Find An Equation Of The Plane Passing Through P = (7,0,0), Q = (0,9,2), R = (10,0,2). (Use Symbolic Notation

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Introduction

In the realm of three-dimensional coordinate geometry, understanding how to find the equation of a plane passing through three non-collinear points is a fundamental skill. Such a task involves a combination of vector operations, including finding vectors, calculating their cross product, and utilizing the point-normal form of a plane equation. This detailed guide aims to provide a comprehensive explanation of how to determine the equation of a plane passing through points P, Q, and R, given as P = (7, 0, 0), Q = (0, 9, 2), and R = (10, 0, 2).

Whether you are a student preparing for exams, a teacher seeking clarity, or a professional applying geometric principles, mastering this process is essential. We will explore each step methodically, using symbolic notation to reinforce understanding, and include relevant formulas, calculations, and tips to ensure clarity.

Understanding the Basic Concepts

Before diving into calculations, it is crucial to review some key concepts related to planes in three-dimensional space:

Points in Space

- Points are represented as ordered triples in the form (x, y, z).

Vectors

- Vectors can be thought of as directed line segments in space, represented as $\vec{v} = \langle v_x, v_y, v_z \rangle$.

Plane Equation

- The general equation of a plane in $\mathbb{R}^3$ can be written in the point-normal form:

$$
\boxed{
\vec{n} \cdot (\vec{r} - \vec{r}_0) = 0
}
$$

where
- $\vec{n}$ is the normal vector to the plane,
- $\vec{r} = \langle x, y, z \rangle$ is the position vector of an arbitrary point (x, y, z) on the plane,
- $\vec{r}_0$ is the position vector of a known point on the plane (e.g., P, Q, or R).

Expanding this gives the scalar equation:

$$
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0
$$

where $\vec{n} = \langle a, b, c \rangle$.

Step-by-Step Procedure to Find the Plane Equation

To find the equation of the plane passing through points P, Q, and R, follow these steps:

1. Identify the Points and Their Coordinates

$$
\begin{cases}
P = (x_1, y_1, z_1) = (7, 0, 0) \\
Q = (x_2, y_2, z_2) = (0, 9, 2) \\
R = (x_3, y_3, z_3) = (10, 0, 2)
\end{cases}
$$

2. Find Two Vectors on the Plane

Construct vectors $\vec{PQ}$ and $\vec{PR}$:

$$
\boxed{
\vec{PQ} = \vec{Q} - \vec{P} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle
}
$$

$$
\boxed{
\vec{PR} = \vec{R} - \vec{P} = \langle x_3 - x_1, y_3 - y_1, z_3 - z_1 \rangle
}
$$

Calculating these:

$$
\begin{aligned}
\vec{PQ} &= \langle 0 - 7, 9 - 0, 2 - 0 \rangle = \langle -7, 9, 2 \rangle \\
\vec{PR} &= \langle 10 - 7, 0 - 0, 2 - 0 \rangle = \langle 3, 0, 2 \rangle
\end{aligned}
$$

3. Compute the Normal Vector via Cross Product

The normal vector $\vec{n}$ to the plane is perpendicular to both $\vec{PQ}$ and $\vec{PR}$. Therefore, it can be found as their cross product:

$$
\boxed{
\vec{n} = \vec{PQ} \times \vec{PR}
}
$$

The cross product $\langle a_1, b_1, c_1 \rangle \times \langle a_2, b_2, c_2 \rangle$ is given by:

$$
\begin{aligned}
\vec{n} &= \left|
\begin{matrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2
\end{matrix}
\right| \\
&= \mathbf{i}(b_1 c_2 - c_1 b_2) - \mathbf{j}(a_1 c_2 - c_1 a_2) + \mathbf{k}(a_1 b_2 - b_1 a_2)
\end{aligned}
$$

Plugging in the components:

$$
a_1 = -7, \quad b_1 = 9, \quad c_1 = 2
$$

$$
a_2 = 3, \quad b_2 = 0, \quad c_2 = 2
$$

Calculations:

$$
\begin{aligned}
\vec{n} &= \mathbf{i}(9 \times 2 - 2 \times 0) - \mathbf{j}(-7 \times 2 - 2 \times 3) + \mathbf{k}(-7 \times 0 - 9 \times 3) \\
&= \mathbf{i}(18 - 0) - \mathbf{j}(-14 - 6) + \mathbf{k}(0 - 27) \\
&= \mathbf{i}(18) - \mathbf{j}(-20) + \mathbf{k}(-27) \\
&= \langle 18, 20, -27 \rangle
\end{aligned}
$$

Thus, the normal vector is:

$$
\boxed{
\vec{n} = \langle 18, 20, -27 \rangle
}
$$

---

4. Write the Equation of the Plane

Using point-normal form:

$$
a(x - x_0) + b(y - y_0) + c(z - z_0) = 0
$$

Choose point P = (7, 0, 0) and the normal vector $\vec{n} = \langle 18, 20, -27 \rangle$:

$$
18(x - 7) + 20(y - 0) - 27(z - 0) = 0
$$

Simplify:

$$
18x - 126 + 20y - 27z = 0
$$

Or, equivalently:

$$
\boxed{
18x + 20y - 27z = 126
}
$$

This is the scalar equation of the plane passing through points P, Q, and R.

Additional Considerations and Tips

Verifying the Plane Equation

To ensure accuracy, verify that points Q and R satisfy the derived plane equation:

- For point Q = (0, 9, 2):

$$
18(0) + 20(9) - 27(2) = 0 + 180 - 54 = 126
$$

- For point R = (10, 0, 2):

$$
18(10) + 20(0) - 27(2) = 180 + 0 - 54 = 126
$$

Both points satisfy the equation, confirming correctness.

Simplification and Normalization

It's often helpful to write the plane equation in a simplified or normalized form:

\[
\frac{18x + 20y - 27z}{\text{GCD of coefficients}} = \text{constant}
\]

The GCD of 18, 20, and 27 is 1 (since they share no common factors), so the equation is already simplified.

Understanding the Geometric Significance

- The normal vector \(\vec{n} \) indicates the orientation of the plane.
- The points P, Q, and R are confirmed to lie on this plane, ensuring the correctness of the equation.

Conclusion

Finding the equation

Frequently Asked Questions

How do you find the normal vector of the plane passing through points P, Q, and R?

You find the normal vector by computing the cross product of two vectors lying on the plane, such as vectors PQ and PR. Specifically, n = (Q - P) × (R - P).

What are the vectors PQ and PR given points P = (7, 0, 0), Q = (0, 9, 2), R = (10, 0, 2)?

PQ = Q - P = (0 - 7, 9 - 0, 2 - 0) = (-7, 9, 2), \quad PR = R - P = (10 - 7, 0 - 0, 2 - 0) = (3, 0, 2).

How is the normal vector n of the plane calculated using vectors PQ and PR?

n = PQ \times PR = (-7, 9, 2) \times (3, 0, 2).

What is the cross product of vectors PQ = (-7, 9, 2) and PR = (3, 0, 2)?

n = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & 9 & 2 \\ 3 & 0 & 2 \end{vmatrix} = (9 \times 2 - 2 \times 0)\mathbf{i} - (-7 \times 2 - 2 \times 3)\mathbf{j} + (-7 \times 0 - 9 \times 3)\mathbf{k} = (18)\mathbf{i} - (-14 - 6)\mathbf{j} + (-0 - 27)\mathbf{k} = 18\mathbf{i} + 20\mathbf{j} - 27\mathbf{k}.

What is the equation of the plane passing through point P = (7, 0, 0) with normal vector n = (18, 20, -27)?

The equation is 18(x - 7) + 20(y - 0) - 27(z - 0) = 0, which simplifies to 18x + 20y - 27z = 126.

How do you write the final equation of the plane in standard form using the calculated normal vector and point P?

The equation is 18x + 20y - 27z = 126.

Can the equation of the plane be expressed in vector form, and if so, what is it?

Yes, the vector form is expressed as: \boxed{\mathbf{r} \cdot \mathbf{n} = d}, where \mathbf{r} = (x, y, z), \mathbf{n} = (18, 20, -27), and substituting point P yields d = 18 \times 7 + 20 \times 0 - 27 \times 0 = 126. So, \mathbf{r} \cdot \mathbf{n} = 126.

Additional Resources

Find An Equation Of The Plane Passing Through P = (7,0,0), Q = (0,9,2), R = (10,0,2))

Introduction

In the realm of three-dimensional geometry, one of the fundamental problems involves determining the equation of a plane given three non-collinear points. This problem is not only academically stimulating but also practically significant in fields such as computer graphics, engineering design, and spatial analysis. Specifically, given three points P, Q, R in three-dimensional space, the task is to find the explicit equation of the plane passing through these points.

This article explores the systematic process of deriving the equation of a plane passing through the points $P = (7, 0, 0)$, $Q = (0, 9, 2)$, and $R = (10, 0, 2)$. The discussion will delve into foundational concepts, algebraic derivations, and geometric interpretations, providing a comprehensive guide suitable for students, educators, and practitioners alike.

Fundamental Concepts and Mathematical Foundations

The Equation of a Plane in 3D Space

A plane in three dimensions can be expressed in various forms, but the most common is the scalar form:

$$
A x + B y + C z + D = 0
$$

where (A, B, C) is a non-zero vector normal (perpendicular) to the plane, and D is a scalar constant.

Determining the Normal Vector

Given three points P, Q, R, the normal vector $\vec{n}$ to the plane can be found using the cross product of two vectors lying on the plane:

$$
\vec{PQ} = \vec{Q} - \vec{P}, \quad \vec{PR} = \vec{R} - \vec{P}
$$

The cross product:

$$
\vec{n} = \vec{PQ} \times \vec{PR}
$$

gives a vector orthogonal to both $\vec{PQ}$ and $\vec{PR}$, hence perpendicular to the plane.

Step-by-Step Derivation of the Plane Equation

Step 1: Compute Vectors $\vec{PQ}$ and $\vec{PR}$

Given the points:

$$
P = (7, 0, 0), \quad Q = (0, 9, 2), \quad R = (10, 0, 2)
$$

calculate:

$$
\vec{PQ} = Q - P = (0 - 7, \, 9 - 0, \, 2 - 0) = (-7, 9, 2)
$$

$$
\vec{PR} = R - P = (10 - 7, \, 0 - 0, \, 2 - 0) = (3, 0, 2)
$$

Step 2: Compute the Cross Product $\vec{n} = \vec{PQ} \times \vec{PR}$

Using the determinant method:

$$
\vec{n} =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-7 & 9 & 2 \\
3 & 0 & 2 \\
\end{vmatrix}
$$

which expands to:

$$
\vec{n} = \mathbf{i} (9 \times 2 - 2 \times 0) - \mathbf{j} (-7 \times 2 - 2 \times 3) + \mathbf{k} (-7 \times 0 - 9 \times 3)
$$

Calculating each component:

- i-component:

$$

$$
9 \times 2 - 2 \times 0 = 18 - 0 = 18
$$

- j-component:

$$
-(-7 \times 2 - 2 \times 3) = -(-14 - 6) = -(-20) = 20
$$

- k-component:

$$
-7 \times 0 - 9 \times 3 = 0 - 27 = -27
$$

Therefore,

$$
\boxed{
\vec{n} = (18, 20, -27)
}
$$

Formulating the Equation of the Plane

The general form:

$$
A x + B y + C z + D = 0
$$

can now be specified with $(A, B, C) = (18, 20, -27)$, the normal vector. To find D, substitute one of the known points, say $P = (7, 0, 0)$:

$$
18 \times 7 + 20 \times 0 - 27 \times 0 + D = 0
$$

$$
126 + 0 + 0 + D = 0 \Rightarrow D = -126
$$

Thus, the equation of the plane is:

$$
\boxed{
18 x + 20 y - 27 z - 126 = 0
}
$$

or, dividing through by 1 (since the coefficients are not all divisible by the same number for simplicity):

$$
18 x + 20 y - 27 z = 126
$$

Geometric Interpretation and Validation

Normal Vector Significance

The normal vector $(18, 20, -27)$ indicates the orientation of the plane in space. Its magnitude:

$$
|\vec{n}| = \sqrt{18^2 + 20^2 + (-27)^2} = \sqrt{324 + 400 + 729} = \sqrt{1453} \approx 38.08
$$

which signifies the steepness and tilt of the plane relative to the coordinate axes.

Verifying the Points Lie on the Plane

- For $P = (7, 0, 0)$:

$$
18 \times 7 + 20 \times 0 - 27 \times 0 = 126 \quad \checkmark
$$

- For $Q = (0, 9, 2)$:

$$
18 \times 0 + 20 \times 9 - 27 \times 2 = 0 + 180 - 54 = 126 \quad \checkmark
$$

- For $R = (10, 0, 2)$:

$$
18 \times 10 + 20 \times 0 - 27 \times 2 = 180 + 0 - 54 = 126 \quad \checkmark
$$

All three points satisfy the equation, confirming its correctness.

Alternative Forms and Applications

Symmetric Form of the Plane Equation

While the standard form is most straightforward for algebraic manipulations, the symmetric form can sometimes provide deeper geometric insights. However, for this particular problem, the standard form suffices.

Practical Applications

- Computer Graphics: Defining surfaces and rendering scenes.
- Engineering Design: Structural analysis and component alignment.
- Navigation and Robotics: Path planning in three-dimensional environments.
- Geographical Information Systems (GIS): Modeling terrain and landforms.

Advanced Considerations

Vector and Scalar Equations in Different Contexts

The derived plane equation can be adapted to various contexts by manipulating coefficients or converting to other forms, such as parametric or point-normal forms, which are especially useful in computational applications.

Handling Special Cases

- Collinearity: When the three points are collinear, the cross product yields the zero vector, indicating no unique plane.
- Degenerate Cases: Points with identical coordinates or lying on a line require careful analysis.

Conclusion

The rigorous derivation of the plane passing through points \(P = (7, 0, 0) \), \(Q = (0, 9, 2) \), and \(R = (10, 0, 2) \) underscores the interplay between algebraic computation and geometric intuition. By leveraging vector operations—particularly the cross product—and substituting known points, we successfully obtained the explicit equation:

\[
\boxed{
18 x + 20 y - 27 z = 126
}
\]

This comprehensive approach not only reinforces foundational concepts in vector calculus but also exemplifies the systematic process vital for advanced spatial analysis. Such methodologies serve as essential tools across scientific disciplines, fostering a deeper understanding of three-dimensional structures and their algebraic representations.

References

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- Anton, H., Bivens, I., & Davis, S. (2012). Calculus: Early Transcendental. Wiley.

Disclaimer: The derivations and interpretations provided are based on standard vector and plane equations in coordinate geometry. For specialized applications, consult domain-specific literature.

Find An Equation Of The Plane Passing Through P 7 0 0 Q 0 9 2 R 10 0 2 Use Symbolic Notation

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