Consider The Rectangle With Vertices At $(5,4),$ $(5,-4),$ $(-5,4),$ $(-5,-4)$. How Many Integer Coordinates

Consider The Rectangle With Vertices At $(5,4),$ $(5,-4),$ $(-5,4),$ $(−5,-4)$ How Many Integer Coordinates

Understanding the distribution of integer coordinates within geometric figures is a fundamental aspect of coordinate geometry. The question of how many lattice points—points with integer coordinates—lie within or on the boundary of a particular shape is both intriguing and mathematically rich. In this article, we will explore the rectangle defined by the vertices at $(5,4)$, $(5,-4)$, $(-5,4)$, and $(-5,-4)$, and determine precisely how many integer-coordinate points it contains. This exploration will involve concepts from coordinate geometry, lattice point counting, and the application of Pick's Theorem.

Overview of the Rectangle's Coordinates

Before diving into counting the lattice points, it is essential to understand the rectangle's geometry.

Coordinates of Vertices

The rectangle is defined by four vertices:

- Top-right vertex: $(5, 4)$
- Bottom-right vertex: $(5, -4)$
- Top-left vertex: $(-5, 4)$
- Bottom-left vertex: $(-5, -4)$

Shape and Dimensions

This rectangle is aligned with the axes, making it an axis-aligned rectangle. The rectangle's width and height are straightforward to compute:

- Width: The difference in the x-coordinates:

$$
|5 - (-5)| = 10
$$

- Height: The difference in the y-coordinates:

$$
|4 - (-4)| = 8
$$

Thus, the rectangle spans from $x = -5$ to $x = 5$, and from $y = -4$ to $y = 4$.

Counting Lattice Points on the Boundary

The first step is to determine the number of integer points lying on the boundary of the rectangle. Since the rectangle's vertices are at integer coordinates, and it is aligned with the axes, the boundary points are precisely those points with integer coordinates along the sides.

Boundary Points on Each Side

The rectangle has four sides:

1. Top side: from $(-5, 4)$ to $(5, 4)$
2. Bottom side: from $(-5, -4)$ to $(5, -4)$
3. Left side: from $(-5, -4)$ to $(-5, 4)$
4. Right side: from $(5, -4)$ to $(5, 4)$

Because the sides are horizontal or vertical lines with integer endpoints, the boundary points on each side are all the integer points along that segment.

Counting boundary points on each side:

- Top side: from $x = -5$ to $x = 5$ at $y=4$

Number of integer points:

$$
x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5
$$

Total points:

$$
(5 - (-5)) + 1 = 10 + 1 = 11
$$

Similarly, the bottom side from $x = -5$ to $x=5$ at $y=-4$ also has 11 points.

- Left side: from $y = -4$ to $y=4$ at $x=-5$

Number of points:

$$
y = -4, -3, -2, -1, 0, 1, 2, 3, 4
$$

Total:

$$
4 - (-4) + 1 = 8 + 1 = 9
$$

Similarly, the right side from $y=-4$ to $y=4$ at $x=5$ also has 9 points.

Avoiding Double Counting of Corner Points

When summing boundary points across all sides, the vertices are counted twice because each corner belongs to two sides. To find the total number of unique boundary points, we use the inclusion-exclusion principle:

Total boundary points:

\[
\text{Sum of points on all sides} - \text{Number of vertices}
\]

But since the vertices are counted twice, the total boundary points are:

\[
(11 + 11 + 9 + 9) - 4 = (40) - 4 = 36
\]

Alternatively, we can verify by directly counting the unique points:

- Horizontal sides (top and bottom): each has 11 points, sharing the corners, so total unique points on top and bottom combined:

\[
11 + 11 - 2 = 20
\]

- Vertical sides: 9 points each, sharing corners, total:

\[
9 + 9 - 2 = 16
\]

Adding these:

\[
20 + 16 = 36
\]

which matches the previous calculation.

Summary of Boundary Points

Total boundary points (B):

\[
\boxed{
B = 36
}

\]

Counting All Lattice Points Inside the Rectangle

Now that we know how many points are on the boundary, we proceed to count the total number of lattice points inside and on the rectangle.

The Set of All Integer Points in the Rectangle

Since the rectangle spans from $x = -5$ to 5, and $y = -4$ to 4, all integer points inside or on it are:

\[
\{(x, y) \mid x \in \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\},\, y \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}\}
\]

Total number of integer points:

\[
(\text{number of } x) \times (\text{number of } y) = 11 \times 9 = 99
\]

All these points are either on the boundary or strictly inside the rectangle, because the boundary points are included in this count.

Confirming Boundary vs. Interior Points

To differentiate between boundary points and interior points, note:

- Boundary points: points with $x = -5$ or $x=5$, or $y = -4$ or $y=4$.
- Interior points: points with $x \in \{-4, -3, -2, -1, 0, 1, 2, 3, 4\}$ and $y \in \{-3, -2, -1, 0, 1, 2, 3\}$.

Number of interior points:

\[
9 \times 7 = 63
\]

since:

- x values: from -4 to 4, total 9
- y values: from -3 to 3, total 7

Total points:

\[
\boxed{
\text{Interior points} = 63
}
\]

and total points (interior + boundary):

$$
63 + 36 = 99
$$

which matches the total count of all integer points within the rectangle.

Applying Pick's Theorem to Confirm the Count

Pick's Theorem provides a relation between the area of a lattice polygon, the number of interior lattice points, and the number of boundary lattice points:

$$
A = I + \frac{B}{2} - 1
$$

where:

- A is the area of the polygon,
- I is the number of interior lattice points,
- B is the number of boundary lattice points.

Calculating the Area

Since the rectangle is axis-aligned:

$$
A = \text{width} \times \text{height} = 10 \times 8 = 80
$$

Verifying with Pick's Theorem

Given:

$$
A = 80, \quad B = 36
$$

Solve for I:

$$
I = A - \frac{B}{2} + 1 = 80 - \frac{36}{2} + 1 = 80 - 18 + 1 = 63
$$

This matches our earlier count of interior points, confirming the consistency of our counts.

Summary and Final Count

Bringing all the calculations together, we can now state:

- The total number of lattice points on or inside the rectangle is:

$$
\boxed{
99
}
$$

- The total number of boundary lattice points:

$$
\boxed{
36
}
$$

- The total number of interior lattice points:

$$
\boxed{
63
}
$$

Additional Insights and Applications

Understanding the distribution of lattice points in geometric figures has numerous applications, including:

- Number theory: Counting solutions to Diophantine equations.
- Combinatorics: Enumerating arrangements within geometric constraints.
- Computational geometry: Algorithms for raster graphics and pixel-based representations.
- Physics and Engineering: Modeling discretized systems or grid-based simulations.

Extending to Other Shapes

While rectangles aligned with axes make counting straightforward, more complex polygons require advanced techniques like Pick's Theorem or Ehrhart polynomials. For non-axis-aligned shapes, lattice point counting can involve more intricate methods, such as the use of the Euler–Maclaurin formula or geometric transformations

Frequently Asked Questions

What are the vertices of the rectangle in the coordinate plane?

The vertices are at (5, 4), (5, -4), (-5, 4), and (-5, -4).

How many integer lattice points are inside or on the boundary of the given rectangle?

There are 90 integer lattice points on or inside the rectangle.

How can we determine the number of integer points on the boundary of the rectangle?

By calculating the points on each side using the line equations and counting the lattice points, then summing and avoiding double counting corners.

What is the total number of lattice points inside the rectangle (excluding boundary)?

The interior contains 81 lattice points.

How many lattice points are on each side of the rectangle?

Each vertical side has 11 lattice points, and each horizontal side has 11 lattice points.

Which formula can be used to find the total number of lattice points in a rectangle with integer vertices?

The total is computed by (width + 1) (height + 1), where width and height are the differences in x and y coordinates.

Calculate the width and height of the rectangle in terms of lattice points.

The width is 10 (from x = -5 to 5), and the height is 8 (from y = -4 to 4).

Using Pick's Theorem, how can we verify the number of interior lattice points?

Pick's Theorem states that Area = I + B/2 - 1, where I is interior points and B is boundary points; solving for I confirms the count.

What is the area of the rectangle in coordinate units?

The area is 80 square units (width 10 times height 8).

Can the number of lattice points be generalized for any

rectangle aligned with the axes?

Yes, for a rectangle with vertices at (x1, y1) and (x2, y2), the total lattice points on or inside are (|x2 - x1| + 1) (|y2 - y1| + 1).

Additional Resources

Consider the rectangle with vertices at (5, 4), (5, -4), (-5, 4), and (-5, -4). This fundamental geometric figure offers a fascinating exploration into the world of coordinate geometry, particularly when counting lattice points—points with integer coordinates—within or on its boundary. Such problems are not just academic exercises; they have practical applications in computer graphics, lattice point enumeration, and mathematical problem-solving. In this comprehensive guide, we will analyze how to determine the number of integer coordinates that lie on or inside this rectangle, applying principles from coordinate geometry, number theory, and combinatorics.

Understanding the Rectangle's Geometry

Coordinates and Shape

The vertices of the rectangle are given as:

- (5, 4)
- (5, -4)
- (-5, 4)
- (-5, -4)

These points form a rectangle aligned with the axes, with:

- Horizontal sides connecting (-5, 4) and (5, 4), and (-5, -4) and (5, -4)
- Vertical sides connecting (-5, 4) and (-5, -4), and (5, 4) and (5, -4)

The rectangle's boundaries are thus:

- Top boundary: y = 4
- Bottom boundary: y = -4
- Left boundary: x = -5
- Right boundary: x = 5

Dimensions

- Width: The difference in x-coordinates: 5 - (-5) = 10
- Height: The difference in y-coordinates: 4 - (-4) = 8

This makes the rectangle a 10-unit wide and 8-unit tall figure on the coordinate plane.

Counting Integer Lattice Points

When asked "How many integer coordinates are within or on the rectangle?", we need to count all points with integer x and y values that lie inside or on the boundary of the rectangle.

Boundary Points

These are points where either x or y (or both) are at the boundary values.

Interior Points

Points with x and y strictly between the boundary values.

Step-by-Step Approach

1. Count Points on the Boundary

The boundary consists of four sides:

- Top side: from (-5, 4) to (5, 4)
- Bottom side: from (-5, -4) to (5, -4)
- Left side: from (-5, 4) to (-5, -4)
- Right side: from (5, 4) to (5, -4)

Because the rectangle is axis-aligned, the points on the boundary are straightforward to count.

2. Count Interior Points

Once the boundary points are known, the interior points are those with x and y strictly between the boundary values:

- x: from -4 to 4
- y: from -3 to 3

(assuming you exclude the boundary points)

Detailed Calculation

Counting Boundary Points

Top boundary: y = 4, x = -5 to 5

- Number of points: from x = -5 to 5 inclusive.
- Count: 5 (negative side) to 5 (positive side) = 11 points.

Similarly, bottom boundary: $y = -4$

- Points: $x = -5$ to 5
- Count: 11 points.

For left boundary: $x = -5$, $y = -4$ to 4

- Count: $y = -4$ to 4 inclusive.
- Count: 4 (negative y) to 4 (positive y) = 9 points.

Similarly, right boundary: $x = 5$

- Count: $y = -4$ to 4
- Count: 9 points.

Adjust for Corner Points

Notice that the four corners are counted twice (once in each adjoining boundary). The corners are:

- (-5, 4)
- (5, 4)
- (-5, -4)
- (5, -4)

Total boundary points before adjustment:

- Top and bottom sides: 11 + 11 = 22 points
- Left and right sides: 9 + 9 = 18 points

Total without adjustment: 22 + 18 = 40 points

But each corner is counted twice (once in a horizontal boundary and once in a vertical boundary). Since there are 4 corners, subtract 4 to avoid double counting:

Total boundary points:

$40 - 4 = 36$

Counting Interior Points

Interior points are those with:

- x-coordinate: integers from -4 to 4 (excluding boundary $x = -5$ and 5)
- y-coordinate: integers from -3 to 3 (excluding boundary $y = -4$ and 4)

Number of integer x-values in interior:

\[

$$
(-4, -3, -2, -1, 0, 1, 2, 3, 4) \Rightarrow 9 \text{ points}
$$

Number of integer y-values in interior:

$$
(-3, -2, -1, 0, 1, 2, 3) \Rightarrow 7 \text{ points}
$$

Total interior points:

$$
9 \times 7 = 63
$$

Final Tally: Total Lattice Points

The total number of integer coordinate points on or inside the rectangle is:

$$
\text{Boundary points} + \text{Interior points} = 36 + 63 = 99
$$

Summary and Key Takeaways

- The rectangle defined by vertices at (5, 4), (5, -4), (-5, 4), and (-5, -4) spans 10 units in width and 8 units in height.
- To find all lattice points on or within this rectangle, count boundary points and interior points separately.
- Boundary points are obtained by counting all points on the four sides, taking care to avoid double counting corners.
- Interior points are those with x and y coordinates strictly between the boundary values.
- The total number of integer points within or on this rectangle is 99.

Additional Insights: Pick's Theorem

For polygons with integer vertices, Pick's Theorem relates the area A, the number of interior lattice points I, and the number of boundary lattice points B:

$$
A = I + \frac{B}{2} - 1
$$

Applying this:

- Area \(A = \text{width} \times \text{height} = 10 \times 8 = 80\)

- Boundary points \(B = 36\)

Calculate interior points:

\[
80 = I + \frac{36}{2} - 1 \Rightarrow 80 = I + 18 - 1 \Rightarrow 80 = I + 17
\]

\[
I = 80 - 17 = 63
\]

which matches our earlier count, confirming the correctness of our calculations.

Conclusion

Consider the rectangle with vertices at (5, 4), (5, -4), (-5, 4), and (-5, -4): it contains 99 lattice points in total—both on the boundary and inside. This problem showcases how coordinate geometry and number theory principles like Pick's Theorem work hand-in-hand to solve lattice point enumeration problems efficiently. Whether you're tackling similar geometric puzzles or applying these techniques in computational geometry, understanding these foundational methods is invaluable.

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