Give A General Formula For All The Solutions. 3sin+5=2sin Select The Correct Choice Below And, If Necessary,

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Understanding and solving trigonometric equations is a fundamental skill in mathematics, especially in the study of periodic functions and their applications. One common equation encountered in trigonometry involves solving for a variable within sine functions. For the equation 3sin + 5 = 2sin, the goal is to find all possible solutions that satisfy the equation over the domain of interest, typically all real numbers or within a specific interval such as [0, 2π].

In this article, we will explore how to derive the general solution for such an equation, select the correct answer from multiple-choice options (if provided), and understand the underlying principles of solving trigonometric equations. Whether you're a student preparing for exams or a teacher creating instructional content, understanding this process is essential.

Understanding the Trigonometric Equation

The given equation is:

3sin + 5 = 2sin

At first glance, it appears to be missing a variable or argument within the sine function. Typically, sine functions are written as sin(θ), where θ is an angle. It's possible the original problem intends to express the equation as:

$3\sin(\theta) + 5 = 2\sin(\theta)$

or simply:

$3\sin(\theta) + 5 = 2\sin(\theta)$

Assuming this is the case, the equation involves a single variable θ, and the goal is to solve for θ.

Rearranging the Equation

To solve for θ, we can start by bringing all terms involving sin(θ) to one side:

$3\sin(\theta) - 2\sin(\theta) + 5 = 0$

which simplifies to:

$(3\sin(\theta) - 2\sin(\theta)) + 5 = 0$

or:

$\sin(\theta) + 5 = 0$

Alternatively, if the original equation was meant to be:

$3\sin(\theta) + 5 = 2\sin(\theta)$

then subtracting $2\sin(\theta)$ from both sides gives:

$3\sin(\theta) - 2\sin(\theta) + 5 = 0$

which simplifies to the same:

$\sin(\theta) + 5 = 0$

Solving for sin(θ)

Now, solve for $\sin(\theta)$:

$\sin(\theta) = -5$

However, it's important to recognize the range of the sine function:

- The sine of any real angle θ always satisfies:

$-1 \leq \sin(\theta) \leq 1$

Since the value -5 is outside this range, this indicates that the equation has no real solutions.

Key Point: If the simplified form yields a value outside the sine function's range, then the original equation has no solutions in the real numbers.

Interpreting the Solutions and Selecting the Correct Choice

Suppose the multiple-choice options for the solutions are as follows:

A) $\sin(\theta) = -5$ (No solution)

B) $\sin(\theta) = 0$

C) $\sin(\theta) = 1$

D) $\sin(\theta) = 4$

Given our calculations, the only feasible solution—if any—is when $\sin(\theta)$ lies within the range [-1, 1].

Analysis of options:

- Option A: $\sin(\theta) = -5$ — outside the valid range, so no solutions.

- Option B: $\sin(\theta) = 0$ — valid; solutions occur at $\theta = 0, \pi, 2\pi$, etc.

- Option C: $\sin(\theta) = 1$ — valid; solutions at $\theta = \pi/2 + 2\pi k$.

- Option D: $\sin(\theta) = 4$ — outside the range, invalid.

Conclusion:

Since the simplified form yields $\sin(\theta) = -5$, which is outside the valid range, the original equation has no solutions over the real numbers.

Deriving the General Solution for Equations with Valid Sine Values

In cases where the simplified form yields a sine value within [-1, 1], the general solutions can be systematically derived.

General Formula for All Solutions of sin(θ) = k

When solving an equation of the form:

$\sin(\theta) = k$

where $-1 \le k \le 1$, the solutions are given by:

1. Principal solutions:

$\theta = \arcsin(k) + 2\pi n$

2. Supplementary solutions:

$\theta = \pi - \arcsin(k) + 2\pi n$

where n is any integer ($n \in \mathbb{Z}$), representing the periodic nature of the sine function.

Example:

Suppose you need to solve $\sin(\theta) = 0.5$.

- $\arcsin(0.5) = \pi/6$

- The solutions are:

$\theta = \pi/6 + 2\pi n$

$\theta = 5\pi/6 + 2\pi n$

for all integers n.

Note: When solving equations, always consider the domain restrictions and whether solutions are valid within the specified interval.

Applying the General Solution to the Original Equation

Returning to the original problem, if the simplified form yields a valid sine value within [-1, 1], the solution process involves:

1. Calculating the inverse sine (arcsin).

2. Writing the principal solutions.

3. Adding the periodic solutions by incorporating $2\pi n$.

However, in our case:

- The simplified form yields $\sin(\theta) = -5$, which is invalid.

- Therefore, no real solutions exist for the equation $3\sin(\theta) + 5 = 2\sin(\theta)$.

Summary of Steps for Solving Similar Trigonometric Equations

To effectively solve equations like $3\sin(\theta) + 5 = 2\sin(\theta)$, follow these steps:

1. Rewrite the equation to isolate the sine term:

- Combine like terms on one side.

2. Simplify the equation:

- Subtract or add terms to express $\sin(\theta)$ in terms of a constant.

3. Check the range of the sine function:

- Ensure the value obtained for $\sin(\theta)$ lies within [-1, 1].

4. Determine the solutions:

- If the value is within the range, find arcsin(k).

- Write all solutions using the general formula:

$\theta = \arcsin(k) + 2\pi n$

$\theta = \pi - \arcsin(k) + 2\pi n$

5. Express the solutions explicitly:

- Include all possible solutions within the desired interval.

6. Verify solutions:

- Substitute back into the original equation to confirm correctness.

Conclusion: How to Approach These Problems

Understanding how to derive a general formula for all solutions of trigonometric equations is vital for mastering the subject. In the specific case of equations like 3sin + 5 = 2sin, the key steps involve algebraic manipulation, recognizing the domain restrictions of the sine function, and applying the general solutions for $\sin(\theta) = k$.

Key Takeaways:

- Always simplify the equation to isolate the sine function.

- Check whether the resulting value for $\sin(\theta)$ is within the range [-1, 1].

- Use the general solution formulas involving arcsin and the periodicity of sine to write all solutions.

- Remember that equations yielding sine values outside the range have no real solutions.

By following these principles, students and mathematicians can confidently approach and solve a wide variety of trigonometric equations, ensuring they select the correct solutions efficiently.

Meta-description:
Learn how to derive the general formula for all solutions of the trigonometric equation 3sin + 5 = 2sin, identify valid solutions, and understand the process step-by-step.

Frequently Asked Questions

What is the general solution for the equation 3sin(x) + 5 = 2sin(x)?

The general solution is $x = 2n\pi + \arcsin(-5/1)$ or $x = \pi - \arcsin(-5/1)$, adjusted for the specific values, but since sin(x) ranges between -1 and 1, and $-5/1 = -5$ is outside this range, there are no real solutions.

Why does the equation 3sin(x) + 5 = 2sin(x) have no real solutions?

Because after simplifying to $\sin(x) = -5/1$, which simplifies to $\sin(x) = -5$, and since sin(x) must be between -1 and 1, the equation has no solutions in real numbers.

How do you determine if an equation involving sine has solutions?

By checking if the value obtained for sin(x) is within the range [-1, 1]. If it is outside this range, there are no real solutions.

What is the step-by-step method to find the general solution of 3sin(x) + 5 = 2sin(x)?

First, simplify the equation to isolate sin(x): $3\sin(x) - 2\sin(x) = -5$, which gives $\sin(x) = -5$. Since -5 is outside the sine range, conclude no real solutions exist.

Can the equation 3sin+5=2sin be solved for complex solutions?

Yes, but the solutions involve complex logarithms and are beyond typical trigonometric solutions. For real solutions, no solutions exist because sin(x) cannot be less than -1 or greater than 1.

What is the general formula for solutions when sin(x) equals a value within [-1, 1]?

If $\sin(x) = k$, where $-1 \leq k \leq 1$, then the solutions are $x = \arcsin(k) + 2n\pi$ and $x = \pi - \arcsin(k) + 2n\pi$, for all integers n.

In the given equation, why is it important to check the sine range before solving?

Because sine values must be between -1 and 1; if the computed value falls outside this range, it indicates no real solutions exist.

What is the significance of the constant 'n' in the general

solution formulas?

The constant 'n' represents all integers, accounting for the periodic nature of the sine function and providing all possible solutions.

Given the equation 3sin+5=2sin, is it necessary to consider multiple solutions?

Yes, because sine is periodic, and when solutions exist within the valid range, they often have infinitely many solutions differing by multiples of 2π.

Additional Resources

Give A General Formula For All The Solutions. 3sin(θ) + 5 = 2sin(θ). Select The Correct Choice Below And, If Necessary,

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Understanding and Solving the Trigonometric Equation: An In-Depth Analysis

Mathematics, especially trigonometry, plays a pivotal role in various scientific and engineering disciplines. Equations involving sine and other trigonometric functions often appear in modeling wave phenomena, oscillations, and periodic behaviors. A fundamental skill in trigonometry is solving equations that involve these functions, especially when seeking a general solution that encompasses all possible solutions within a specific interval or the entire set of real numbers.

In this article, we focus on the specific problem:
3sin(θ) + 5 = 2sin(θ)

By dissecting this equation, understanding its structure, and applying algebraic and trigonometric principles, we will derive a general formula that provides all solutions for the variable θ. We will also explore the importance of choosing the correct solution set and understanding the implications of the solutions in various contexts.

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1. Introduction to the Equation and Its Significance

1.1 The Role of Trigonometric Equations in Mathematics

Trigonometric equations are equations where the unknown appears inside a trigonometric function such as sine, cosine, tangent, etc. These equations are fundamental in analyzing periodic phenomena, including sound waves, electromagnetic waves, and mechanical oscillations. Solving such equations often involves algebraic manipulation combined with understanding the properties of trigonometric functions.

1.2 The Specific Equation: 3sin(θ) + 5 = 2sin(θ)

The equation under consideration combines linear and trigonometric components:

$$ 3\sin(\theta) + 5 = 2\sin(\theta) $$

This simple-looking equation actually encapsulates the process of isolating the variable and finding all solutions that satisfy the relationship between sine and constant values.

1.3 Why Find a General Solution?

A general solution provides a complete set of solutions that account for all possible angles θ satisfying the equation, including those that recur periodically due to the nature of sine. This is particularly important in applications where the behavior repeats over cycles, and understanding the totality of solutions is crucial for accurate modeling.

---

2. Step-by-Step Solution of the Equation

2.1 Simplification and Algebraic Manipulation

The initial step involves isolating the sine term:

$$
3\sin(\theta) + 5 = 2\sin(\theta)
$$

Subtract $2\sin(\theta)$ from both sides:

$$
3\sin(\theta) - 2\sin(\theta) + 5 = 0
$$

Simplify:

$$
\sin(\theta) + 5 = 0
$$

Rearranged:

$$
\sin(\theta) = -5
$$

2.2 Analyzing the Feasibility of the Solution

This is a critical step. Since the sine function's range is $[-1, 1]$, the equation $\sin(\theta) = -5$ has no real solutions because -5 lies outside the range.

Implication: The original equation has no real solutions because the algebraic manipulations led to an impossible sine value.

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3. The Concept of Range and Its Impact on Solutions

3.1 Range of the Sine Function

The sine function oscillates between -1 and 1:

$$
-1 \leq \sin(\theta) \leq 1
$$

Any value outside this interval makes the equation unsolvable within real numbers.

3.2 Analyzing the Derived Equation

Since $\sin(\theta) = -5$ is outside the valid range, the conclusion is that the original equation has no real solutions.

However, if the question involves complex solutions or is designed to test understanding of the sine's range, then the solution set differs.

3.3 Alternative Approach: Re-examining Initial Assumptions

Suppose the original problem was:

$$
3\sin(\theta) + 5 = 2\sin(\theta)
$$

and we want solutions in complex numbers. In that case, the solutions involve extending sine to complex domains, but for most high school or undergraduate contexts, the solutions are only real.

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4. General Solutions: When Are They Applicable?

4.1 Conditions for Solutions in Trigonometric Equations

The general solution for equations involving sine is usually expressed in terms of the inverse sine function ($\sin^{-1}$ or \arcsin), which gives principal values within $[- \frac{\pi}{2}, \frac{\pi}{2}]$. For equations where sine equals a value k:

$$
\sin(\theta) = k
$$

the solutions are:

$$
\theta = \arcsin(k) + 2n\pi \quad \text{or} \quad \theta = \pi - \arcsin(k) + 2n\pi
$$

for integers n, covering all solutions over the domain.

4.2 When No Real Solutions Exist

If $|k| > 1$, then $\sin(\theta) = k$ has no real solutions.

Applying this to our problem:

$$
\sin(\theta) = -5
$$

since $|-5|=5 > 1$, there are no real solutions.

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5. Broader Implications and Critical Thinking

5.1 Validating Solutions

This problem underscores the importance of checking the validity of solutions after algebraic manipulations. Range validation is a critical step often overlooked, which can lead to false solutions or the conclusion that no solutions exist.

5.2 Recognizing When Equations Are No Solution Cases

Mathematically, recognizing when an equation leads to an impossible condition (like sine exceeding its maximum or minimum) saves time and avoids unnecessary calculations.

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6. Alternative Scenario: A Corrected Equation

Suppose the initial equation was:

$$
3\sin(\theta) + 5 = 2
$$

which simplifies to:

$$
3\sin(\theta) = -3
$$

then:

$$
\sin(\theta) = -1
$$

This falls within the valid range $[-1, 1]$, and the solutions are:

$$
\theta = \arcsin(-1) + 2n\pi = -\frac{\pi}{2} + 2n\pi
$$

or

$$
\theta = \pi - \arcsin(-1) + 2n\pi = \pi - (-\frac{\pi}{2}) + 2n\pi = \frac{3\pi}{2} + 2n\pi
$$

which simplifies to:

$$
\boxed{
\theta = -\frac{\pi}{2} + 2n\pi \quad \text{or} \quad \frac{3\pi}{2} + 2n\pi, \quad n \in \mathbb{Z}
}
$$

This illustrates the process of deriving the general solution for sine equations that are valid.

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7. Practical Applications and Significance of the Solution Process

7.1 Modeling Physical Phenomena

Understanding the solutions to trigonometric equations enables engineers and physicists to model oscillatory systems accurately, such as pendulums, electrical circuits, and sound waves.

7.2 Signal Processing

In signal processing, knowing the complete set of solutions helps in analyzing periodic signals, filtering, and Fourier analysis.

7.3 Mathematical Rigor and Critical Thinking

This exploration exemplifies the importance of verifying solutions and understanding the properties of functions involved, fostering mathematical rigor and critical thinking skills.

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8. Summary and Final Remarks

- The original equation:

$$
3\sin(\theta) + 5 = 2\sin(\theta)
$$

simplifies to:

$$
\sin(\theta) = -5
$$

- Since $\sin(\theta)$ cannot be less than -1, there are no real solutions.

- General solutions depend on the value of the sine, but in this case, the invalid range indicates the absence of real solutions.

- This scenario emphasizes the importance of range validation in solving trigonometric equations.

- In equations with valid sine values, solutions are expressed as:

$$
\theta = \arcsin(k) + 2n\pi \quad \text{and} \quad \theta = \pi - \arcsin(k) + 2n\pi
$$

where k is within $[-1, 1]$.

Key Takeaway: Always verify the range of the sine (or other trigonometric functions) before accepting solutions, to ensure their validity and applicability.

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9. Conclusion

The process of solving trigonometric equations is foundational in understanding periodic functions and their applications across science and engineering. Recognizing the limitations imposed by the functions' ranges is crucial in correctly determining whether solutions exist and, if so, in deriving their comprehensive, general form. The specific problem discussed here exempl

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