Given That Y = Et Is A Solution Of The Equation, Ty" + (3t-1)y' + (2t - 1)y=0, T>0 Find A Second Linearly

Understanding the Differential Equation and Its Context

Given That Y = Et Is A Solution Of The Equation, Ty" + (3t-1)y' + (2t - 1)y=0, T>0 Find A Second Linearly introduces a fundamental problem in the study of differential equations—finding a second linearly independent solution to a given second-order linear differential equation. This problem is central to understanding the complete solution set of such equations, which frequently appear in physics, engineering, and applied mathematics.

The differential equation provided:

$$ T y'' + (3t - 1) y' + (2t - 1) y = 0 $$

where $T > 0$, is a linear second-order differential equation with variable coefficients. The existence of one known solution $Y = e^t$ suggests the potential to find a second solution, which, along with the first, forms a fundamental set of solutions. This allows the general solution to be expressed as a linear combination of these solutions.

In this article, we will explore methods to find the second linearly independent solution, interpret the differential equation's structure, and understand the significance of such solutions in the broader context of differential equations.

Analyzing the Given Differential Equation

Form of the Equation

The given differential equation is:

$$ T y'' + (3t - 1) y' + (2t - 1) y = 0 $$

- Type: It's a second-order linear differential equation with variable coefficients.
- Coefficients: The coefficients depend explicitly on t, indicating that standard methods for constant coefficient equations are inapplicable

directly.
- Solution approach: We need to leverage known solutions, reduction of order, or special methods tailored for variable coefficient equations.

Known Solution and Its Implications

Given that:

$$ Y = e^t $$

is a solution, this is a valuable piece of information. It indicates that the differential equation admits exponential solutions, and we can use this known solution to find the second solution via the method of reduction of order.

Method to Find the Second Linearly Independent Solution

Reduction of Order Technique

Reduction of order is a powerful technique when one solution y_1 is known, and we seek a second solution y_2 such that:

$$ y_2 = v(t) y_1(t) $$

where $v(t)$ is a function to be determined.

Steps involved:

1. Assume:

$$ y_2 = v(t) e^t $$

2. Compute derivatives:

$$ y_2' = v' e^t + v e^t $$
$$ y_2'' = v'' e^t + 2 v' e^t + v e^t $$

3. Substitute into the original differential equation:

$$ T y_2'' + (3t - 1) y_2' + (2t - 1) y_2 = 0 $$

4. Simplify to derive an equation for $v(t)$.

This process effectively reduces the problem to solving a first-order differential equation for v'.

Applying Reduction of Order to Our Equation

Let's proceed step-by-step:

- Substitute $y_2 = v(t) e^t$
- Derivatives:

$$y_2' = v' e^t + v e^t$$
$$y_2'' = v'' e^t + 2 v' e^t + v e^t$$

- Plug into the original equation:

$$
T (v'' e^t + 2 v' e^t + v e^t) + (3t - 1)(v' e^t + v e^t) + (2t - 1) v e^t = 0
$$

- Factor out e^t:

$$
e^t [T v'' + 2 T v' + T v + (3t - 1) v' + (3t - 1) v + (2t - 1) v] = 0
$$

- Since $e^t \neq 0$, we get:

$$
T v'' + (2 T + 3t - 1) v' + [T + (3t - 1) + (2t - 1)] v = 0
$$

- Simplify the coefficient of v:

$$
T + 3t - 1 + 2t - 1 = T + 5t - 2
$$

Thus, the reduced equation becomes:

$$
T v'' + (2 T + 3t - 1) v' + (T + 5t - 2) v = 0
$$

This is a first-order differential equation in v' and v.

Solving the Reduced Equation for $v(t)$

Transforming the Equation

To solve for $v(t)$, we can consider substitution methods or look for particular solutions. Because the coefficients depend explicitly on t, applying an integrating factor or substitution may be effective.

Alternatively, since $y_1 = e^t$ is known, one standard approach is to express the second solution as:

$$ y_2 = y_1 \int \frac{e^{-\int P(t) dt}}{y_1^2} dt $$

where $P(t)$ is the coefficient in the standard form of the differential equation.

Standard form:

Divide the original equation by T:

$$
y'' + \frac{3t - 1}{T} y' + \frac{2t - 1}{T} y = 0
$$

In standard form:

$$
y'' + p(t) y' + q(t) y = 0
$$
with

$$
p(t) = \frac{3t - 1}{T}
$$
$$
q(t) = \frac{2t - 1}{T}
$$

The integrating factor method involves computing:

$$
\mu(t) = e^{\int p(t) dt} = e^{\frac{1}{T} \int (3t - 1) dt} = e^{\frac{1}{T} \left(\frac{3}{2} t^2 - t \right)}
$$

Now, the second solution:

\[

$$
y_2 = y_1 \int \frac{e^{-\int p(t) dt}}{y_1^2} dt
$$

Given that $y_1 = e^t$, which simplifies the expression.

Calculating the integral:

$$
y_2 = e^t \int \frac{e^{-\frac{1}{T} (\frac{3}{2} t^2 - t)}}{e^{2t}} dt = e^t \int e^{-\frac{1}{T} (\frac{3}{2} t^2 - t)} e^{-2t} dt
$$

Combine exponents:

$$
y_2 = e^t \int e^{-\frac{3 t^2}{2 T} + \frac{t}{T} - 2 t} dt
$$

which simplifies to:

$$
y_2 = e^t \int e^{-\frac{3 t^2}{2 T} + \left(\frac{1}{T} - 2 \right) t } dt
$$

This integral generally does not have a closed-form solution in elementary functions, but it can be expressed in terms of special functions (e.g., error functions) or approximated numerically.

Summary of the Method to Find the Second Solution

Key Steps Recap

1. Identify the known solution: $y_1 = e^t$.
2. Convert the original differential equation into standard form.
3. Compute the integrating factor:

$$
\mu(t) = e^{\int p(t) dt} = e^{\frac{1}{T} (\frac{3}{2} t^2 - t)}
$$

4. Use reduction of order to find y_2:

$$
y_2(t) = y_1(t) \int \frac{e^{-\int p(t) dt}}{y_1^2} dt
$$

which simplifies to an integral involving exponential functions with quadratic terms.

5. Express the second solution: as an integral that, depending on the context, can be evaluated or approximated.

Significance of the Second Linearly Independent Solution

The second solution, together with the known solution y_1, forms a fundamental set of solutions for the differential equation. The general solution is:

$$
y(t) = C_1 y_1(t) + C_2 y_2(t)
$$

where C_1 and C_2 are arbitrary constants.

This complete solution provides insight into the behavior of the system described by the differential equation, such as oscillations, exponential growth or decay, or more complex dynamics depending on the form of

Frequently Asked Questions

What type of differential equation is given: Ty'' + (3t - 1)y' + (2t - 1)y = 0?

It is a second-order linear differential equation with variable coefficients.

Given that Y = Et is a solution, how can we verify this solution for the differential equation?

Substitute Y = e^t into the equation and verify if the equation is satisfied for all t.

What method can be used to find a second linearly independent solution for this differential equation?

The reduction of order method can be employed, assuming a solution of the form y = v(t) e^t.

How do you apply the reduction of order technique to this differential equation?

Assume y = v(t) e^t, then find derivatives y' and y'' and substitute into the original equation to find v(t).

What is the general form of the second linearly independent solution once reduction of order is applied?

It will be of the form y2 = v(t) e^t, where v(t) is obtained by solving a simplified differential equation.

Why is it necessary to find a second solution for this differential equation?

Because the differential equation is second order, two linearly independent solutions are needed to form the general solution.

What is the significance of the parameter T in the differential equation?

T is a positive parameter affecting the coefficients of the equation, influencing the form of solutions.

Can the solution Y = e^t be considered a particular solution or a fundamental solution?

It is a fundamental solution that forms part of the general solution; finding a second independent solution completes the general solution.

What are the typical steps to find the second linearly independent solution for this type of differential equation?

Identify the known solution, apply reduction of order, solve the resulting equations for v(t), then write the second solution.

Are there special functions or transformations that can simplify solving this differential equation?

Depending on the form, substitutions or special functions like Bessel or confluent hypergeometric functions might be used, but reduction of order is often the direct approach.

Additional Resources

Given That Y = Et Is A Solution Of The Equation, Ty" + (3t-1)y' + (2t - 1)y=0, T>0 Find A Second Linearly Independent Solution

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Introduction

In the realm of differential equations, discovering solutions and understanding their properties remains a central task. The equation under scrutiny,

$$
T y'' + (3t - 1) y' + (2t - 1) y = 0,
$$

where $T > 0$, presents an intriguing case for analysis. Particularly, the problem states that $y = E_t$ (which is often interpreted as the exponential function e^{t} or some similar exponential form) is a solution. The primary goal is to find a second linearly independent solution, which is fundamental in establishing the general solution to the differential equation.

This investigation aims to methodically analyze the problem, explore solution techniques, and derive the second independent solution, emphasizing the importance of linear independence and solution structure.

Understanding the Differential Equation

1. Nature of the Equation

The given differential equation is a second-order linear differential equation with variable coefficients:

$$
T y'' + (3t - 1) y' + (2t - 1) y = 0,
$$

where $T > 0$. Such equations often resist straightforward solution methods like constant coefficient techniques and typically require methods tailored to variable coefficients, such as reduction of order or series solutions.

2. Significance of the Known Solution $y = E_t$

The problem states that $y = E_t$ is a solution. Assuming E_t refers to the exponential function e^{t}, this is a natural starting point because exponential functions often serve as solutions to linear differential equations, especially when coefficients are polynomials in t.

Given that, verify if $y = e^{t}$ indeed satisfies the differential equation.

Verifying the Known Solution

1. Substitution into the Equation

Compute derivatives:

$$
\begin{aligned}
y &= e^{t}, \\
y' &= e^{t}, \\
y'' &= e^{t}.
\end{aligned}
$$

Substitute into the original equation:

$$
T e^{t} + (3t - 1) e^{t} + (2t - 1) e^{t} = 0,
$$

which simplifies to:

$$
e^{t} \left[T + (3t - 1) + (2t - 1) \right] = 0.
$$

Simplify the terms inside brackets:

$$
T + 3t - 1 + 2t - 1 = T + 5t - 2.
$$

Thus, the equation reduces to:

$$
e^{t} [T + 5t - 2] = 0.
$$

Since $e^{t} \neq 0$ for all t, the solution $y = e^{t}$ satisfies the differential equation only if

$$
T + 5t - 2 = 0

\]

for all t, which is impossible unless T is variable-dependent, conflicting with the problem statement that T is a positive constant.

2. Clarification of the Solution

This apparent contradiction suggests that perhaps $y = e^{t}$ is not a general solution but rather a particular solution at specific T or under certain conditions.

Alternatively, the notation E_t might refer to a different exponential function, such as the Mittag-Leffler function or an exponential with a different argument.

Assumption: For the purpose of this analysis, assume the problem intends for $y = e^{k t}$ for some constant k. Let's test the general exponential form $y = e^{k t}$.

Finding the Characteristic Equation for the Known Solution

1. Substituting $y = e^{k t}$

Compute derivatives:

$$
\begin{aligned}
y &= e^{k t}, \\
y' &= k e^{k t}, \\
y'' &= k^{2} e^{k t}.
\end{aligned}
$$

Substituting into the differential equation:

$$
T k^{2} e^{k t} + (3t - 1) k e^{k t} + (2t - 1) e^{k t} = 0.
$$

Dividing through by $e^{k t}$:

$$
T k^{2} + (3t - 1) k + (2t - 1) = 0.
$$

Since this depends explicitly on t, the exponential $e^{k t}$ does not generally satisfy the equation unless the coefficients vanish identically, which is impossible unless the coefficients are constant.

2. Implication

The previous analysis indicates that the assumption $y = e^{t}$ is not an exact solution unless specific conditions on T and t are met.

Reassessing the Approach: Reduction of Order

Given the ambiguity, a standard method to find the second solution when one solution y_1 is known is the reduction of order. This technique applies to linear differential equations when one solution is known, and the goal is to find a second, linearly independent solution.

1. General Form of Reduction of Order

Suppose $y_1(t)$ is a known solution. Then the second solution $y_2(t)$ can be expressed as:

$$
y_2(t) = y_1(t) \cdot v(t),
$$

where $v(t)$ is an unknown function to be determined.

2. Derivation of the Reduction of Order Equation

Applying this substitution into the original differential equation yields a second-order reduction:

$$
\text{(Standard reduction of order process)}.
$$

However, to proceed, we need an explicit known solution $y_1(t)$.

Clarifying the Known Solution and Its Role

Given the original problem's phrasing, it's possible that the solution $y = e^{t}$ is known under specific scenarios or that $y = e^{t}$ satisfies a simplified version of the differential equation when parameters are chosen accordingly.

1. Hypotheses on the form of the solution

Suppose the problem intends that the differential equation is:

$$
T y'' + (3t - 1) y' + (2t - 1) y = 0,
$$

\]

and that for some particular T, $y = e^{t}$ is a solution.

Given previous calculations, this is only true if:

$$
T + 5t - 2 = 0,
$$

which cannot be valid for all t unless T depends on t, contradicting the problem statement.

2. Alternative interpretation

Perhaps the problem contains a typographical error or is referencing a different known solution, such as a special function, or is considering the case where the coefficients are functions of t but with specific properties.

Methodology to Find the Second Solution

Assuming $y_1(t)$ is known, the standard approach is the reduction of order, which proceeds as follows:

1. Setup

Let $y_1(t)$ be a known solution. The second solution is:

$$
y_2(t) = y_1(t) \cdot v(t),
$$

with $v(t)$ to be determined.

2. Derive the formula for $v(t)$

Substituting $y_2(t)$ into the original differential equation leads to a second-order linear ODE for $v(t)$, which simplifies to:

$$
v' = \frac{W}{y_1^2},
$$

where W is the Wronskian of the solutions.

Alternatively, the reduction of order formula states:

\[

v' = \frac{C}{y_1^2},
\]

for some constant \(C \), and integrating gives:

\[
v(t) = \int \frac{C}{y_1^2} dt.
\]

Thus, the second solution is:

\[
y_2(t) = y_1(t) \int \frac{1}{y_1^2(t)} dt,
\]

up to a multiplicative constant.

Practical Approach: Applying Reduction of Order

1. Identify a Known Solution \(y_1(t) \)

Given the uncertainties, select a plausible solution based on the structure of the differential equation.

Suppose, for the sake of illustration, that \(y_1(t) = e^{\lambda t} \) is a solution for some \(\lambda \). However, previous analysis suggests that \(y = e^{t} \) does not satisfy the original equation unless specific conditions are met.

2. Alternative: Guess a solution of the form \(y = t^{m} \)

Since coefficients contain \(t \), polynomial or power solutions might be more appropriate.

3. Series solutions or special functions

If the coefficients are polynomial, series solutions or special functions (e.g.,

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