Identify The Inequalities A, B , And C For Which The Given Ordered Pair Is A Solution.A. X+y 2 B. Y (3/2)x-1

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Introduction

Understanding inequalities and their solutions is a fundamental aspect of algebra that plays a crucial role in various fields such as mathematics, engineering, economics, and sciences. When working with inequalities, a common task involves determining whether a specific ordered pair (x, y) satisfies a given inequality, thus qualifying as a solution. This process is vital in graphing inequalities, solving systems of inequalities, and analyzing feasible regions in optimization problems.

In this article, we will focus on a specific problem: identifying the inequalities A, B, and C for which the given ordered pair is a solution. The problem statement presents two inequalities:

- Inequality A: $x + y \leq 2$
- Inequality B: $y \geq \frac{3}{2}x - 1$

The goal is to understand how to verify if a particular ordered pair satisfies these inequalities and how to determine if it is a solution within certain inequality systems. We will explore the methodology involved in solving such problems, interpret the inequalities graphically, and extend the discussion to systems of inequalities, feasible regions, and their applications.

Understanding the Given Inequalities

Before diving into the solution process, it is essential to interpret the given inequalities and understand their geometric representations.

Inequality A: $x + y \leq 2$

This inequality represents all points (x, y) on or below the line $x + y = 2$. The line itself acts as the boundary, and the inequality indicates the region on or beneath this boundary.

- Boundary line: $x + y = 2$
- Test point: To determine which side of the line satisfies the inequality, pick a test point not on the line, such as (0,0):
- $0 + 0 = 0 \leq 2$ → True
- Therefore, the region including (0,0) is part of the solution set.

Inequality B: $y \geq \frac{3}{2}x - 1$

This inequality signifies all points (x, y) on or above the line $y = \frac{3}{2}x - 1$. The boundary is the line itself, and the inequality points to the region above or on this line.

- Boundary line: $y = \frac{3}{2}x - 1$
- Test point: Again, test with (0,0):
- $0 \geq \frac{3}{2} \times 0 - 1 = -1$ → $0 \geq -1$ → True
- So, the region including (0,0) is part of the solution.

Determining if a Given Ordered Pair Is a Solution

Suppose we are given an ordered pair, say, $(x, y) = (a, b)$. To verify if this pair satisfies the inequalities, follow these steps:

1. Substitute $x = a$ and $y = b$ into each inequality.
2. Check if the inequalities hold true:
- For Inequality A: $a + b \leq 2$
- For Inequality B: $b \geq \frac{3}{2}a - 1$
3. Determine whether the pair satisfies both inequalities simultaneously.

For example, consider the pair $(x, y) = (1, 1)$:

- Substitute into Inequality A:
$1 + 1 = 2 \leq 2$ → True
- Substitute into Inequality B:
$1 \geq \frac{3}{2} \times 1 - 1 = \frac{3}{2} - 1 = 0.5$
$1 \geq 0.5$ → True

Since both inequalities are satisfied, $(1, 1)$ is a solution to the system.

Similarly, test $(x, y) = (2, 0)$:

- Inequality A: $2 + 0 = 2 \leq 2$ → True
- Inequality B: $0 \geq \frac{3}{2} \times 2 - 1 = 3 - 1 = 2$ → $0 \geq 2$ → False

Thus, $(2, 0)$ does not satisfy both inequalities and is not a solution.

Graphical Representation of the Inequalities

Visualizing inequalities helps in understanding the solution space and the relationships between different inequalities.

Graphing Inequality A: $x + y \leq 2$

- Draw the boundary line $x + y = 2$.
- Shade the region below or on the line, since the inequality is "less than or equal to".

Graphing Inequality B: $y \geq \frac{3}{2}x - 1$

- Draw the boundary line $y = \frac{3}{2}x - 1$.
- Shade the region above or on this line.

The intersection of the shaded regions from both inequalities represents the feasible solution set satisfying both inequalities simultaneously.

System of Inequalities and Feasible Region

When multiple inequalities are combined, they form a system of inequalities. The solution to the system is the feasible region—the intersection of all individual solution regions.

Steps to Find the Feasible Region

1. Graph each inequality on the same coordinate plane.
2. Identify the regions satisfying each inequality.
3. The feasible region is where all these regions overlap.

Example: Graphical Solution of the System

- Graph the boundary lines $x + y = 2$ and $y = \frac{3}{2}x - 1$.
- Shade the respective solution regions:
- $x + y \leq 2$: below or on the line.
- $y \geq \frac{3}{2}x - 1$: above or on the line.
- The overlapping region is the set of all points satisfying both inequalities.

Applications of Inequalities and Solution Sets

Understanding inequalities and their solutions is essential in numerous practical applications:

- Linear Programming: Finding optimal solutions within a feasible region defined by inequalities.
- Economics: Budget constraints and profit maximization.
- Engineering: Safety margins and design constraints.
- Science: Modeling physical phenomena with bounds and restrictions.

Conclusion

Determining whether an ordered pair satisfies a system of inequalities is a vital skill in algebra. It involves substituting the pair into each inequality and verifying the inequalities' conditions. Graphical representation enhances understanding by visualizing feasible regions and solution sets, especially when dealing with systems of inequalities.

In the specific case of the inequalities $x + y \leq 2$ and $y \geq \frac{3}{2}x - 1$, the solution process involves testing the point in question, analyzing the boundary lines, and visualizing the intersection of the solution regions. Mastering these concepts enables students and professionals to solve complex problems involving inequalities effectively and apply them in real-world scenarios.

By developing a solid understanding of inequalities, their graphical representations, and solution verification methods, you will enhance your mathematical problem-solving skills and be better equipped to handle advanced topics in algebra, optimization, and beyond.

Frequently Asked Questions

Given the ordered pair (4, 2), does it satisfy the inequality $X + Y > 5$? If not, why?

Substitute $X=4$ and $Y=2$ into the inequality: $4 + 2 > 5$, which simplifies to $6 > 5$. Since this is true, the ordered pair (4, 2) satisfies the inequality $X + Y > 5$.

For the inequality $Y \leq (3/2)X - 1$, does the point (2, 2) satisfy it?

Substitute $X=2$ and $Y=2$: $2 \leq (3/2)2 - 1 \rightarrow 2 \leq 3 - 1 \rightarrow 2 \leq 2$, which is true. Therefore, (2, 2) satisfies the inequality $Y \leq (3/2)X - 1$.

If the ordered pair (1, 3) is a solution for the inequality $X - 2Y < -5$, does it satisfy this inequality?

Substitute $X=1$ and $Y=3$: $1 - 23 < -5 \rightarrow 1 - 6 < -5 \rightarrow -5 < -5$. Since -5 is not less than -5, the pair (1, 3) does not satisfy the inequality.

Identify the inequalities A, B, and C from the description: A is $X + Y > 4$, B is $Y \leq (3/2)X - 1$, and C is $X - 2Y < -3$. Which of these are satisfied by the point (3, 2)?

Substitute $X=3$, $Y=2$:
- For A: $3 + 2 > 4 \rightarrow 5 > 4$ (true)
- For B: $2 \leq (3/2)3 - 1 \rightarrow 2 \leq 4.5 - 1 \rightarrow 2 \leq 3.5$ (true)
- For C: $3 - 22 < -3 \rightarrow 3 - 4 < -3 \rightarrow -1 < -3$ (false)

Thus, the point (3, 2) satisfies inequalities A and B, but not C.

Determine whether the ordered pair (0, 1) satisfies the inequalities A: X + Y ≥ 1, B: Y > (3/2)X - 2, and C: X - 2Y ≤ -2.

Substitute X=0, Y=1:
- For A: 0 + 1 ≥ 1 → 1 ≥ 1 (true)
- For B: 1 > (3/2)0 - 2 → 1 > -2 (true)
- For C: 0 - 21 ≤ -2 → 0 - 2 ≤ -2 → -2 ≤ -2 (true)

Therefore, the point (0, 1) satisfies all three inequalities.

Additional Resources

Understanding and Identifying Inequalities from Given Solutions: A Comprehensive Guide

When working with inequalities in algebra, a fundamental skill is being able to determine which inequalities are satisfied by a particular ordered pair. This process is essential in various fields such as mathematics, economics, engineering, and data science, where understanding the feasible regions or solutions in a coordinate system plays a vital role. In this detailed review, we will explore how to identify the inequalities for which a given ordered pair is a solution, using the specific examples of inequalities A and B, and a particular ordered pair.

Introduction to Inequalities and Ordered Pairs

Before diving into the specific inequalities, let's establish a solid foundation on what inequalities are and how ordered pairs relate to them.

What is an Inequality?
An inequality is a mathematical statement that compares two expressions using symbols such as `<`, `>`, `≤`, or `≥`. For example, `x + y ≤ 5` indicates that the sum of `x` and `y` is less than or equal to 5.

What is an Ordered Pair?
An ordered pair `(x, y)` represents a point in the coordinate plane, where `x` is the horizontal coordinate, and `y` is the vertical coordinate. To determine whether a point is a solution to an inequality, we substitute the `x` and `y` values into the inequality and check if the resulting statement is true.

The Goal
Given an ordered pair and a set of inequalities, our goal is to determine which inequalities the point satisfies. Conversely, given an inequality, we can also find all points that satisfy it—these points form the solution region.

Analyzing the Given Inequalities and the Ordered Pair

Suppose we are provided with the following inequalities:

- Inequality A: \(x + y \leq 2 \)
- Inequality B: \(y \geq \left(\frac{3}{2}\right) x - 1 \)

And an ordered pair: \((x_0, y_0)\) (which you can think of as specific coordinates, e.g., (1, 0), (2, 1), etc.).

Our task is:

- To verify whether this particular pair satisfies each inequality.
- To identify the inequalities for which the pair is a solution.

Step-by-Step Methodology for Verification

To systematically determine whether the ordered pair is a solution, follow these steps:

Step 1: Write down the inequalities explicitly
Ensure clarity on the inequalities' forms:

- Inequality A: \(x + y \leq 2 \)
- Inequality B: \(y \geq \left(\frac{3}{2}\right) x - 1 \)

Step 2: Substitute the ordered pair into each inequality
Suppose the ordered pair is \((x_0, y_0)\). Substituting these values:

- For Inequality A: Check if \(x_0 + y_0 \leq 2 \)
- For Inequality B: Check if \(y_0 \geq \left(\frac{3}{2}\right) x_0 - 1 \)

Step 3: Evaluate the inequalities
Perform the calculations:

- Compute \(x_0 + y_0 \).
- Compute \(\left(\frac{3}{2}\right) x_0 - 1 \) and compare with \(y_0 \).

Step 4: Determine the solution status
- If the inequality holds true (e.g., \(x_0 + y_0 \leq 2 \)), then the pair satisfies that inequality.
- If not, then the pair does not satisfy it.

Deep Dive into Inequality A: $x + y \leq 2$

Let's analyze Inequality A thoroughly.

Geometric Interpretation
The inequality $x + y \leq 2$ defines a region in the coordinate plane:

- The boundary line is $x + y = 2$.
- The region of solutions is below or on this line, because the inequality is less than or equal to.

Graphical Perspective
- The line $x + y = 2$ cuts through the coordinate plane with intercepts at:

- x-intercept: when $y=0$, then $x=2$.
- y-intercept: when $x=0$, then $y=2$.

- The feasible region includes all points where $x + y \leq 2$, which is the half-plane beneath or on the line.

Algebraic Analysis
To verify if a specific point (x_0, y_0) is in this region:

- Calculate $S = x_0 + y_0$.
- Check if $S \leq 2$.

Example:
Suppose the point is (1, 0.5):

- $1 + 0.5 = 1.5 \leq 2$, so yes, the point satisfies Inequality A.

Counterexample:
Suppose the point is (2, 1):

- $2 + 1 = 3 \not\leq 2$, so no, it does not satisfy Inequality A.

Summary of Critical Points
- The boundary line: $x + y = 2$.
- The inequality: $x + y \leq 2$ signifies the region below or on this line.
- Points satisfy the inequality if their sum $x + y$ is less than or equal to 2.

Deep Dive into Inequality B: $y \geq \left(\frac{3}{2}\right) x - 1$

Now, let's thoroughly analyze Inequality B.

Geometric Interpretation
- The boundary line is $y = \left(\frac{3}{2}\right) x - 1$.
- The solution region includes points above or on this line since the inequality is $y \geq$ the expression.

Graphical Perspective
- The line has a slope of $\frac{3}{2}$ and a y-intercept at $y = -1$.
- The feasible region is the half-plane above or on the line.

Algebraic Analysis
To determine if a point (x_0, y_0) satisfies Inequality B:

- Calculate the right side: $R = \left(\frac{3}{2}\right) x_0 - 1$.
- Compare y_0 with R:

- If $y_0 \geq R$, the point satisfies the inequality.
- Otherwise, it does not.

Example:
Suppose the point is (2, 2):

- $R = \frac{3}{2} \times 2 - 1 = 3 - 1 = 2$.
- $y_0 = 2 \geq 2$, so yes, the point satisfies Inequality B.

Counterexample:
Suppose the point is (0, -2):

- $R = \frac{3}{2} \times 0 - 1 = -1$.
- $y_0 = -2 \not\geq -1$, so no.

Boundary Line and Region
- The boundary line: $y = \left(\frac{3}{2}\right) x - 1$.
- The solution region: above or on the line.

Applying the Process to a Specific Example

Given the inequalities:
- Inequality A: $x + y \leq 2$
- Inequality B: $y \geq \left(\frac{3}{2}\right) x - 1$

Suppose the ordered pair is (1, 0).

Step 1: Substitute into Inequality A
- $x + y = 1 + 0 = 1$
- Check: $1 \leq 2$? Yes.
Conclusion: The pair satisfies Inequality A.

Step 2: Substitute into Inequality B
- $R = \left(\frac{3}{2}\right) \times 1 - 1 = 1.5 - 1 = 0.5$
- Compare $y = 0$ with $R = 0.5$:

- Is $0 \geq 0.5$? No.

Conclusion: The pair does not satisfy Inequality B.

Final assessment:
The point (1, 0) satisfies Inequality A but not Inequality B.

General Strategy for Identifying Inequalities for Any Given Point

When dealing with multiple inequalities and a specific point, follow this structured approach:

1. List all inequalities clearly.
2. Substitute the point's coordinates into each inequality.
3. Perform the calculations meticulously.
4. Interpret the results:

Identify The Inequalities A B And C For Which The Given Ordered Pair Is A Solution A X Y 2 B Y 3 2 X 1

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