An Urn Contains 5 Tiles, Each Containing One Of The Numbers From 1 To 5. An Experiment Consists Of Selecting

Introduction

An urn contains 5 tiles, each containing one of the numbers from 1 to 5. An experiment consists of selecting a tile from the urn, possibly under various conditions such as with or without replacement, and analyzing the outcomes. This simple setup serves as an excellent foundation for exploring fundamental concepts in probability theory, including probability distributions, sample spaces, and combinatorial calculations. In this article, we delve into the various scenarios and calculations associated with this experiment, providing a comprehensive understanding of the principles involved.

Understanding the Basic Setup

The Composition of the Urn

The urn contains five distinct tiles:

- Tile 1: Number 1
- Tile 2: Number 2
- Tile 3: Number 3
- Tile 4: Number 4
- Tile 5: Number 5

Each tile is unique, ensuring that each number from 1 to 5 is represented exactly once. This setup forms the basis for exploring simple and compound probability scenarios.

The Nature of the Selection Process

The experiment involves selecting one or more tiles from the urn under specified conditions, such as:
- Selecting a single tile (single draw)
- Selecting multiple tiles in sequence (multiple draws)
- Whether the selection is with replacement (the tile is put back after selection)
- Or without replacement (the tile is not returned after selection)

Understanding these conditions is crucial because they influence the probability calculations and the sample space.

Sample Space and Outcomes

Single Draw: Basic Outcomes

When selecting one tile from the urn:
- The sample space consists of 5 outcomes: {1, 2, 3, 4, 5}
- Each outcome has an equal probability of 1/5 assuming a fair selection.

Multiple Draws: Expanding the Sample Space

If the experiment involves selecting multiple tiles, the sample space grows significantly, depending on whether the selection is with or without replacement.

- With Replacement:
Each draw is independent, and the sample space for a sequence of n draws is 5^n.
For example, selecting 2 tiles with replacement yields $5^2 = 25$ possible ordered outcomes.

- Without Replacement:
The sample space consists of all permutations of the 5 tiles for each number of draws, provided the number of draws does not exceed 5.

Probability Calculations in Different Scenarios

Single Draw Probabilities

- The probability of drawing any specific number (say, 3) is $\frac{1}{5}$.

Multiple Draws with Replacement

- Probabilities are independent across draws.
- For example, the probability of drawing a 2 followed by a 4 is:
$\frac{1}{5} \times \frac{1}{5} = \frac{1}{25}$.

Multiple Draws without Replacement

- Probabilities depend on previous outcomes.
- For example, the probability of drawing 2 first and then 4:
- Probability of first drawing 2: $\frac{1}{5}$
- Probability of then drawing 4 (from remaining 4 tiles): $\frac{1}{4}$
- Total probability: $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$.

Probability of Specific Events

- The probability of drawing a tile with an even number (2 or 4):

$\frac{2}{5}$.
- The probability of drawing a tile with a number greater than 3 (i.e., 4 or 5): $\frac{2}{5}$.

Combinatorial Analysis and Outcomes

Number of Possible Outcomes

- For a single draw: 5 outcomes.
- For k draws with replacement: 5^k outcomes.
- For k draws without replacement: $\frac{5!}{(5 - k)!}$ if $k \leq 5$.

Sample Problems and Solutions

1. **Probability of drawing a sequence 1, 3, 5 in three draws with replacement:**

   - Since each draw is independent with 5 options: $\left(\frac{1}{5}\right)^3 = \frac{1}{125}$.

2. **Probability of drawing exactly two even numbers in three draws with replacement:**

   - Number of favorable sequences:
   - Choose 2 positions out of 3 for even numbers: $\binom{3}{2} = 3$.
   - Each even number (2 or 4) has probability $\frac{2}{5}$.
   - Each odd number (1, 3, 5) has probability $\frac{3}{5}$.
   - Total probability: $\binom{3}{2} \times \left(\frac{2}{5}\right)^2 \times \left(\frac{3}{5}\right) = 3 \times \frac{4}{25} \times \frac{3}{5} = 3 \times \frac{4}{25} \times \frac{3}{5}$.
   - Simplify: $3 \times \frac{4}{25} \times \frac{3}{5} = 3 \times \frac{4 \times 3}{25 \times 5} = 3 \times \frac{12}{125} = \frac{36}{125}$.

Expected Values and Probabilities

Expected Value of a Single Draw

- Since each number 1 to 5 has an equal chance:

$$
E = \frac{1 + 2 + 3 + 4 + 5}{5} = \frac{15}{5} = 3.
$$

- The expected value of the outcome of a single draw is 3.

Expected Value for Multiple Draws

- For independent draws with replacement, the expected value remains 3 per draw.
- For multiple draws, the total expected sum is simply the number of draws multiplied by 3.

Extensions and Variations

Replacing the Tiles and Changing Probabilities

- If the tiles are weighted differently, the probabilities change accordingly.
- For example, if each tile has a different probability p_i, the calculations must incorporate these weights.

Drawing Without Replacement and Its Implications

- This scenario models situations where each outcome is unique and impacts subsequent probabilities.
- It aligns with real-world situations like drawing cards from a deck without replacing them.

Multiple Sequential Draws and Conditional Probabilities

- Analyzing the probability of complex sequences requires understanding conditional probabilities.
- For example, the probability of drawing a 5 given that a 3 was drawn first without replacement.

Conclusion

The simple act of selecting tiles from an urn containing the numbers 1 to 5 offers a rich ground for understanding core probability concepts. Whether considering single or multiple draws, with or without replacement, the calculations involve fundamental principles of combinatorics, permutations, and probability theory. By exploring various scenarios and outcomes, one gains a deeper appreciation for how probability models real-world uncertainty and random events. These foundational insights not only serve as building blocks for advanced topics but also have practical applications in fields ranging from statistics and data science to gaming and decision-making processes.

Frequently Asked Questions

What is the probability of drawing the tile with number 3 from

the urn?

Since there are 5 tiles numbered from 1 to 5, the probability of drawing the tile with number 3 is 1/5.

If two tiles are drawn without replacement, what is the probability that both are even numbers?

There are two even numbers (2 and 4). Probability of drawing an even number first is 2/5. After drawing one, the probability of drawing the other is 1/4. So, total probability = (2/5) (1/4) = 1/10.

What is the probability of selecting a tile with a number greater than 3?

Numbers greater than 3 are 4 and 5, so 2 out of 5 tiles. Probability = 2/5.

In how many ways can one tile be selected from the urn?

Since there are 5 tiles, there are 5 possible ways to select one tile.

What is the probability of selecting the tile with number 1 or 2?

Number of favorable outcomes = 2 (tiles with 1 and 2). Total outcomes = 5. Probability = 2/5.

If an experiment involves selecting two tiles with replacement, what is the probability both are odd numbers?

Odd numbers are 1, 3, and 5. Probability of selecting an odd number in one draw = 3/5. Since replacement occurs, probability both are odd = (3/5) (3/5) = 9/25.

What is the total number of possible outcomes when selecting two tiles without replacement?

Number of ways to select 2 tiles out of 5 without replacement is 5C2 = 10.

What is the probability of selecting a tile with an even number first and an odd number second (without replacement)?

First draw even: 2/5. Remaining tiles after removing one even number: 4 tiles left, with 3 odd numbers remaining. Probability second draw odd: 3/4. Total probability = (2/5) (3/4) = 3/10.

Can the experiment be considered a binomial experiment?

Why or why not?

No, because the selections are without replacement, which makes the outcomes dependent, violating the binomial experiment's requirement of independent trials with replacement or identical conditions.

What is the probability of selecting a tile with the number 4 in two consecutive draws without replacement?

Probability of selecting 4 first: 1/5. Then, remaining tiles are 4, and the probability of selecting 4 again (which is impossible without replacement since only one tile with 4 exists) is 0. Therefore, the probability that 4 is selected in both draws is 0.

Additional Resources

An Urn Contains 5 Tiles, Each Containing One Of The Numbers From 1 To 5. An Experiment Consists Of Selecting

Introduction to the Experiment

The described experiment involves a simple yet intriguing probabilistic setup: an urn containing five distinct tiles, each inscribed with a unique number from 1 to 5. The process of selecting tiles from this urn, whether with or without replacement, opens a broad spectrum of probability calculations and statistical analyses. This experiment, while seemingly straightforward, serves as an excellent foundation for understanding core principles of probability, combinatorics, and statistical reasoning.

This detailed review aims to explore every facet of this experiment, including its fundamental concepts, variations, analyses, and implications for probability theory. We will dissect the experiment step-by-step, considering different scenarios, calculating probabilities, and discussing the theoretical underpinnings that make this experiment an ideal pedagogical tool for understanding randomness and chance.

Understanding the Basic Setup

Components of the Experiment

The core elements include:

- The Urn: A container holding five unique tiles.
- Tiles: Each tile bears a distinct number from 1 to 5.
- Selection Process: Drawing tiles, which can be conducted in various ways (with or without

replacement).

Initial Assumptions and Clarifications

- Uniformity: Each tile has an equal chance of being selected at each draw.
- Independence: The outcome of each draw depends solely on the selection process, not on previous selections, unless specified.
- Replacement Policy: Whether tiles are replaced after each draw significantly impacts probability calculations.
- Number of Draws: The number of selections made from the urn influences the complexity and the nature of possible outcomes.

Having these assumptions clearly outlined provides the foundational framework for analyzing the experiment.

Variations of the Experiment

Different variations of the experiment lead to different probability models. Here, we explore the most common and instructive scenarios.

Scenario 1: Drawing Without Replacement

In this case, once a tile is drawn, it is not returned to the urn. The key points include:

- Sample Space: The set of all possible arrangements or sequences in which tiles are drawn.
- Number of Draws: Typically, up to five, since there are five tiles.
- Implications: The probabilities are dependent on previous draws because the composition of the urn changes after each pick.

Scenario 2: Drawing With Replacement

Here, after each draw, the tile is returned to the urn, maintaining the total of five tiles:

- Sample Space: All possible sequences of draws, with repetitions allowed.
- Independence: Each draw is independent since the composition of the urn remains constant.
- Probabilities: Calculated based on fixed probabilities per draw.

Scenario 3: Multiple Draws and Outcomes

- Single Draw: The simplest case, 5 possible outcomes, each with probability 1/5.
- Multiple Draws: For example, drawing twice, thrice, or more, leading to combinations and

permutations of the numbers.

Probability Calculations in Different Scenarios

Understanding the probabilities in each variation is fundamental. Here, we analyze the calculations for the most common cases.

Single Draw: Basic Probability

- Outcome: Selecting any one tile.
- Probability: Since all tiles are equally likely,

$$
P(\text{drawing a specific tile}) = \frac{1}{5}
$$

for each number from 1 through 5.

Multiple Draws Without Replacement

Suppose we want to find the probability of drawing a specific sequence, such as 2, then 4, then 1, in three draws:

- First draw (tile 2): $P = \frac{1}{5}$
- Second draw (tile 4): Since tile 2 is removed, remaining tiles are 4, so

$$
P = \frac{1}{4}
$$

- Third draw (tile 1): Remaining tiles are 3, so

$$
P = \frac{1}{3}
$$

- Combined probability:

$$
P = \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{1}{60}
$$

This calculation demonstrates how the total probability diminishes with each successive draw due to decreasing options.

Multiple Draws With Replacement

If the tiles are replaced after each draw, the probabilities for each draw remain constant at $\frac{1}{5}$. For example, the probability of drawing the sequence 2, 4, 1:

$$
P = \frac{1}{5} \times \frac{1}{5} \times \frac{1}{5} = \frac{1}{125}
$$

This illustrates the independence of each draw and the constant probability across trials.

Calculating Probabilities for Specific Events

- Event A: Drawing a specific number at least once in multiple draws.
- Event B: Drawing all five numbers in a sequence.
- Event C: Drawing only even or odd numbers.

Example: Probability of drawing at least one '3' in three independent draws with replacement:

- Probability of not drawing '3' in a single draw: $\frac{4}{5}$
- Probability of not drawing '3' in all three draws:

$$
\left(\frac{4}{5} \right)^3 = \frac{64}{125}
$$

- Therefore, probability of drawing at least one '3':

$$
1 - \frac{64}{125} = \frac{61}{125}
$$

This demonstrates how complement probabilities are often easier to compute.

Combinatorial Aspects and Outcome Counting

Beyond simple probability calculations, understanding the combinatorial elements helps analyze the nature of possible outcomes.

Permutations and Arrangements

- Without Replacement: The number of arrangements of k tiles selected from 5 is:

\[

$$
P(5, k) = \frac{5!}{(5 - k)!}
$$

- With Replacement: The number of sequences of length k:

$$
5^k
$$

Each sequence corresponds to a specific ordered outcome.

Sample Space Size

- Without Replacement: For k draws:

$$
\text{Number of outcomes} = \frac{5!}{(5 - k)!}
$$

- With Replacement: For k draws:

$$
\text{Number of outcomes} = 5^k
$$

These counts are crucial for calculating exact probabilities of complex events.

Expected Values and Variance

Understanding the expected value (mean) and variance of outcomes provides insight into the behavior of the experiment over many repetitions.

Expected Value in a Single Draw

Since each number from 1 to 5 has probability $\frac{1}{5}$:

$$
E = \sum_{i=1}^{5} i \times \frac{1}{5} = \frac{1 + 2 + 3 + 4 + 5}{5} = \frac{15}{5} = 3
$$

The average value of a single draw is 3.

Expected Sum Over Multiple Draws

- With Replacement: For k draws, the expected total sum:

$$
E_{\text{total}} = k \times E = 3k
$$

- Without Replacement: The expected sum remains the same because each number is equally likely, and the total sum of all tiles is 15; the expected sum over k draws (without replacement) can be derived from hypergeometric distribution properties, but typically, the expectation remains centered around the mean.

Variance of a Single Draw

- Variance:

$$
Var = E[X^2] - (E[X])^2
$$

- Compute $E[X^2]$:

$$
E[X^2] = \sum_{i=1}^5 i^2 \times \frac{1}{5} = \frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2}{5} = \frac{1 + 4 + 9 + 16 + 25}{5} = \frac{55}{5} = 11
$$

- Variance:

$$
Var = 11 - 3^2 = 11 - 9 = 2
$$

This quantifies the variability of individual outcomes.

Real-World Applications and Educational Significance

This experiment, though simple, serves as a microcosm for larger probabilistic models, including:

- Card Games: Drawing cards without replacement.
- Lottery Draws: Random selection of numbers.
- Quality Control: Sampling items in production.
- Decision Making: Probabilistic reasoning in uncertain environments.

Its educational value is immense, providing a hands-on approach to understanding fundamental concepts like independence, dependence, permutations, combinations, and probability distributions.

Advanced Topics and Extensions

Building

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