# Find The First Three Nonzero Terms In The Taylor Polynomial Approximation To The DE V" +9y + 5y3 = Cos(10t),

**Find The First Three Nonzero Terms In The Taylor Polynomial Approximation To The DE V" + 9y + 5y^3 = Cos(10t)**

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## Introduction

The task of approximating solutions to differential equations using Taylor polynomials is a fundamental technique in mathematical analysis and applied mathematics. When dealing with nonlinear differential equations, such as the one presented:

$$
V'' + 9y + 5y^3 = \cos(10t),
$$

it becomes particularly interesting to find the Taylor polynomial expansion of the solution around a specific point, typically $t = 0$. This process involves expressing the solution $y(t)$ as a power series and determining its coefficients. The goal here is to find the first three nonzero terms of this Taylor polynomial, providing a local approximation to the solution near $t = 0$.

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## Understanding the Differential Equation

### Equation Breakdown

The differential equation:

$$
V'' + 9y + 5y^3 = \cos(10t),
$$

can be interpreted as a nonlinear second-order differential equation with a forcing function $\cos(10t)$.

- $V''$ denotes the second derivative of $y$ with respect to $t$.

- The term $9y$ suggests a linear component similar to a simple harmonic oscillator.
- The nonlinear term $5y^3$ introduces complexity, making the solution non-trivial.
- The right-hand side $\cos(10t)$ acts as an external forcing function.

Our goal is to approximate the solution $y(t)$ near $t=0$ using a Taylor polynomial, which involves expanding $y(t)$ as:

$$
y(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots
$$

and computing the coefficients $a_0, a_1, a_2, \ldots$.

---

# Initial Conditions and Assumptions

To proceed with the Taylor expansion, initial conditions are necessary. Typically, for such problems, initial conditions are specified at $t = 0$:

$$
y(0) = y_0, \quad y'(0) = y_1.
$$

In the absence of explicit initial conditions, a common approach is to assume:

$$
y(0) = 0, \quad y'(0) = 0,
$$

which simplifies calculations and allows focus on the structure of the solution. If different initial conditions are provided, the coefficients would be adjusted accordingly.

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# Methodology for Finding the Taylor Polynomial

## Step 1: Express $y(t)$ as a Power Series

Assume:

$$
y(t) = \sum_{n=0}^\infty a_n t^n,
$$

where:

- $a_0 = y(0)$,
- $a_1 = y'(0)$,
- higher coefficients are determined via derivatives evaluated at $t=0$.

The derivatives are:

$$
y'(t) = \sum_{n=1}^\infty n a_n t^{n-1},
$$
$$
y''(t) = \sum_{n=2}^\infty n(n-1) a_n t^{n-2}.
$$

---

## Step 2: Find Derivatives at $t = 0$

Using the above:

$$
y(0) = a_0,
$$
$$
y'(0) = a_1,
$$
$$
y''(0) = 2 a_2.
$$

These will be used to find the first few coefficients.

---

## Step 3: Formulate the Differential Equation in Series Form

We substitute the series representations into the original differential equation:

$$
y'' + 9 y + 5 y^3 = \cos(10t).
$$

Expressed as:

\[

$$
\sum_{n=2}^\infty n(n-1) a_n t^{n-2} + 9 \sum_{n=0}^\infty a_n t^n + 5 \left( \sum_{n=0}^\infty a_n t^n \right)^3 = \cos(10t).
$$

---

## Step 4: Expand $y^3$ as a Power Series

The cubic term involves convolution of the series:

$$
y^3 = \left( \sum_{n=0}^\infty a_n t^n \right)^3.
$$

Using the multinomial theorem, the coefficients for $y^3$ are computed via convolution sums:

$$
b_n = \sum_{i=0}^n \sum_{j=0}^{n-i} a_i a_j a_{n - i - j}.
$$

---

## Step 5: Expand the Forcing Function $\cos(10t)$

The Maclaurin series for $\cos(10t)$:

$$
\cos(10t) = \sum_{k=0}^\infty \frac{(-1)^k (10t)^{2k}}{(2k)!} = 1 - \frac{(10t)^2}{2!} + \frac{(10t)^4}{4!} - \cdots.
$$

The first few terms:

$$
\cos(10t) \approx 1 - 50 t^2 + \frac{(10)^4}{4!} t^4 + \cdots,
$$

but for the first three nonzero terms, the key contributions are:

$$
\text{Terms up to } t^4.
$$

---

# Calculating the First Three Nonzero Terms

## Step 1: Determine $a_0$ and $a_1$

- Since we are looking for a local approximation near $t=0$, and the differential equation involves $y(0)$ and $y'(0)$, initial conditions are critical.

- Assuming initial conditions:

$$
y(0) = y_0, \quad y'(0) = y_1,
$$

these give:

$$
a_0 = y_0,
$$
$$
a_1 = y_1.
$$

- For simplicity, and to find the structure of the solution, assume initial rest:

$$
a_0 = 0, \quad a_1 = 0,
$$

which simplifies the process.

---

## Step 2: Find $a_2$ using the differential equation at $t=0$

At $t=0$:

$$
V''(0) + 9 y(0) + 5 y(0)^3 = \cos(0) = 1.
$$

Since $y''(0) = 2 a_2$, and with $y(0) = 0$, $y'(0) = 0$, the equation reduces to:

$$
2 a_2 + 0 + 0 = 1,
$$

thus:

$$
a_2 = \frac{1}{2}.
$$

---

## Step 3: Find $a_3$ via derivatives or substitution

Next, differentiate the original equation to obtain an expression for $y'''(0)$ or directly expand the series.

Alternatively, differentiate the entire series expression twice and evaluate at $t=0$:

- $y''(t)$ derivative:

$$
y'''(t) = \sum_{n=3}^\infty n(n-1)(n-2) a_n t^{n-3}.
$$

At $t=0$:

$$
y'''(0) = 6 a_3,
$$
since $n=3$:

$$
3 \times 2 \times 1 \times a_3 = 6 a_3.
$$

Now, differentiate the original differential equation with respect to $t$:

$$
V''' + 9 y' + 15 y^2 y' = -10 \sin(10 t),
$$

which at $t=0$ becomes:

$$
V'''(0) + 9 y'(0) + 15 y(0)^2 y'(0) = 0,
$$

since $\sin(0) = 0$.

Given $y(0) = 0, y'(0) = 0$:

$$

$$
V'''(0) = y'''(0) = 6 a_3,
$$

but this alone does not fully determine $a_3$. Alternatively, substitute the series into the original equation and equate coefficients for the $t^{0}$ and $t^{2}$ terms.

---

### Step 4: Match coefficients with the series expansion of the RHS

The left side series expansion, up to $t^2$:

$$
V'' + 9 y + 5 y^3,
$$

with \( y(t) \approx a_

## Frequently Asked Questions

### What is the differential equation given in the problem for finding the Taylor polynomial?

The differential equation is V'' + 9y + 5y^3 = cos(10t).

### How do we determine the first three nonzero terms in the Taylor polynomial approximation for this differential equation?

We expand the solution y(t) into a Taylor series around a point (usually t=0), then substitute into the differential equation, equate coefficients, and solve iteratively to find the first three nonzero terms.

### What is the significance of finding the first three nonzero terms in the Taylor polynomial for this DE?

These terms provide an approximate solution near the expansion point, capturing the behavior of the solution with increasing accuracy for small t.

### Which initial conditions are typically needed to

## determine the Taylor polynomial in such a differential equation?

Initial conditions like y(0) and y'(0) are needed to determine the constants in the Taylor series expansion.

## What is the general approach to compute the first three nonzero terms of the Taylor polynomial for the given differential equation?

The approach involves assuming a power series solution, substituting into the DE, matching coefficients for powers of t, and solving for the coefficients of the first three terms.

# Additional Resources

Find The First Three Nonzero Terms In The Taylor Polynomial Approximation To The DE V" + 9y + 5y^3 = Cos(10t)

When approaching differential equations, especially nonlinear ones like $V'' + 9y + 5y^3 = \cos(10t)$, Taylor polynomial approximations serve as powerful tools for understanding the behavior of solutions around specific points, typically around $t = 0$. This article delves into the process of finding the first three nonzero terms in the Taylor polynomial for this differential equation, providing a comprehensive guide that covers the theoretical foundation, step-by-step calculations, and insightful analysis of the method's strengths and limitations.

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# Understanding the Differential Equation

## The Given Equation

The differential equation in question is:

$$
V'' + 9y + 5y^3 = \cos(10t),
$$

where $V = y(t)$, and the primes denote derivatives with respect to $t$. It is a nonlinear second-order differential equation due to the $y^3$ term.

Key features:

- Nonlinearity: The presence of $y^3$ makes the equation nonlinear, complicating direct

solutions.
- Forcing term: $\cos(10t)$ acts as an external forcing function.
- Objective: Find the Taylor polynomial approximation of $y(t)$ around $t=0$, focusing on the first three nonzero terms.

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# Methodology for Deriving the Taylor Polynomial

## 1. Initial Conditions and Assumptions

To determine the Taylor series, initial conditions such as $y(0)$ and $y'(0)$ are typically required. If not provided, you may assume initial conditions for the purpose of illustration, such as:

$$
y(0) = y_0, \quad y'(0) = y_1.
$$

For simplicity, assume $y(0) = 0$ and $y'(0) = 0$, unless specified otherwise.

## 2. Taylor Series Expansion

The solution $y(t)$ can be expressed as a Taylor series around $t=0$:

$$
y(t) = a_0 + a_1 t + a_2 t^2 + a_3 t^3 + \cdots,
$$

where coefficients $a_n$ are to be determined.

The goal is to find the first three nonzero terms, which typically correspond to the lowest powers of $t$ with nonzero coefficients.

## 3. Deriving Derivatives at $t=0$

To compute the Taylor coefficients, we need derivatives of $y(t)$ at $t=0$. These are obtained by differentiating the differential equation and evaluating at $t=0$.

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# Step-by-Step Solution

## Step 1: Determine $y(0)$ and $y'(0)$

Assuming initial conditions:

$$
y(0) = 0, \quad y'(0) = 0.
$$

These are common starting points, but they can be modified if initial data are known.

---

## Step 2: Find $y''(0)$

From the differential equation:

$$
V'' + 9 y + 5 y^3 = \cos(10t).
$$

At $t=0$:

$$
y''(0) + 9 y(0) + 5 y(0)^3 = \cos(0) = 1.
$$

Since $y(0) = 0$:

$$
y''(0) + 0 + 0 = 1 \Rightarrow y''(0) = 1.
$$

---

## Step 3: Find $y'''(0)$

Differentiate the original equation:

$$
V''' + 9 y' + 15 y^2 y' = -10 \sin(10 t),
$$

where we've used the chain rule on $5 y^3$:

$$
\frac{d}{dt} (5 y^3) = 15 y^2 y'.
$$

Evaluate at $t=0$:

$$
y'''(0) + 9 y'(0) + 15 y(0)^2 y'(0) = -10 \sin(0) = 0.
$$

Given $y'(0) = 0$ and $y(0) = 0$:

$$
y'''(0) + 0 + 0 = 0 \Rightarrow y'''(0) = 0.
$$

---

## Step 4: Find $y^{(4)}(0)$

Differentiate the original equation a second time:

$$
V^{(4)} + 9 y'' + 15 (2 y y'^2 + y^2 y'') = -100 \cos(10 t),
$$

since the derivative of $-10 \sin(10 t)$ is $-100 \cos(10 t)$.

At $t=0$:

$$
y^{(4)}(0) + 9 y''(0) + 15 [2 y(0) y'(0)^2 + y(0)^2 y''(0)] = -100 \cos(0) = -100.
$$

Plugging in known initial derivatives:

$$
y^{(4)}(0) + 9 \times 1 + 15 [0 + 0] = -100,
$$

$$
y^{(4)}(0) + 9 = -100,
$$

$$
y^{(4)}(0) = -109.
$$

---

## Step 5: Constructing the Taylor Series

Now, the derivatives at $t=0$:

- $y(0) = 0$,
- $y'(0) = 0$,
- $y''(0) = 1$,
- $y'''(0) = 0$,
- $y^{(4)}(0) = -109$.

The Taylor series expansion up to the 4th derivative term:

$$
y(t) = y(0) + y'(0) t + \frac{y''(0)}{2!} t^2 + \frac{y'''(0)}{3!} t^3 + \frac{y^{(4)}(0)}{4!} t^4 + \cdots,
$$

which simplifies to:

$$
y(t) = 0 + 0 \times t + \frac{1}{2} t^2 + 0 \times t^3 - \frac{109}{24} t^4 + \cdots.
$$

---

# Identifying the First Three Nonzero Terms

Based on the series:

$$
y(t) \approx \frac{1}{2} t^2 - \frac{109}{24} t^4 + \cdots,
$$

the first three nonzero terms are:

1. $\frac{1}{2} t^2$,
2. $-\frac{109}{24} t^4$,
3. (Next nonzero term would come from higher derivatives, typically involving more complex calculations).

Since the third nonzero term appears at $t^4$, the first three nonzero terms are:

\[
\boxed{
\frac{1}{2} t^2 - \frac{109}{24} t^4 + \text{(next nonzero term at higher order).}

}
\]

---

# Analysis and Insights

## Features of the Taylor Approximation in This Context

- Local Behavior: The Taylor polynomial provides a good approximation near \( t=0 \), capturing the solution's behavior locally.
- Handling Nonlinearity: Despite the nonlinearity, the method leverages derivatives at a point, although calculations become more involved at higher orders.
- Incorporation of Forcing: The external force \( \cos(10t) \) influences derivatives, seen in the derivatives of the differential equation.

## Pros and Cons of Taylor Series Approximation

Pros:

- Analytical insight: Provides explicit approximate solutions near a point.
- Simplicity for small \( t \): Easy to compute and interpret for small \( t \).
- Foundation for numerical methods: Useful in developing initial conditions for numerical solvers.

Cons:

- Limited radius of convergence: Valid only near the expansion point \( t=0 \).
- Complex derivatives for nonlinear equations: Higher derivatives rapidly become complicated, especially with nonlinear terms.
- Potential inaccuracies: As \( t \) moves away from zero, the approximation deteriorates.

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# Conclusion and Final Remarks

Finding the first three nonzero terms of the Taylor polynomial approximation to the nonlinear differential equation \( V'' + 9 y + 5 y^3 = \cos(10t) \) involves a systematic process of evaluating derivatives at \( t=0 \

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