\, t^2] = 18.00\, t$ (m/s)

Thus,

$V(t) = 5.00\, I\hat{} + 18.00\, t\, J\hat{}$

This indicates:

- The x-velocity remains constant at 5.00 m/s.
- The y-velocity increases linearly with time, starting at zero when t=0.

---

3. Acceleration Components

Acceleration is the derivative of velocity:

- $A(t) = dV/dt = a_x I\hat{} + a_y J\hat{}$

Calculating:

- $a_x = 0$ (since V_x is constant)
- $a_y = d/dt [18.00 t] = 18.00 m/s^2$

Therefore,

$A(t) = 0 I\hat{} + 18.00 m/s^2 J\hat{}$

This shows:

- No acceleration in the x-direction.
- Constant acceleration in the y-direction, consistent with the quadratic position term.

Implications of the Motion

The given position function describes a scenario where:

- The object moves uniformly in the x-direction.
- The object experiences constant acceleration in the y-direction.

This is akin to a projectile launched with an initial x-velocity and under a constant acceleration (such as gravity, if we interpret the acceleration as gravity).

Key Calculations and Interpretations

1. Position at Specific Times

Suppose we want to calculate the position at t = 2.0 seconds:

- $x(2.0) = 2.0 + 5.00\ 2.0 = 2.0 + 10.0 = 12.0$ meters
- $y(2.0) = 9.00\ (2.0)^2 = 9.00\ 4 = 36.0$ meters

So, at t=2 seconds, the object is at (12.0 m, 36.0 m).

2. Velocity at Specific Times

At t=2.0 seconds:

- $V_x = 5.00$ m/s (constant)
- $V_y = 18.00 \ 2.0 = 36.0$ m/s

Total velocity vector:

$V(2.0) = 5.00 \, \hat{I} + 36.0 \, \hat{J}$ m/s

3. Acceleration Confirmation

Since $a_y = 18.00$ m/s², the y-velocity increases by 18.00 m/s every second:

- Initial y-velocity at t=0: 0 m/s.
- At t=1 s: $V_y = 18.00$ m/s.
- At t=2 s: $V_y = 36.0$ m/s, as calculated.

This confirms the consistent acceleration.

Graphical Interpretation

Visualizing the motion:

- The x-position increases linearly, resulting in a straight line when plotted against time.
- The y-position follows a parabola, opening upwards, due to the quadratic dependence on time.
- The velocity in x remains constant, while the y-component of velocity increases linearly.
- The acceleration vector points purely in the y-direction, indicating vertical acceleration (e.g., gravity).

Practical Applications and Insights

Analyzing such motion is fundamental in various fields:

- Projectile motion: The equations resemble the motion of projectiles launched in a uniform gravitational field, except here, the acceleration is specified independently.
- Engineering: Designing trajectories for vehicles or robotics where movement in multiple dimensions involves constant velocity and acceleration.
- Physics education: Demonstrating the principles of kinematics with combined linear and accelerated

motion.

Summary of Key Concepts

- The position vector combines linear and quadratic terms, representing different types of motion.
- The velocity vector derives directly from the position, revealing constant velocity in x and linearly increasing velocity in y.
- The acceleration vector remains constant in the y-direction, indicating uniform acceleration.
- Understanding these components allows prediction of the object's future position and velocity at any given time.

Final Thoughts

The position function R = [2.0 M + (5.00 M/s)t] I^ + [3.0 M (3.00 M/s^2)t^2] J^ provides a comprehensive snapshot of an object's motion, blending uniform motion with uniformly accelerated motion. By analyzing each component carefully, students and professionals can develop a nuanced understanding of two-dimensional kinematics, crucial for real-world applications ranging from projectile trajectory prediction to motion planning in robotics.

Whether you're a physics student tackling projectile motion problems or a professional designing motion paths for autonomous vehicles, mastering the interpretation of vector position functions like this is essential.

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Back to Home