Consider Triangle ABC With Vertices A(0,0), B (0,6), And C (4,0). The Image Of Triangle ABC After A Dilation

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Understanding the effects of geometric transformations is fundamental in coordinate geometry, especially when analyzing how shapes change under various operations. One such transformation, dilation, enlarges or reduces a figure relative to a fixed point called the center of dilation. In this article, we will explore how triangle ABC, with vertices at A(0,0), B(0,6), and C(4,0), transforms after undergoing a dilation. We will examine the concept of dilation in detail, compute the image of the triangle under dilation, and discuss how different scale factors influence the size and position of the resulting triangle.

What Is Dilation in Coordinate Geometry?

Dilation is a type of similarity transformation that alters the size of a figure but preserves its shape and the proportional relationships between its points. The key components involved in a dilation are:

Center of Dilation

- The fixed point around which the dilation occurs.
- Denoted as P, typically represented with coordinates (x_p, y_p).

Scale Factor (k)

- A positive real number indicating the degree of enlargement or reduction.
- If $k > 1$, the figure enlarges.
- If $0 < k < 1$, the figure shrinks.
- If $k = 1$, the shape remains unchanged (identity transformation).

Effect of Dilation on Coordinates

Given a point (x, y), the image (x', y') after dilation with center (x_p, y_p) and scale factor k is calculated as:

$$
x' = x_p + k(x - x_p)
$$

$$
y' = y_p + k(y - y_p)
$$

This formula stretches or compresses the point's position relative to the center.

Analyzing Triangle ABC Before Dilation

Let's first visualize and understand the original triangle ABC with vertices:
- $A(0,0)$
- $B(0,6)$
- $C(4,0)$

Plotting the Vertices

- Point A is at the origin.
- Point B is directly above A at (0,6).
- Point C is to the right of A at (4,0).

Properties of Triangle ABC

- It is a right triangle with the right angle at A.
- The hypotenuse stretches between B and C.
- The base AC lies along the x-axis, and the height AB along the y-axis.
- The area of triangle ABC can be calculated as:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 6 = 12
$$

Understanding these properties helps predict how the triangle will change after dilation.

Performing Dilation on Triangle ABC

Suppose we perform a dilation centered at the origin $(0,0)$ with a scale factor k. We will examine how the vertices transform and how the overall shape and size of the triangle are affected.

Calculating the Image of Each Vertex

Using the dilation formula with $(x_p, y_p) = (0,0)$:

$$
x' = 0 + k(x - 0) = kx
$$
$$
y' = 0 + ky
$$

Applying this to each vertex:

- Vertex A(0,0):
$$
A' = (k \times 0,\, k \times 0) = (0,0)
$$
The origin remains fixed under dilation centered at the origin.

- Vertex B(0,6):
$$
B' = (k \times 0,\, k \times 6) = (0, 6k)
$$
The point B moves vertically along the y-axis.

- Vertex C(4,0):
$$
C' = (k \times 4,\, k \times 0) = (4k, 0)
$$
The point C moves horizontally along the x-axis.

Effects of Different Scale Factors

- For $k = 2$, the triangle doubles in size:
$$
A' = (0,0), \quad B' = (0,12), \quad C' = (8,0)
$$
The new triangle is similar to the original but larger, with area scaled by $k^2 = 4$.

- For $k = 0.5$, the triangle shrinks to half its size:
$$
A' = (0,0), \quad B' = (0,3), \quad C' = (2,0)
$$
The triangle is smaller but maintains shape proportions.

- For $k = 1$, the triangle remains unchanged, as expected.

Visualizing the Dilation of Triangle ABC

Plotting the original and dilated triangles helps visualize the transformation:

- The original triangle ABC has vertices at (0,0), (0,6), and (4,0).
- The dilated triangle's vertices depend on the scale factor k, moving outward or inward from the origin.
- When $k > 1$, the triangle enlarges, with vertices moving further from the origin.
- When $0 < k < 1$, the shape shrinks towards the origin.

This visualization reinforces the understanding that dilation preserves the shape's angles and proportionality but alters its size and position relative to the center.

Impact of Center of Dilation on Triangle ABC

While we've considered dilation centered at the origin, changing the center of dilation affects the image differently.

Dilation Center at a Different Point

Suppose the center of dilation is at point $P(x_p, y_p) \neq (0,0)$. The image of each vertex becomes:

$$
x' = x_p + k(x - x_p)
$$
$$
y' = y_p + k(y - y_p)
$$

This shifts the entire triangle in a more complex way, depending on the location of P.

Example: Dilation Center at Point D(2,2)

Applying a dilation with center $D(2,2)$ and scale factor $k = 2$:

- Vertex A(0,0):
$$
A' = (2 + 2(0-2),\, 2 + 2(0-2)) = (2 - 4,\, 2 - 4) = (-2, -2)
$$
- Vertex B(0,6):
$$
B' = (2 + 2(0-2),\, 2 + 2(6-2)) = (2 - 4,\, 2 + 8) = (-2, 10)
$$
- Vertex C(4,0):
$$
C' = (2 + 2(4-2),\, 2 + 2(0-2)) = (2 + 4,\, 2 - 4) = (6, -2)
$$

This results in a larger, shifted triangle that maintains shape similarity but is relocated in the coordinate plane.

Applications and Real-World Examples of Dilation

Dilation is not just a theoretical concept; it has numerous practical applications:

- Scaling images in computer graphics and digital art.
- Creating similar shapes in engineering and architecture.
- Modeling phenomena where size changes but shape remains constant, such as biological growth patterns.
- Designing maps and blueprints where proportional scaling is necessary.

Understanding how dilation affects figures like triangle ABC helps in solving problems involving similar figures, scale models, and transformations.

Conclusion

Transforming triangle ABC with vertices A(0,0), B(0,6), and C(4,0) through dilation offers rich insights into geometric transformations and coordinate geometry. The key points include:

- Dilation enlarges or reduces figures relative to a fixed center with a specific scale factor.
- The coordinates of each vertex are scaled proportionally relative to the center.
- When centered at the origin, the vertices' transformations are straightforward, scaling directly with k.
- Changing the center of dilation shifts the entire image, affecting the triangle's position in the coordinate plane.
- The shape remains similar, preserving angles and proportional side lengths, but the size varies depending on the scale factor.

Mastering dilation provides a foundational understanding of similarity transformations, essential for advanced studies in geometry, design, and real-world problem-solving.

Keywords: triangle dilation, coordinate geometry, similar figures, scale factor, transformation, triangle ABC, geometric transformation

Frequently Asked Questions

What are the coordinates of triangle ABC before dilation?

The vertices are A(0,0), B(0,6), and C(4,0).

How does a dilation transform the vertices of triangle ABC?

A dilation scales each vertex relative to a fixed center point, enlarging or reducing the triangle proportionally while preserving its shape.

If the dilation is centered at the origin with a scale factor of 2, what are the new coordinates of the triangle's vertices?

The new vertices are A'(0,0), B'(0,12), and C'(8,0).

What is the effect of the dilation on the size and shape of triangle ABC?

The dilation enlarges or reduces the triangle proportionally, maintaining its shape but changing its size based on the scale factor.

How can you verify that the dilated triangle is similar to the original triangle ABC?

By checking that the corresponding angles are equal and the sides are proportional according to the scale factor, confirming similarity.

Additional Resources

Consider Triangle ABC With Vertices A(0,0), B(0,6), And C(4,0). The Image Of Triangle ABC After A Dilation

Understanding the transformations of geometric figures is a fundamental aspect of geometry, providing insights into how shapes can be manipulated while preserving certain properties. Among these transformations, dilation stands out as a powerful operation that enlarges or reduces figures relative to a fixed point, known as the center of dilation, by a specific scale factor. This article explores the transformation of triangle ABC, with vertices at A(0,0), B(0,6), and C(4,0), after it undergoes a dilation. We will analyze the effect of dilation on its size, shape, and position, and delve into the underlying principles that govern this transformation.

Understanding Triangle ABC: Coordinates and Properties

Vertices and Basic Characteristics

Triangle ABC is defined by three points in the coordinate plane:

- Vertex A at (0,0)
- Vertex B at (0,6)
- Vertex C at (4,0)

Plotting these points provides an initial visualization:

- $A(0,0)$ is at the origin.
- $B(0,6)$ lies directly above A along the y-axis.
- $C(4,0)$ is 4 units to the right of A along the x-axis.

This configuration indicates a right triangle, with right angle at vertex A, since:

- AB is vertical, extending from $(0,0)$ to $(0,6)$.
- AC is horizontal, extending from $(0,0)$ to $(4,0)$.
- BC connects $B(0,6)$ to $C(4,0)$.

Calculating the lengths of sides:

- AB: distance between A and B = $|0 - 0| + |6 - 0| = 6$ units.
- AC: distance between A and C = $|4 - 0| + |0 - 0| = 4$ units.
- BC: distance between B and C = $\sqrt{[(4 - 0)^2 + (0 - 6)^2]} = \sqrt{(16 + 36)} = \sqrt{52} \approx 7.21$ units.

The right angle at A confirms the right-angled nature of the triangle, and these side lengths establish a clear scale for transformations.

Area and Perimeter

The area of triangle ABC can be calculated using the formula for a right triangle:

$$ \text{Area} = \frac{1}{2} \times \text{base} \times \text{height} $$

Using AB as height and AC as base:

$$ \text{Area} = \frac{1}{2} \times 4 \times 6 = 12 \text{ square units} $$

Perimeter:

$$ \text{Perimeter} = AB + BC + AC \approx 6 + 7.21 + 4 = 17.21 \text{ units} $$

Understanding these properties provides a baseline for comparing the original triangle to its dilated image.

Principles of Dilation in Geometry

Definition and Key Concepts

Dilation is a transformation that produces an image that is a scaled version of the original figure, either enlarged or reduced, with respect to a fixed point called the center of dilation. The operation involves:

- Center of dilation (O): The fixed point about which the figure is scaled.
- Scale factor (k): A positive real number determining how much the figure is enlarged or reduced.

Mathematically, for a point $P(x, y)$, its image $P'(x', y')$ after dilation with center $O(x_o, y_o)$ and scale factor k is given by:

$$
x' = x_o + k (x - x_o)
$$

$$
y' = y_o + k (y - y_o)
$$

This formula signifies that each point moves along the line connecting it to the center O, scaled relative to that point.

Key properties of dilation include:

- Preservation of shape: The image is similar to the original; angles are preserved.
- Change in size: Distances are scaled by k.
- Center of similarity: All points and their images lie along lines passing through the center.

Impact of Dilation on Geometric Figures

The principal effects of dilation on a triangle are:

- Scaling of side lengths: Each side length is multiplied by k.
- Area change: The area of the image is scaled by k^2.
- Preservation of angles: The figure remains similar to the original.
- Position and orientation: The shape's position depends on the center of dilation and scale factor.

By understanding these principles, one can predict and analyze how triangle ABC transforms under dilation.

Applying Dilation to Triangle ABC

Choosing the Center of Dilation

The choice of the center O significantly influences the resulting image. Common options include:

- Origin (0,0): Convenient, as A is at the origin.
- Vertices of the triangle: For instance, center at A, B, or C.
- Other points: For example, the centroid, incenter, or circumcenter.

For this analysis, let's assume the dilation is centered at vertex A (0,0) because A is at the origin, simplifying calculations.

Determining the Scale Factor

The scale factor k can be any positive real number:

- $k > 1$: The triangle enlarges.
- $0 < k < 1$: The triangle reduces in size.
- $k = 1$: The triangle remains unchanged.

Suppose the scale factor is $k = 2$, indicating the triangle will double in size.

Calculating the Coordinates of the Image

Using the dilation formula with $O(0,0)$:

$$
P'(x', y') = (k \times x, k \times y)
$$

Applying this to each vertex:

- A(0,0): $A' = (2 \times 0, 2 \times 0) = (0,0)$
- B(0,6): $B' = (0, 12)$
- C(4,0): $C' = (8, 0)$

The image triangle $A'B'C'$ has vertices at:

- $A'(0,0)$
- $B'(0,12)$
- $C'(8,0)$

This triangle is similar to the original but scaled by a factor of 2, with its size doubled.

Analyzing the Image Triangle: Geometric Properties

Side Lengths of the Dilated Triangle

Applying the distance formula to the new vertices:

- A'B': between (0,0) and (0,12):

$$
\sqrt{(0 - 0)^2 + (12 - 0)^2} = 12
$$

- A'C': between (0,0) and (8,0):

$$
\sqrt{(8 - 0)^2 + (0 - 0)^2} = 8
$$

- B'C': between (0,12) and (8,0):

$$
\sqrt{(8 - 0)^2 + (0 - 12)^2} = \sqrt{64 + 144} = \sqrt{208} \approx 14.42
$$

Comparing with original side lengths:

Original Side	Length	Dilated Side	Length
AB	6	A'B'	12
AC	4	A'C'	8
BC	≈7.21	B'C'	≈14.42

All sides are scaled by $k = 2$, confirming the dilation's effect.

Area of the Dilated Triangle

Given the original area was 12 square units, the new area:

$$
\text{Area}' = k^2 \times \text{Area} = 2^2 \times 12 = 4 \times 12 = 48 \text{ square units}
$$

This quadrupling of the area is consistent with the properties of dilation, which scale area by the square of the scale factor.

Shape and Similarity Preservation

Since the dilation is a similarity transformation:

- The original and image triangles are similar.
- Corresponding angles are equal.
- The ratios of corresponding sides match the scale factor.

The shape remains congruent in form, but size varies according to the scale factor.

Visualizing and Interpreting the Dilation

Graphical Representation

Visualizing the transformation helps solidify understanding:

- Original triangle ABC can be plotted with vertices at (0,0), (0,6), and (4,0).
- The image triangle A'B'C' appears precisely scaled, with vertices at

Consider Triangle Abc With Vertices A 0 0 B 0 6 And C 4 0 The Image Of Triangle Abc After A Dilation

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