Find Both The Vector Equation And The Parametric Equations Of The Line Through (0,0,0) That Is Perpendicular

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Understanding the equations of lines in three-dimensional space is fundamental in vector calculus and spatial geometry. When asked to find the vector and parametric equations of a line passing through the origin (0,0,0) that is perpendicular to a given plane or vector, it involves a clear grasp of vector operations, particularly the dot product, and the geometric interpretation of lines and planes. This guide will walk you through the process of deriving both the vector and parametric equations of such a line, emphasizing clarity, step-by-step procedures, and practical examples.

Introduction to Lines in 3D Space

A line in three-dimensional space can be represented in multiple ways. The most common forms are:

Vector Equation of a Line

- Expresses the line as all points r that can be written as:

$r = r_0 + t v$

where:
- r_0 is a point on the line,
- v is a direction vector,
- t is a scalar parameter.

Parametric Equations of a Line

- Breaks down the vector equation into component equations:

$x = x_0 + t v_x$

$y = y_0 + t v_y$

$z = z_0 + t v_z$

where (x_0, y_0, z_0) is a point on the line, and (v_x, v_y, v_z) are the components of the direction vector.

Understanding these forms is crucial because they provide different perspectives on the geometric object and facilitate various calculations such as intersections, distances, and perpendicularity tests.

Determining the Direction of the Perpendicular Line

Suppose you are given a plane or a vector, and you need to find a line passing through the origin that is perpendicular to this plane or vector.

Perpendicular to a Plane

- The line perpendicular to a plane passing through (0,0,0) will be along the plane's normal vector.
- The normal vector n of the plane is perpendicular to the plane's surface.

Perpendicular to a Given Vector

- The line's direction vector v must be parallel or anti-parallel to the given vector a to be perpendicular to it. This is because the dot product of perpendicular vectors is zero:

$a \cdot v = 0$

- Therefore, the direction vector of the line is orthogonal to the given vector.

Step-by-Step Procedure to Find the Line

The process involves identifying the direction vector and the point through which the line passes. Since the line passes through (0,0,0), that point is fixed.

Step 1: Identify the Given Data

- The specific plane or vector to which the line is perpendicular.
- For example:
- A plane with equation $Ax + By + Cz + D = 0$.
- A vector a = .

Step 2: Find the Direction Vector of the Line

- If perpendicular to a plane:
- Use the plane's normal vector n = .

- If perpendicular to a vector:
- The direction vector v must satisfy $a \cdot v = 0$.
- Find any vector orthogonal to a.

Step 3: Write the Vector Equation

- Use point (0,0,0) as r_0.
- Use the found direction vector v.
- The vector equation becomes:

$$r = t\, v$$

Step 4: Derive the Parametric Equations

- From the vector equation, extract component forms:

$$x = t\, v_x$$

$$y = t\, v_y$$

$$z = t\, v_z$$

This provides a complete parametric description of the line.

Examples of Finding the Line's Equations

Let's explore practical examples to cement understanding.

Example 1: Line Perpendicular to a Plane

Suppose the plane equation is $2x + 3y - z + 4 = 0$.

Step 1: Identify the normal vector
- The normal vector n = <2, 3, -1>.

Step 2: Direction vector of the line
- Since the line is perpendicular to the plane, its direction vector v = n = <2, 3, -1>.

Step 3: Write the vector equation
- The line passes through (0,0,0) with direction vector <2, 3, -1>:

$$r = t \langle 2, 3, -1 \rangle$$

Step 4: Write parametric equations
- $x = 2t$
- $y = 3t$

- $z = -t$

This describes all points on the line perpendicular to the plane passing through the origin.

Example 2: Line Perpendicular to a Given Vector

Suppose the given vector is a = <1, 2, 3>.

Step 1: Find a vector orthogonal to a
- To find v such that $a \cdot v = 0$, choose arbitrary values for two components and solve for the third.

Let’s choose $v_x = 2$, $v_y = -1$:

- Compute v_z:

$12 + 2(-1) + 3v_z = 0$

$2 - 2 + 3 v_z = 0$

$0 + 3 v_z = 0$

$v_z = 0$

Step 2: Direction vector
- v = <2, -1, 0>

Step 3: Vector equation
- r = t <2, -1, 0>

Step 4: Parametric equations
- $x = 2t$
- $y = -t$
- $z = 0$

This line is perpendicular to the vector <1, 2, 3> and passes through the origin.

Additional Considerations and Tips

- Choosing a perpendicular vector: When finding a vector perpendicular to a given vector, you can:
- Use algebraic methods to solve for components.
- Use cross products if you have two vectors and want a vector perpendicular to both.

- Ensuring the line passes through the origin: Since the point is (0,0,0), the vector equation simplifies to r = t v.

- Direction vector importance: The direction vector defines the orientation of the line;

choosing different perpendicular vectors results in the same line but scaled differently.

- Visualization: Sketching the plane, vectors, and lines can help understand the spatial relationships.

Applications of Finding Lines Perpendicular to a Plane or Vector

- Engineering and Physics: Calculating normal lines for surface analysis.
- Computer Graphics: Determining viewing angles and light reflections.
- Mathematics Education: Reinforcing understanding of vector operations and geometry.
- Robotics: Planning paths perpendicular to objects or surfaces.

Summary

To find both the vector and parametric equations of a line passing through (0,0,0) that is perpendicular to a given plane or vector:

1. Identify the normal vector or the vector to which the line must be perpendicular.
2. Use the property that the dot product of perpendicular vectors is zero to find an appropriate direction vector.
3. Formulate the vector equation using the point (0,0,0) and the direction vector.
4. Derive the parametric equations from the vector equation for explicit component-wise descriptions.

This process combines fundamental concepts of vector algebra with geometric intuition, enabling precise and versatile descriptions of lines in three-dimensional space.

Conclusion

Mastering how to find the equations of lines perpendicular to planes or vectors in 3D space is a vital skill in advanced mathematics, physics, and engineering disciplines. By understanding the relationship between vectors, dot products, and geometric configurations, you can efficiently derive the equations of such lines, facilitating deeper insights into the spatial relationships of objects and surfaces. Practice with various examples will reinforce these concepts, enhancing your ability to interpret and solve complex spatial problems with confidence.

Frequently Asked Questions

How do you find the vector equation of a line passing through the origin that is perpendicular to a given plane?

First, identify the normal vector of the plane, which is perpendicular to it. Since the line is perpendicular to the plane and passes through the origin, its direction vector is the same as the plane's normal vector. The vector equation is then $r(t) = t\,n$, where n is the normal vector.

What are the parametric equations of a line passing through (0,0,0) and perpendicular to the plane 2x + 3y - z = 0?

The normal vector of the plane is (2, 3, -1). Since the line is perpendicular to the plane and passes through the origin, its parametric equations are: $x = 2t$, $y = 3t$, $z = -t$.

How can I express the line through the origin that is perpendicular to a given vector?

If the line is perpendicular to a specific vector v, then the line's direction vector is parallel to v. The vector equation is $r(t) = t\,v$, and the parametric equations are derived from the components of v as $x = v_x t$, $y = v_y t$, $z = v_z t$.

What is the general form of the vector and parametric equations for a line through the origin perpendicular to a given vector?

The vector equation is $r(t) = t\,v$, where v is the given vector. The parametric equations are $x = v_x t$, $y = v_y t$, $z = v_z t$, with t spanning all real numbers.

If a line passes through (0,0,0) and is perpendicular to both the xy-plane and a given vector, how do you find its equations?

Perpendicular to the xy-plane means the line's direction vector is along the z-axis, e.g., (0, 0, 1). If it is also perpendicular to a vector v, then the line's direction vector must be orthogonal to v, so you find a vector perpendicular to both the z-axis and v. The equations are then constructed using this direction vector, starting at the origin.

Additional Resources

Find Both The Vector Equation And The Parametric Equations Of The Line Through (0,0,0) That Is Perpendicular

In the realm of three-dimensional geometry, understanding the equations that describe lines in space is foundational. Whether for engineering, physics, computer graphics, or advanced mathematics, being able to formulate these equations precisely is essential. One particularly interesting problem involves determining the equations of a line that passes through the origin—specifically, the point (0,0,0)—and is perpendicular to a given plane or direction. This task requires a blend of vector algebra and coordinate geometry, allowing us to derive both the vector form and the parametric equations of such a line.

In this article, we will explore the process of finding the vector and parametric equations of a line passing through the origin and perpendicular to a specified plane or vector. We will begin by reviewing the fundamental concepts of lines in three-dimensional space, then proceed step-by-step through the calculation process, finally illustrating with examples to clarify each stage.

Understanding the Foundations: Lines, Vectors, and Perpendicularity in 3D Space

Before delving into the specific problem, it’s crucial to understand the fundamental concepts involved:

1. The Vector Equation of a Line

A line in 3D space can be represented in vector form as:

$r(t) = r_0 + t\,v$

where:
- r(t) is the position vector of any point on the line as a function of parameter t.
- r_0 is a fixed point through which the line passes; in this case, the origin, so $r_0 = (0, 0, 0)$.
- v is a direction vector parallel to the line.
- t is a real parameter.

When the line passes through the origin, the vector equation simplifies to:

$r(t) = t\,v$

2. The Parametric Equations of a Line

From the vector equation, we can derive the parametric equations which explicitly express the coordinates (x, y, z) as functions of the parameter t:

- $x(t) = v_x t$
- $y(t) = v_y t$
- $z(t) = v_z t$

where v_x, v_y, v_z are the components of the direction vector v.

3. Perpendicularity in Space

A line is perpendicular (or orthogonal) to a plane if its direction vector is orthogonal to the plane’s normal vector. Conversely, if you are given a plane, the line perpendicular to it passing through the origin has a direction vector aligned with the plane’s normal vector.

Key Point: The direction vector v of the line must satisfy:

$v \cdot n = 0$

where:
- v is the direction vector of the line.
- n is the normal vector to the plane.

If instead, a specific vector or line is given, the same principle applies: the line's direction vector is orthogonal to that vector.

Step-by-Step: Deriving the Equations of the Perpendicular Line

Suppose we are given a specific plane or vector, and we need to find the line through the origin perpendicular to it. The process generally involves:

- Identifying the normal vector or the vector to which the line is perpendicular.
- Using this vector as the direction vector of the line.
- Expressing the line in both vector and parametric forms.

Let's break down these steps in detail.

Step 1: Identify the Normal Vector or Direction

- If given a plane: The normal vector n can typically be read from the plane's equation in the form $Ax + By + Cz = D$. The coefficients A, B, and C form the normal vector $n = (A, B, C)$.

- If given a vector: The line must be perpendicular to that vector, so the direction vector v of the line is parallel to that vector.

Example: Suppose the plane is given by $2x + 3y - z = 5$. The normal vector is $n = (2, 3, -1)$.

Step 2: Choose the Direction Vector of the Line

- For a line perpendicular to the plane, the direction vector v = n.

- For a line perpendicular to a given vector a, then v = a.

Note: Since the line passes through the origin, the point $r_0 = (0, 0, 0)$, and the vector equation becomes:

r(t) = t v

Step 3: Write the Vector Equation of the Line

Using the point (0, 0, 0) and the direction vector v, the vector equation is:

r(t) = t v

For the example with n = (2, 3, -1):

r(t) = t (2, 3, -1)

Step 4: Derive the Parametric Equations

From the vector equation, the parametric equations are:

- x(t) = 2t
- y(t) = 3t
- z(t) = -t

These equations describe the line passing through the origin and perpendicular to the plane 2x + 3y - z = 5.

Practical Examples and Illustration

Let’s look at some concrete examples to cement these concepts.

Example 1: Line Perpendicular to a Plane

Given:

Plane: $4x - y + 2z = 7$

Find the vector and parametric equations of the line passing through (0, 0, 0) and perpendicular to this plane.

Solution:

1. Identify the normal vector:

$n = (4, -1, 2)$

2. Use this normal vector as the direction vector:

$v = (4, -1, 2)$

3. Write the vector equation:

$r(t) = t\,(4, -1, 2)$

4. Derive parametric equations:

$x(t) = 4t$

$y(t) = -t$

$z(t) = 2t$

This line passes through the origin and is perpendicular to the given plane.

Example 2: Line Perpendicular to a Given Vector

Given:

Vector: $a = (1, 2, 3)$

Find the equations of the line through (0, 0, 0) perpendicular to this vector.

Solution:

1. The direction vector:

$v = (1, 2, 3)$

2. The vector equation:

$r(t) = t\,(1, 2, 3)$

3. Parametric equations:

$x(t) = t$

$y(t) = 2t$

$z(t) = 3t$

Additional Considerations and Special Cases

While the above methods cover most scenarios, it’s important to be aware of special cases and additional considerations:

- If the given plane is parallel to a coordinate axis: The normal vector simplifies, making the equations easier to derive.

- If the line is to be perpendicular to more than one plane: The direction vector must be orthogonal to the normals of all these planes, which involves solving a system of equations.

- Multiple lines passing through the origin: There are infinitely many lines passing through a point, but only those with direction vectors orthogonal to a given vector or plane's normal are considered.

Conclusion: Mastering the Equations of Perpendicular Lines in 3D

Understanding how to derive both the vector and parametric equations of a line passing through the origin and perpendicular to a given plane or vector is a fundamental skill in three-dimensional geometry. By identifying the normal vector or the vector to which the line must be perpendicular, and then constructing the line's equations accordingly, one gains a powerful tool for spatial analysis.

This process hinges on the core principles of vector algebra—particularly the dot product—and the geometric interpretations of perpendicularity. Whether applied in academic contexts, engineering design, or computer graphics, mastering these derivations enhances spatial reasoning and problem-solving capabilities.

In practical terms, always start with the given data—be it a plane’s equation or a specific vector—identify the normal or direction, and then systematically formulate the equations. With practice, this approach becomes an intuitive part of your mathematical toolkit, enabling you to navigate the complex three-dimensional space with confidence.

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