Estimate The Area Under The Graph Of F(x) =10 Sqrt X From X = 0 To X = 4 Using Four Approximating Rectangles

Estimate The Area Under The Graph Of F(x) = 10√x From X = 0 To X = 4 Using Four Approximating Rectangles

Understanding how to approximate the area under a curve is a fundamental concept in calculus, especially when dealing with functions that do not lend themselves easily to direct integration. In this article, we will explore how to estimate the area under the graph of the function $f(x) = 10 \sqrt{x}$ from $x=0$ to $x=4$ using four approximating rectangles. This approach provides a practical introduction to the concept of Riemann sums, a cornerstone in the development of integral calculus.

Introduction to Area Estimation Under Curves

Before delving into the specifics of our function, it's essential to understand why and how we estimate areas under curves. The definite integral $\int_a^b f(x) \, dx$ represents the exact area under the graph of $f(x)$ from $x=a$ to $x=b$. However, in many cases, especially with complex functions, calculating this integral analytically can be challenging.

Approximate methods, such as Riemann sums, involve dividing the interval $[a, b]$ into smaller subintervals and summing the areas of rectangles that approximate the area under the curve. As the number of rectangles increases, the approximation becomes more accurate, approaching the true value of the integral.

Understanding the Function $f(x) = 10 \sqrt{x}$

Our function, $f(x) = 10 \sqrt{x}$, is a continuous, increasing function on the interval $[0, 4]$. The square root function grows slowly as x increases, and multiplying by 10 scales the function vertically.

Graphically, $f(x)$ starts at 0 when $x=0$ and rises to $10 \times 2 = 20$ when $x=4$. The task is to estimate the area under this curve from 0 to 4 using four rectangles.

Dividing the Interval into Four Subintervals

To use four rectangles, we first divide the interval $[0, 4]$ into four equal parts:

- Subinterval 1: $[0, 1]$

- Subinterval 2: $[1, 2]$
- Subinterval 3: $[2, 3]$
- Subinterval 4: $[3, 4]$

The width of each subinterval, Δx, is:

$$
\Delta x = \frac{4 - 0}{4} = 1
$$

This consistent width simplifies calculations and helps illustrate the approximation process.

Choosing Sample Points for Rectangles

In Riemann sum approximations, the height of each rectangle depends on the value of the function at specific points within each subinterval. Common choices include:

- Left endpoints
- Right endpoints
- Midpoints

Each choice yields a different approximation. For this article, we'll explore all three methods to compare their results.

1. Left Endpoint Approximation

In the left endpoint method, the height of each rectangle is determined by the value of $f(x)$ at the left endpoint of each subinterval:

- For $[0,1]$, height = $f(0)$
- For $[1,2]$, height = $f(1)$
- For $[2,3]$, height = $f(2)$
- For $[3,4]$, height = $f(3)$

Calculating each:

$$
\begin{aligned}
f(0) &= 10 \sqrt{0} = 0 \\

f(1) &= 10 \sqrt{1} = 10 \\
f(2) &= 10 \sqrt{2} \approx 10 \times 1.4142 \approx 14.142 \\
f(3) &= 10 \sqrt{3} \approx 10 \times 1.7321 \approx 17.321
\end{aligned}
\]

Estimate of the area:

\[
A_{L} = \sum_{i=1}^4 f(x_{i-1}) \times \Delta x = (f(0) + f(1) + f(2) + f(3)) \times 1
\]

\[
A_{L} = (0 + 10 + 14.142 + 17.321) \times 1 = 41.463
\]

2. Right Endpoint Approximation

Here, heights are based on the right endpoints:

- \([0,1]\): \(f(1)\)
- \([1,2]\): \(f(2)\)
- \([2,3]\): \(f(3)\)
- \([3,4]\): \(f(4)\)

Calculations:

\[
\begin{aligned}
f(1) &= 10 \times 1 = 10 \\
f(2) &= 14.142 \quad (\text{as above}) \\
f(3) &= 17.321 \\
f(4) &= 10 \times 2 = 20
\end{aligned}
\]

Area estimate:

\[
A_{R} = (f(1) + f(2) + f(3) + f(4)) \times 1 = (10 + 14.142 + 17.321 + 20) = 61.463
\]

3. Midpoint Approximation

Using midpoints, we evaluate \(f(x)\) at the center of each subinterval:

- \([0,1]\): \(x=0.5\)

- $[1,2]$: $x=1.5$
- $[2,3]$: $x=2.5$
- $[3,4]$: $x=3.5$

Calculations:

$$
\begin{aligned}
f(0.5) &= 10 \sqrt{0.5} \approx 10 \times 0.7071 \approx 7.071 \\
f(1.5) &= 10 \sqrt{1.5} \approx 10 \times 1.2247 \approx 12.247 \\
f(2.5) &= 10 \sqrt{2.5} \approx 10 \times 1.5811 \approx 15.811 \\
f(3.5) &= 10 \sqrt{3.5} \approx 10 \times 1.8708 \approx 18.708
\end{aligned}
$$

Estimate:

$$
A_{M} = (7.071 + 12.247 + 15.811 + 18.708) \times 1 = 53.837
$$

Summary of Approximations

Method	Approximate Area
Left Endpoint	41.463
Right Endpoint	61.463
Midpoint	53.837

The true area under $f(x) = 10 \sqrt{x}$ from 0 to 4 can be computed exactly using definite integration.

Calculating the Exact Area Using Integration

The exact value of the area is:

$$
\int_0^{4} 10 \sqrt{x} \, dx
$$

Rewrite as:

$$
10 \int_0^{4} x^{1/2} \, dx
$$

The integral:

$$
10 \times \left[\frac{2}{3} x^{3/2} \right]_0^{4}
$$

Calculate:

$$
= \frac{20}{3} \left[4^{3/2} - 0 \right]
$$

Since:

$$
4^{3/2} = (4^{1/2})^3 = (2)^3 = 8
$$

Thus:

$$
\text{Exact area} = \frac{20}{3} \times 8 = \frac{160}{3} \approx 53.333
$$

Comparison and Analysis

The exact area is approximately 53.333. Comparing this with our Riemann sum approximations:

- Left endpoint estimate: 41.463 (underestimate)
- Midpoint estimate: 53.837 (very close to actual)
- Right endpoint estimate: 61.463 (overestimate)

This illustrates the strengths and limitations of each method. The midpoint approximation provides the most accurate estimate among the three, especially with a moderate number of rectangles.

Conclusion

Estimating the area under a curve using rectangles is a foundational technique that bridges the understanding of Riemann sums and definite integrals. By dividing the interval into four parts and choosing different sample points, we see how approximation methods converge toward the exact area as the number of rectangles increases.

For the function

Frequently Asked Questions

How do you set up the problem to estimate the area under the graph of $F(x) = 10\sqrt{x}$ from $x = 0$ to $x = 4$ using four rectangles?

You partition the interval [0, 4] into four equal subintervals, determine the height of each rectangle using the function value at a chosen sample point within each subinterval (such as left, right, or midpoint), and then sum the areas of these rectangles to approximate the total area.

What is the width of each rectangle when dividing the interval from 0 to 4 into four equal parts?

The width of each rectangle is 1 unit since $(4 - 0) / 4 = 1$.

How do you compute the height of each rectangle when using left endpoints for the approximation?

For left endpoints, the height of each rectangle is obtained by evaluating $F(x)$ at the left boundary of each subinterval: at $x = 0, 1, 2$, and 3 respectively.

What is the approximate area under the graph using four rectangles with right endpoints?

Using right endpoints at $x = 1, 2, 3$, and 4, compute $F(1)$, $F(2)$, $F(3)$, and $F(4)$, then sum their products with the width (1) to estimate the area.

Why might using midpoints or different sample points improve the accuracy of the estimate?

Using midpoints or other sample points can provide a better approximation because they often reduce the error associated with the shape and curvature of the graph, leading to a more accurate estimate of the area.

How does increasing the number of rectangles affect the accuracy of the area estimate?

Increasing the number of rectangles decreases the width of each rectangle, which generally improves the approximation by more closely conforming to the shape of the curve, thus reducing the approximation error.

Additional Resources

Estimate the Area Under the Graph of $F(x) = 10\sqrt{x}$ from $x = 0$ to $x = 4$ Using Four Approximating Rectangles

Understanding how to approximate the area under a curve is a fundamental concept in calculus, with applications spanning physics, engineering, economics, and beyond. Here, we explore the method of estimating this area for the function $F(x) = 10\sqrt{x}$ over the interval [0, 4] using four approximating rectangles. This approach not only provides a practical way to estimate integrals before delving into more precise calculus techniques but also offers insight into the concepts of Riemann sums and the fundamental theorem of calculus.

In this comprehensive review, we will break down the problem into digestible parts, analyze the function's behavior, select appropriate partitioning strategies, compute the areas of rectangles, and discuss the implications and limitations of this approximation method.

Understanding the Function and Its Graph

Function Definition and Basic Properties

The function under consideration is:

$$F(x) = 10 \sqrt{x}$$

This is a scaled square root function, which is known for its characteristic shape: it starts at zero when $x=0$, increases monotonically, but at a decreasing rate as x increases. The square root function $\sqrt{x}$ is concave down, and multiplying by 10 scales the graph vertically.

Key properties:

- Domain: [0, 4]
- Range: $[0, 10\sqrt{4}] = [0, 10\ 2] = [0, 20]$
- Behavior: As x increases from 0 to 4, F(x) increases from 0 to 20, but the rate of increase diminishes.

Graphical Insight

Plotting F(x) over [0, 4] reveals a curve starting at (0, 0), rising steeply initially, then gradually leveling off as x approaches 4. The shape resembles a gentle upward curve that flattens out, characteristic of square root functions.

Understanding this shape is essential for approximation because it informs where the function's value is higher or lower within subintervals and influences the choice of rectangles' heights.

Partitioning the Interval: Dividing into Four Subintervals

Choosing the Subintervals

To approximate the area under F(x) from 0 to 4 using four rectangles, we must partition the interval [0, 4] into four subintervals of equal length:

$$ \text{Total interval length} = 4 - 0 = 4 $$
$$ \text{Number of rectangles} = 4 $$
$$ \text{Width of each rectangle} (\Delta x) = \frac{4}{4} = 1 $$

Thus, the subintervals are:

1. [0, 1]
2. [1, 2]
3. [2, 3]
4. [3, 4]

This uniform partition simplifies calculations and provides a straightforward framework for applying different approximation rules.

Advantages and Limitations of Equal Subintervals

Using equal widths offers simplicity and consistency. However, because the function's rate of change varies across the interval, uniform partitions may not always yield the most accurate estimates, especially if the function is highly nonlinear. Nonetheless, for initial approximation and educational purposes, equal subdivisions are effective.

Selecting the Approximation Method: Left, Right, or Midpoint Rectangles

Types of Riemann Sum Approximations

When estimating the area under a curve with rectangles, the choice of the point within each subinterval at which the function is evaluated impacts the approximation's accuracy. The three common methods include:

- Left Riemann Sum: Height of each rectangle is determined by the function value at the left endpoint.
- Right Riemann Sum: Height determined by the function value at the right endpoint.
- Midpoint Riemann Sum: Height determined by the function value at the midpoint.

Each method offers different approximations:

- Left sums tend to underestimate for increasing functions.
- Right sums tend to overestimate in such cases.
- Midpoint sums often provide more accurate estimates, especially for smooth functions.

Given that $F(x) = 10\sqrt{x}$ is increasing over [0,4], the left sum will likely underestimate, while the right sum will overestimate.

Calculating the Area Using Four Rectangles

Step 1: Determine Function Values at Subinterval Endpoints and Midpoints

Since the function is increasing:

- Left endpoints: $x=0, 1, 2, 3$
- Right endpoints: $x=1, 2, 3, 4$
- Midpoints: $x=0.5, 1.5, 2.5, 3.5$

Calculate $F(x)$ at these points:

x	$F(x) = 10\sqrt{x}$	Numerical value
0	$10\sqrt{0} = 0$	0
1	$10\sqrt{1} = 10$	10
2	$10\sqrt{2} \approx 10\ 1.4142 \approx 14.142$	14.142
3	$10\sqrt{3} \approx 10\ 1.732 \approx 17.320$	17.320
4	$10\sqrt{4} = 10\ 2 = 20$	20
0.5	$10\sqrt{0.5} \approx 10\ 0.7071 \approx 7.071$	7.071
1.5	$10\sqrt{1.5} \approx 10\ 1.2247 \approx 12.247$	12.247
2.5	$10\sqrt{2.5} \approx 10\ 1.5811 \approx 15.811$	15.811
3.5	$10\sqrt{3.5} \approx 10\ 1.8708 \approx 18.708$	18.708

Step 2: Compute the Areas of Rectangles for Each Method

Since each rectangle has width $\Delta x = 1$, the areas are:

- Left Riemann Sum:

$$
\text{Sum}_L = \sum_{i=1}^{4} f(\text{left endpoint}_i) \times \Delta x
$$

$$
= [F(0) + F(1) + F(2) + F(3)] \times 1
$$

$$
= (0 + 10 + 14.142 + 17.320) \times 1 = 41.462
$$

- Right Riemann Sum:

$$
\text{Sum}_R = [F(1) + F(2) + F(3) + F(4)] \times 1
$$

$$
= (10 + 14.142 + 17.320 + 20) = 61.462
$$

- Midpoint Riemann Sum:

$$
\text{Sum}_M = [F(0.5) + F(1.5) + F(2.5) + F(3.5)] \times 1
$$

$$
= (7.071 + 12.247 + 15.811 + 18.708) = 53.837
$$

Step 3: Interpretation of Results

- The left sum (~41.462) underestimates the true area because the function is increasing and the rectangles are anchored at the left endpoints.
- The right sum (~61.462) overestimates the area, for the same reason.
- The midpoint sum (~53.837) provides an estimate more centered and often closer to the actual value.

Estimating the Actual Area: Comparing with Exact Calculations

Calculating the Exact Area via Integration

While the approximation methods give us valuable insights, the true area under the curve can be computed precisely using calculus:

$$
\text{Area} = \int_{0}^{4} 10 \sqrt{x} \, dx
$$

Expressing √x as $x^{1/2}$:

$$
= 10 \int_{0}^{4} x^{1/2} dx
$$

Using the power rule:

$$
= 10 \left[\frac{x^{3/2}}{\frac{3}{2}} \right]_0^{4} = 10 \times \left(\frac{2}{3} \right) [x^{3/2}]_0^{4}
$$

Calculate $x^{3/2}$ at the bounds:

$$
4^{3/2} = (4^{1/2})^3 = (2)^3 = 8
$$
$$
0^{3/2} = 0
$$

Therefore,

$$
\text{Area} = 10 \times \frac{2}{3} \times (8 - 0) =
$$

Estimate The Area Under The Graph Of F X 10 Sqrt X From X 0

To X 4 Using Four Approximating Rectangles

Estimate The Area Under The Graph Of F X 10 Sqrt X From X 0 To X 4 Using Four Approximating Rectangles

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