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Introduction

Determining the total mass of a geometric region is a fundamental problem in physics and mathematics, especially within the contexts of calculus and engineering. When dealing with a triangular region, the task involves integrating a density function over the specified area. In this article, we will explore how to find the total mass of a triangular region with given dimensions, emphasizing the methods, formulas, and steps required to arrive at the solution. We will assume a uniform density unless otherwise specified, which simplifies the process to calculating the area and multiplying by the density. If the density varies, the approach involves setting up a proper integral with the density function.

Understanding the Problem

What Is the Triangular Region?

A triangle is a polygon with three sides and three angles. To determine its mass, we need to understand its dimensions, shape, and position in the coordinate plane. Typically, the problem provides:

- The lengths of the sides or base and height
- Coordinates of the vertices
- The density function (uniform or variable)

Assumptions for This Problem

Since the problem statement mentions all lengths are in centimeters and asks for the total mass, and no specific density function is provided, the most straightforward assumption is:

- The density is uniform throughout the region.
- The triangle is right-angled or has specified vertices.

If the problem provides a diagram (which it references as "shown below"), it would specify dimensions such as base, height, or side lengths. For demonstration, we will assume a specific case: a right-angled triangle with a base and height.

Step 1: Identifying the Dimensions of the Triangle

Example Dimensions

Suppose the triangle has:

- Base length, b cm
- Height length, h cm

Coordinates of Vertices

Assuming the triangle is positioned in the coordinate plane with vertices at:

- $A(0,0)$
- $B(b,0)$
- $C(0,h)$

This configuration makes calculations straightforward.

Step 2: Calculating the Area of the Triangle

Area Formula

The area A of a triangle with base b and height h is:

$$
A = \frac{1}{2} \times b \times h
$$

Example Calculation

If $b = 8$ cm and $h = 6$ cm, then:

$$
A = \frac{1}{2} \times 8 \times 6 = 24 \text{ cm}^2
$$

Step 3: Determining the Density Function

Uniform Density

If the density ρ (mass per unit area) is uniform, then:

$$
\rho = \text{constant}
$$

The total mass M is:

$$
M = \rho \times \text{Area}
$$

Variable Density (Optional)

If the density varies over the region, represented as $\rho(x, y)$, then the total mass requires integrating over the area:

$$
M = \iint_{R} \rho(x, y) \, dA
$$

Where R is the region of the triangle.

Step 4: Calculating Total Mass

Case 1: Uniform Density

Suppose the density $\rho = 2 \text{ g/cm}^2$. Then:

$$
M = \rho \times A = 2 \times 24 = 48 \text{ grams}
$$

Case 2: Variable Density

If $\rho(x, y) = k \times y$, for example, then:

$$
M = \iint_{R} \rho(x, y) \, dA
$$

Using the coordinate system, the limits of integration are:

- x from 0 to b
- y from 0 to the line connecting $(b, 0)$ and $(0, h)$:

Equation of the hypotenuse:

$$
y = -\frac{h}{b} x + h
$$

The integral becomes:

$$
M = \int_{x=0}^{b} \int_{y=0}^{- \frac{h}{b} x + h} \rho(x, y) \, dy \, dx
$$

Plugging in $\rho(x, y) = y$:

$$
M = \int_{0}^{b} \int_{0}^{- \frac{h}{b} x + h} y \, dy \, dx
$$

Calculating the inner integral:

$$
\int_{0}^{- \frac{h}{b} x + h} y \, dy = \frac{1}{2} \left(- \frac{h}{b} x + h \right)^2
$$

Then, the total mass:

$$
M = \frac{1}{2} \int_{0}^{b} \left(- \frac{h}{b} x + h \right)^2 dx
$$

This integral can be evaluated to find the total mass for the variable density case.

Step 5: General Approach for Arbitrary Triangles

Using Coordinate Geometry

For triangles not aligned with axes or with irregular shapes, coordinate geometry and calculus are essential.

Steps:

1. Identify vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$.
2. Determine the boundary equations for the sides.
3. Set up double integrals over the region with appropriate limits.
4. Insert the density function $\rho(x, y)$.
5. Compute the integral to find the total mass.

Step 6: Examples and Practice Problems

Example 1: Right Triangle with Uniform Density

Suppose a triangle with vertices at:

- $A(0,0)$
- $B(10,0)$
- $C(0,5)$

Density $\rho = 3 \text{ g/cm}^2$.

Solution:

- Area:

$$
A = \frac{1}{2} \times 10 \times 5 = 25 \text{ cm}^2
$$

- Total mass:

$$
M = 3 \times 25 = 75 \text{ grams}
$$

Example 2: Triangle with Variable Density

Density varies linearly with x:

$$
\rho(x, y) = 2x
$$

Using the same triangle as above:

- x from 0 to 10
- For each x, y from 0 to $-\frac{1}{2} x + 5$

Set up the integral:

$$
M = \int_{x=0}^{10} \int_{y=0}^{- \frac{1}{2} x + 5} 2x \, dy \, dx
$$

Inner integral:

$$
\int_{0}^{- \frac{1}{2} x + 5} 2x \, dy = 2x \left(- \frac{1}{2} x + 5 \right) = 2x \times \left(- \frac{1}{2} x + 5 \right)
$$

Simplify:

$$

$$
2x \times \left(- \frac{1}{2} x + 5 \right) = -x^2 + 10x
$$

Total mass:

$$
M = \int_{0}^{10} (-x^2 + 10x) \, dx
$$

Compute:

$$
\int_{0}^{10} -x^2 \, dx = -\frac{x^3}{3} \bigg|_{0}^{10} = -\frac{1000}{3}
$$

$$
\int_{0}^{10} 10x \, dx = 5x^2 \bigg|_{0}^{10} = 5 \times 100 = 500
$$

Adding:

$$
M = -\frac{1000}{3} + 500 = \frac{-1000 + 1500}{3} = \frac{500}{3} \approx 166.67 \text{ grams}
$$

Conclusion

Calculating the total mass of a triangular region involves understanding its geometry, defining the appropriate coordinate system, and applying calculus techniques to integrate the density function over the region. When the density is uniform, the process simplifies to multiplying the area by the density constant. For variable densities, setting up and evaluating a double integral is necessary, often requiring establishing the limits based on the region's boundary equations.

By mastering these methods, one can solve a wide range of problems involving mass calculations in various geometric shapes, which are essential in physics, engineering, and applied mathematics. Whether dealing with simple right triangles or complex irregular regions, the principles outlined here provide a systematic approach to accurately determining total mass.

References

- Stewart, J. (2015). Calculus: Early Transcendentals. Cengage Learning.
- Anton, H., Bivens, I., & Davis, S. (2012).

Frequently Asked Questions

What is the first step to find the total mass of the triangular region shown in the diagram?

Identify the density function or density distribution across the triangular region and determine the limits for integration based on the triangle's dimensions.

How do you set up the integral to find the total mass of the triangle?

You integrate the density function over the triangular area, choosing appropriate limits for variables like x and y, based on the triangle's vertices.

What role does the density function play in calculating the total mass?

The density function specifies how mass is distributed across the region; integrating it over the area sums up all the infinitesimal mass elements to find the total mass.

If all lengths are in centimeters, what units will the total mass be in assuming a uniform density?

The units of mass will depend on the units of density; if density is given in mass per square centimeter, the total mass will be in units of mass, such as grams or kilograms.

How does the shape of the triangle influence the calculation of total mass?

The shape determines the limits of integration and the form of the density function, which collectively influence the setup and evaluation of the integral for total mass.

What methods can be used to evaluate the integral for total mass in this problem?

Methods include direct integration, substitution, or using symmetry properties, depending on the form of the density function and the shape of the triangle.

Why is it important to confirm the density function before calculating the total mass?

Because the density function defines how mass is distributed, confirming its form ensures accurate setup of the integral and correct calculation of the total mass.

Additional Resources

Correct. (1 Point) Find The Total Mass Of The Triangular Region Shown Below. All Lengths Are In Centimeters.
Understanding how to find the total mass of a triangular region is a fundamental problem in the field of calculus and physics, particularly when dealing with mass distributions across irregular shapes. This problem involves integrating a density function over a specified geometric region—in this case, a triangle. Whether you're a student preparing for an exam, a teacher designing assessments, or a professional applying mathematical principles to real-world problems, mastering this process is essential for accurate calculations and deeper comprehension of integral calculus applications.

In this comprehensive guide, we will explore step-by-step how to determine the total mass of a triangular region given its dimensions and density distribution. We will cover the necessary concepts, methods, and common pitfalls, providing clarity and confidence in tackling similar problems.

Understanding the Problem: Key Components and Terminology

Before delving into calculations, it’s crucial to understand the problem components:

- Triangular Region: A two-dimensional geometric shape with three sides and three angles, defined by specific coordinates or boundary lines.
- All Lengths Are In Centimeters: The given dimensions are in centimeters, which is important for unit consistency.
- Density Function: Often, the problem involves a density function $\rho(x, y)$ that may vary across the region, requiring integration.
- Total Mass: The integral of the density function over the region, expressed mathematically as

$$
\text{Mass} = \iint_{R} \rho(x, y) \, dA
$$

where R is the region of interest.

Step 1: Visualizing and Defining the Triangular Region

1.1 Sketching the Triangle

Begin by sketching the triangle based on given dimensions. Typically, the problem provides:

- Coordinates of vertices
- Lengths of sides
- Coordinates of boundary points

Example:

Suppose the triangle has vertices at points $A(0,0)$, $B(4,0)$, and $C(0,3)$. This forms a right triangle with legs of lengths 4 cm and 3 cm.

1.2 Determining Coordinates and Boundaries

Identify the boundary lines:

- AB: from $(0,0)$ to $(4,0)$, along the x-axis
- AC: from $(0,0)$ to $(0,3)$, along the y-axis
- BC: from $(4,0)$ to $(0,3)$

Find the equations of the boundary lines:

- AB: $y = 0$
- AC: $x = 0$
- BC:

Calculate the slope:

$$
m = \frac{3 - 0}{0 - 4} = -\frac{3}{4}
$$

Equation passing through $B(4,0)$:

$$
y - 0 = -\frac{3}{4}(x - 4) \Rightarrow y = -\frac{3}{4}x + 3
$$

1.3 Describing the Region

The region R is bounded by:

- x from 0 to 4
- For each x, y from 0 up to the line $y = -\frac{3}{4}x + 3$

Step 2: Choosing the Appropriate Coordinate System and Integration Limits

2.1 Cartesian Coordinates

Given the boundary lines, it's straightforward to set up double integrals in Cartesian coordinates.

2.2 Limits of Integration

- Outer integral (for x): from 0 to 4
- Inner integral (for y): from 0 to $-\frac{3}{4}x + 3$

Thus, the double integral becomes:

$$
\text{Mass} = \int_{x=0}^{4} \int_{y=0}^{-\frac{3}{4}x + 3} \rho(x, y) \, dy \, dx
$$

2.3 Simplification Based on Density Function

- If the density ρ is constant, the integration simplifies.
- If ρ varies with x, y, or both, incorporate the specific function.

Step 3: Defining the Density Function

The problem may specify:

- Constant density: $\rho(x, y) = \rho_0$
- Variable density: e.g., $\rho(x, y) = kx + my + c$

3.1 Constant Density Case

Assuming $\rho(x, y) = \rho_0$, a constant, the total mass simplifies to:

$$
\text{Mass} = \rho_0 \times \text{Area of the triangle}
$$

Calculate the area:

$$
\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times 3 = 6 \text{ cm}^2
$$

So,

$$
\text{Mass} = \rho_0 \times 6

\]

3.2 Variable Density Case

When density varies, set up the integral explicitly:

$$
\text{Mass} = \int_{x=0}^{4} \int_{y=0}^{-\frac{3}{4}x + 3} \left(kx + my + c \right) dy \, dx
$$

Step 4: Performing the Integration

4.1 Integrate with Respect to y

For a general density function:

$$
\int_{0}^{-\frac{3}{4}x + 3} \left(kx + my + c \right) dy
$$

Break into parts:

$$
kx \int_{0}^{-\frac{3}{4}x + 3} dy + m \int_{0}^{-\frac{3}{4}x + 3} y \, dy + c \int_{0}^{-\frac{3}{4}x + 3} dy
$$

Calculate each:

- $kx \left[y \right]_0^{-\frac{3}{4}x + 3} = kx \left(-\frac{3}{4}x + 3 \right)$
- $m \left[\frac{y^2}{2} \right]_0^{-\frac{3}{4}x + 3} = m \frac{ \left(-\frac{3}{4}x + 3 \right)^2 }{2}$
- $c \left[y \right]_0^{-\frac{3}{4}x + 3} = c \left(-\frac{3}{4}x + 3 \right)$

4.2 Integrate with Respect to x

Now, integrate the resulting expression over x from 0 to 4:

$$
\text{Mass} = \int_{0}^{4} \left[kx \left(-\frac{3}{4}x + 3 \right) + m \frac{ \left(-\frac{3}{4}x + 3 \right)^2 }{2} + c \left(-\frac{3}{4}x + 3 \right) \right] dx
$$

This integral can be computed term-by-term, applying standard calculus techniques, including expansion and polynomial integration.

Step 5: Computing the Integral and Final Result

5.1 Simplify the Expression

Expand the terms:

- $kx \left(-\frac{3}{4}x + 3 \right) = -\frac{3}{4} k x^2 + 3k x$
- $\left(-\frac{3}{4}x + 3 \right)^2 = \left(\frac{9}{16} x^2 - 2 \times \frac{3}{4} x \times 3 + 9 \right) = \frac{9}{16}x^2 - \frac{18}{4} x + 9$

Simplify:

$$
\frac{9}{16} x^2 - \frac{18}{4} x + 9 = \frac{9}{16}x^2 - \frac{9}{2} x + 9
$$

Now, the integral becomes:

$$
\int_{0}^{4} \left[-\frac{3}{4} k x^2 + 3k x + m \frac{ \frac{9}{16}x^2 - \frac{9}{2} x + 9 }{2} + c \left(-\frac{3}{4}x + 3 \right) \right] dx
$$

Distribute m over the numerator:

\[
m \left(\frac{9}{16}x^2 - \frac{9}{2} x + 9 \right)/2

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