Determine The Total Time That Will Have Elapsed When 12.5 Grams Of The Original Co-60 Sample At The Hospital

Determine The Total Time That Will Have Elapsed When 12.5 Grams Of The Original Co-60 Sample At The Hospital

Understanding the decay process of radioactive isotopes is essential, especially in medical settings where isotopes like Cobalt-60 (Co-60) are widely used for cancer radiotherapy and other diagnostic procedures. When a hospital receives a Co-60 source, it is crucial for medical physicists and radiologists to estimate how much time has passed since the isotope's initial activity or to predict the remaining activity over time. In this article, we will explore how to determine the total time that has elapsed when a specific amount of Co-60 remains, focusing on a scenario where only 12.5 grams of the original sample are left.

Understanding Cobalt-60 and Its Radioactive Decay

Before diving into calculations, it’s vital to understand the nature of Co-60 and how it decays.

What Is Cobalt-60?

- Cobalt-60 is a radioactive isotope of cobalt, with a half-life of approximately 5.27 years.
- It emits beta particles and gamma rays during decay, making it useful for radiotherapy and sterilization.
- The isotope is produced artificially in nuclear reactors and used in medical applications.

Radioactive Decay Basics

- Radioisotopes decay exponentially over time.
- The activity (or amount) of a radioactive sample decreases by half every half-life period.
- The decay follows the formula:

$$
N(t) = N_0 \times e^{-\lambda t}
$$

where:
- $N(t)$ = remaining amount at time t
- N_0 = original amount
- λ = decay constant
- t = elapsed time

Calculating the Decay Constant for Co-60

The decay constant (λ) relates to the half-life ($T_{1/2}$) via:

$$
\lambda = \frac{\ln 2}{T_{1/2}}
$$

Given:
- $T_{1/2}$ for Co-60 = 5.27 years

Calculating:
$$
\lambda = \frac{\ln 2}{5.27} \approx \frac{0.6931}{5.27} \approx 0.1314 \text{ per year}
$$

This decay constant indicates that approximately 13.14% of the remaining Co-60 decays each year.

Relating Remaining Mass to Time Elapsed

The amount of Co-60 remaining after time t is given by:

$$
N(t) = N_0 \times e^{-\lambda t}
$$

Suppose:
- The initial mass of the Co-60 sample is N_0
- The current mass is $N(t)$

Given:
- $N(t) = 12.5$ grams

To find the elapsed time t, rearranged formula:

$$
t = -\frac{1}{\lambda} \times \ln \left(\frac{N(t)}{N_0} \right)
$$

Determining Initial Mass N_0

In practical scenarios, the initial mass might be known, or the initial activity or mass is given. If not specified, we can analyze the problem assuming a known initial amount.

Example:
- Original sample $N_0 = 100$ grams
- Remaining amount $N(t) = 12.5$ grams

Then:

$$
t = -\frac{1}{0.1314} \times \ln \left(\frac{12.5}{100} \right)
$$

Calculations:

$$
\frac{12.5}{100} = 0.125
$$

$$
\ln 0.125 = -2.0794
$$

$$
t = -\frac{1}{0.1314} \times (-2.0794) \approx 7.94 \text{ years}
$$

Interpretation:
It would take approximately 7.94 years for the original 100 grams of Co-60 to decay to 12.5 grams.

Adjusting for Different Initial Amounts

If the initial mass is different, simply substitute the known value into the formula. For example:

- Initial mass: 50 grams
- Remaining: 12.5 grams

$$
t = -\frac{1}{0.1314} \times \ln \left(\frac{12.5}{50} \right)
$$

$$
\frac{12.5}{50} = 0.25
$$

$$
\ln 0.25 = -1.3863
$$

$$
t = -\frac{1}{0.1314} \times (-1.3863) \approx 10.55 \text{ years}
$$

This indicates that it takes about 10.55 years for the sample to decay from 50 grams to 12.5 grams.

Practical Application in Hospital Settings

Knowing how to determine the elapsed time based on remaining Co-60 mass helps hospital physicists:

- Track the age of the radioactive source
- Plan for source replacement or re-calibration
- Ensure safety protocols are maintained
- Comply with radiation safety regulations

Estimating Time When the Exact Initial Mass Is Unknown

If the initial amount of Co-60 is not known, other methods such as measuring the current activity and comparing it with the known activity at calibration can be used. Since activity is directly proportional to the number of radioactive atoms:

$$
A(t) = A_0 \times e^{-\lambda t}
$$

where:
- $A(t)$ = current activity
- A_0 = initial activity

By measuring current activity and knowing initial activity, the elapsed time can be calculated similarly.

Summary of Key Steps to Determine Elapsed Time:

1. Know or estimate the initial amount of Co-60 (N_0)
2. Measure the current amount or activity ($N(t)$)
3. Calculate the decay constant (λ) from the half-life
4. Use the decay formula to find the elapsed time:

$$
t = -\frac{1}{\lambda} \times \ln \left(\frac{N(t)}{N_0} \right)
$$

Conclusion

Determining the total time elapsed since the initial preparation of a Co-60 source based on remaining mass is essential for proper management and safety in medical facilities. By understanding the exponential decay law, calculating the decay constant, and applying the appropriate formulas, hospital physicists can accurately estimate the age of a radioactive sample. Whether dealing with known initial amounts or current measurements, these calculations ensure that medical radiation practices remain safe, effective, and compliant with regulatory standards.

Additional Resources

- Radioactive Decay Law and Applications — A comprehensive guide on understanding decay processes.
- Safety Guidelines for Handling Co-60 in Medical Settings — Best practices for medical professionals.
- Calculating Half-life and Decay Constants — Tutorials for students and professionals alike.

Understanding the decay of Co-60 and accurately estimating elapsed time ensures ongoing safety and efficacy in radiation therapy and medical diagnostics. Proper application of these principles supports optimal patient care and radiation safety compliance.

Frequently Asked Questions

How do I calculate the total time elapsed for a 12.5g Co-60 sample given its half-life?

To determine the elapsed time, you need to know the initial amount and the current amount. Using the half-life of Co-60 (approximately 5.27 years), apply the decay formula: $N = N_0 (1/2)^{(t / T_{1/2})}$. Rearranged to find t: $t = T_{1/2} \log_2(N_0 / N)$.

What is the half-life of Co-60, and how does it relate to the decay process of the hospital sample?

Cobalt-60 has a half-life of about 5.27 years. This means that after 5.27 years, half of the initial sample decays. Knowing this helps estimate the time elapsed based on the remaining activity or mass.

If the initial Co-60 sample was 20 grams, how long has 12.5 grams of it been decaying in the hospital?

Since the sample has halved from 20 grams to 12.5 grams, it indicates it has taken less than one half-life. Specifically, the time elapsed is approximately 5.27 years, as 12.5 grams is half of 25 grams; initial mass was 20 grams, so the decay corresponds to roughly one half-life period, approximately 5.27 years.

Can I determine the exact elapsed time solely based on the remaining mass of Co-60 in the hospital sample?

Yes, if you know the initial mass and the current mass, and the half-life of Co-60, you can calculate the elapsed time using the decay formula. However, measurement precision and initial data accuracy are essential for an exact calculation.

Why is understanding the decay of Co-60 important in a hospital setting?

Understanding Co-60 decay helps hospital staff determine the remaining activity for radiation safety, scheduling proper disposal, and ensuring accurate dosimetry for treatments involving radioactive sources.

Additional Resources

Determine The Total Time That Will Have Elapsed When 12.5 Grams Of The Original Co-60 Sample At The Hospital

In the realm of medical physics and radiology, understanding the decay of radioactive isotopes is crucial for ensuring the safety and efficacy of diagnostic and therapeutic procedures. One such isotope, Cobalt-60 (Co-60), plays a pivotal role in cancer radiotherapy and sterilization processes. Hospitals that utilize Co-60 sources must carefully monitor the residual activity of their radioactive materials to maintain safety standards, optimize treatment schedules, and plan for source replacement. A fundamental question often encountered is: How much time must pass for a Co-60 sample, initially weighing 12.5 grams, to decay to a certain residual activity?

This article delves into the principles of radioactive decay, specifically focusing on Co-60, and provides a comprehensive methodology to determine the total elapsed time based on initial and remaining sample quantities. Through detailed explanations, relevant formulas, and practical examples, readers will gain a clear understanding of how to evaluate the decay timeline of Co-60 in a hospital setting.

Understanding Radioactive Decay and Its Significance in Medical Applications

Radioactive decay is a spontaneous process whereby unstable atomic nuclei lose energy by emitting radiation, transforming into more stable nuclei. For medical applications, particularly with Co-60, decay rates directly influence the source's strength, treatment planning, and safety protocols.

Key concepts include:

- Half-Life: The period required for half of the radioactive nuclei in a sample to decay. For Co-60, the half-life is approximately 5.27 years.
- Decay Constant (λ): A parameter that characterizes the rate of decay, expressed in inverse time units (e.g., per year). It relates directly to the half-life through the formula:

\[

$$
\lambda = \frac{\ln(2)}{T_{1/2}}
$$

- Activity (A): The number of decays per unit time, measured in Becquerels (Bq) or Curies (Ci). It diminishes over time as the isotope decays.

Understanding these parameters is crucial in calculating how long a specific amount of Co-60 will remain above a certain activity threshold, influencing decisions in hospital radiation safety and source management.

Fundamental Principles and Formulas for Radioactive Decay

To determine the elapsed time for a Co-60 sample to decay from its initial weight to a specific residual weight, it's essential to understand the mathematical relationship governing decay.

The decay law states:

$$
N(t) = N_0 e^{-\lambda t}
$$

Where:
- $N(t)$ = number of radioactive nuclei (or mass proportional to nuclei) remaining after time t,
- N_0 = initial number of nuclei,
- λ = decay constant,
- t = elapsed time.

Since the mass of the sample is directly proportional to the number of nuclei, the same relation applies to mass:

$$
m(t) = m_0 e^{-\lambda t}
$$

Where:
- $m(t)$ = mass at time t,
- m_0 = initial mass.

Rearranged, the formula to compute elapsed time based on initial and remaining mass is:

$$
t = \frac{1}{\lambda} \ln \left(\frac{m_0}{m(t)} \right)
$$

This formula enables precise calculation of the elapsed time when the initial mass and current mass are known, provided the decay constant is known.

Applying the Decay Law to Co-60 in a Hospital Setting

Given:
- Initial mass of Co-60: $m_0 = 12.5\, \text{grams}$,
- Target residual mass: $m(t)$ (unknown),
- Half-life of Co-60: $T_{1/2} = 5.27\, \text{years}$.

Step 1: Calculate the decay constant (λ)

$$
\lambda = \frac{\ln(2)}{T_{1/2}} = \frac{0.6931}{5.27\, \text{years}} \approx 0.1314\, \text{year}^{-1}
$$

Step 2: Determine the remaining mass ($m(t)$)

Suppose, for example, the hospital needs to know how long it takes for the Co-60 source to decay from 12.5 grams to 6.25 grams (half the initial weight). This is a common scenario in source management, as it corresponds to one half-life.

Step 3: Calculate elapsed time (t)

Using the formula:

$$
t = \frac{1}{\lambda} \ln \left(\frac{m_0}{m(t)} \right)
$$

Substituting the known values:

$$
t = \frac{1}{0.1314} \ln \left(\frac{12.5}{6.25} \right) = \frac{1}{0.1314} \ln(2) \approx \frac{0.6931}{0.1314} \approx 5.27\, \text{years}
$$

This confirms that after approximately 5.27 years, the 12.5 g Co-60 sample will decay to 6.25 g, aligning with the isotope’s half-life.

Extending the Calculation to Other Residual Masses

While the half-life provides a convenient reference, real-world scenarios often involve different residual amounts. For instance, if the hospital wants to know how long it takes for the sample to decay to a specific activity level, or a different mass, the same formula applies.

Example: Determine the time required for the sample to decay from 12.5 g to 3.125 g (which is one-eighth of the original mass).

- Since $m(t) = m_0/8$,

$$
t = \frac{1}{0.1314} \ln \left(\frac{12.5}{3.125} \right) = \frac{1}{0.1314} \ln(4) \approx \frac{1.3863}{0.1314} \approx 10.54\, \text{years}
$$

This corresponds to two half-lives because:

- After 5.27 years: 12.5 g → 6.25 g,
- After another 5.27 years: 6.25 g → 3.125 g.

Thus, the decay behavior is exponential, with the mass halving every 5.27 years.

Practical Implications for Hospital Radiation Safety and Maintenance

Understanding decay timelines is vital for hospitals managing Co-60 sources. The decay affects:

- Source Activity and Treatment Planning: As the source weakens, longer exposure times or higher source activity may be needed to deliver prescribed doses.
- Safety Protocols: Decayed sources require careful handling and eventual replacement to prevent undue radiation exposure.
- Regulatory Compliance: Hospitals must report residual activity and decay timelines to regulatory bodies, ensuring safe storage and disposal.

Key considerations include:

- Regular monitoring of source activity to determine the current residual mass.
- Planning for source replacement when activity falls below operational thresholds.
- Ensuring staff are trained to interpret decay calculations and safety procedures.

Sample calculation summary:

Initial mass	Residual mass	Time elapsed (years)
12.5 g	6.25 g	5.27
12.5 g	3.125 g	10.54
12.5 g	1.5625 g	15.81

This table highlights the exponential nature of decay and emphasizes the importance of precise calculations for operational planning.

Additional Factors Influencing Decay Calculations

While the theoretical model provides a solid foundation, real-world factors can influence decay-related assessments:

- Measurement Uncertainties: Variations in weighing or detecting activity can introduce errors.
- Source Geometry and Shielding: The physical configuration can affect activity measurements.
- Environmental Conditions: Temperature, humidity, and storage conditions may impact source stability.

Nevertheless, the core decay law remains valid, providing a reliable basis for planning and safety management.

Conclusion: Integrating Decay Calculations into Hospital Operations

Determining the total elapsed time for a Co-60 sample to decay from its initial weight to a specific residual amount is a straightforward application of the exponential decay law. By knowing the half-life of Co-60 and employing the decay constant, hospital physicists and safety officers can accurately estimate how long a source has been decaying and plan accordingly.

This understanding ensures that:

- Treatment protocols remain effective,
- Safety standards are maintained,
- Regulatory requirements are met,
- Resources are optimized for source replacement and disposal.

In an era where precise radiation management is paramount, mastering these calculations is essential for ensuring both patient safety and operational efficiency. Hospitals that integrate these decay principles into their routine practices can better navigate the complexities of radioactive source management, ultimately contributing to safer and more effective healthcare delivery.

In summary:

- The decay constant for Co-60 is approximately 0.1314 year\(^{-1}\).

Determine The Total Time That Will Have Elapsed When 12 5 Grams Of The Original Co 60 Sample At The Hospital

Determine The Total Time That Will Have Elapsed When 12 5 Grams Of The Original Co 60 Sample At The Hospital

Related Articles

- According To Which Of The Following Theories Is Language Acquired By Imitation And Reinforcement Of Specific
- What Makes Producers So Unique Compared To Other Organisms In Any Ecosystem? A. They Are The Only Organisms
- Liquidity Investing Company Owns 500 Shares Of High Flier Stock, And In Accordance With US GAAP, Is Required

Back to Home