Problem Set C (1) For Demand Function La = A-bP And Supply Function Qs = DP-C, Using Cramer's Rule Determine

Problem Set C (1) For Demand Function La = A-bP And Supply Function Qs = DP-C, Using Cramer's Rule Determine

Understanding the core principles of market equilibrium is fundamental in economics, especially when analyzing how demand and supply interact to determine the price and quantity in a market. In this problem set, we examine a specific demand function La = A - bP and a supply function Qs = DP - C, and we aim to find the equilibrium price and quantity using Cramer's Rule, a powerful algebraic method for solving systems of linear equations. This comprehensive guide will walk you through the derivation, solution process, and interpretation of results, providing clarity on both the mathematical and economic aspects.

Introduction to Demand and Supply Functions

Before diving into the solution, it's essential to understand the nature of the demand and supply functions provided:

Demand Function: La = A - bP

- La: The quantity demanded, which depends inversely on the price, P.
- A: A positive constant representing the maximum demand when the price is zero.
- b: The slope coefficient indicating how demand decreases as price increases.

Supply Function: Qs = DP - C

- Qs: The quantity supplied, which depends positively on the price, P.
- D: The slope coefficient indicating how supply increases with price.
- C: A constant representing fixed costs or other factors affecting supply.

The equilibrium in the market occurs when quantity demanded equals quantity supplied, i.e., La = Qs. Setting these equal leads to a system of equations that can be solved for P and Q.

Formulating the System of Equations

To find the equilibrium, we set La equal to Qs:

\[
A - bP = DP - C
\]

Rearranging terms:

\[
A + C = DP + bP
\]

which simplifies to:

\[
A + C = (D + b)P
\]

This is a linear equation in P. To find the equilibrium quantity, substitute P back into either the demand or supply function:

\[
Q = La = A - bP
\]
or
\[
Q = Qs = DP - C
\]

However, for the purpose of applying Cramer's Rule, we often need to set up a system of equations involving two unknowns, which in this case can be P and Q if we express both functions in terms of these variables.

Expressing the System of Equations

Given the demand and supply functions, the system can be written as:

\[
\begin{cases}

La = A - bP \\
Qs = DP - C
\end{cases}
\]

At equilibrium:

\[
A - bP = Q \quad (1)
\]
\[
Q = DP - C \quad (2)
\]

Substituting equation (2) into (1):

\[
A - bP = DP - C
\]

which simplifies to the earlier form:

\[
A + C = (D + b)P
\]

Now, to apply Cramer's Rule, it’s helpful to express the problem as a system of two equations with two unknowns, P and Q.

Setting Up the Equations for Cramer's Rule

From the previous steps, we have:

\[
\begin{cases}
Q = A - bP \quad \text{(Demand equation)} \\
Q = DP - C \quad \text{(Supply equation)}
\end{cases}
\]

Rearranged into standard form:

\[
\begin{cases}
bP + Q = A \\
-DP + Q = C
\end{cases}
\]

Expressed as:

\[
\begin{cases}
bP + 1 \times Q = A \\
- D P + 1 \times Q = C
\end{cases}
\]

This is a system of linear equations in variables P and Q:

\[
\begin{bmatrix}
b & 1 \\
- D & 1
\end{bmatrix}
\begin{bmatrix}
P \\
Q
\end{bmatrix}
=
\begin{bmatrix}
A \\
C
\end{bmatrix}
\]

Applying Cramer's Rule involves calculating determinants of matrices to solve for P and Q.

Applying Cramer's Rule

Cramer's Rule states that for a system:

\[

\begin{cases}
a_{11} P + a_{12} Q = b_1 \\
a_{21} P + a_{22} Q = b_2
\end{cases}
\]

the solutions are:

\[
P = \frac{\det(M_P)}{\det(M)}
\]
\[
Q = \frac{\det(M_Q)}{\det(M)}
\]

where:

- \(\det(M)\) is the determinant of the coefficient matrix:

\[
M =
\begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]

- \(\det(M_P)\) is the determinant of the matrix formed by replacing the first column with the constants:

\[
M_P =
\begin{bmatrix}
b_1 & a_{12} \\
b_2 & a_{22}
\end{bmatrix}
\]

- \(\det(M_Q)\) is the determinant of the matrix formed by replacing the second column with the constants:

\[
M_Q =
\begin{bmatrix}

a_{11} & b_1 \\
a_{21} & b_2
\end{bmatrix}
\]

Applying these to our system:

\[
\det(M) = b \times 1 - (-D) \times 1 = b + D
\]

\[
\det(M_P) = A \times 1 - C \times 1 = A - C
\]

\[
\det(M_Q) = b \times C - (-D) \times A = bC + D A
\]

Therefore:

\[
P = \frac{A - C}{b + D}
\]

\[
Q = \frac{b C + D A}{b + D}
\]

This provides explicit formulas for the equilibrium price and quantity.

Interpreting the Results

The solutions derived via Cramer's Rule give valuable insights into market behavior:

- Equilibrium Price (P):

\[
P^ = \frac{A - C}{b + D}
\]

- Equilibrium Quantity (Q):

$$
Q^{} = \frac{b C + D A}{b + D}
$$

These expressions depend on the parameters of the demand and supply functions:

- An increase in A (maximum demand) tends to raise the equilibrium price and quantity.
- A higher C (fixed costs or supply-side constants) tends to lower the equilibrium price but can increase the quantity depending on the relative sizes of D and b.
- The slopes b and D influence how sensitive demand and supply are to price changes, affecting the equilibrium point accordingly.

Conditions for Validity and Economic Implications

For the solutions to be meaningful in an economic context:

- The denominator $(b + D)$ must not be zero, i.e., $b + D \neq 0$. This ensures the system is solvable.
- The calculated P and Q should be positive, representing realistic market conditions. Negative values would imply non-physical solutions, indicating the need to reassess the parameters or the model assumptions.

Additionally, understanding how changes in parameters affect the equilibrium allows policymakers and businesses to make informed decisions:

- Pricing Strategies: Adjusting prices based on demand responsiveness.
- Market Interventions: Modifying taxes, subsidies, or policies to influence demand or supply.

Conclusion

Employing Cramer's Rule to solve the demand and supply equilibrium problem offers a clear, algebraic method to determine the market's equilibrium price and quantity. By translating the demand and supply functions into a system of linear equations, calculating determinants, and applying the rule, economists and students can obtain precise solutions that illuminate the interplay between various market parameters. Mastery of this approach enhances analytical capabilities and deepens understanding of market dynamics, proving invaluable in both academic and practical economic analyses.

Summary of Key Steps:

1. Formulate demand and supply functions into a system of linear equations.
2. Express the system in matrix form.
3. Calculate the determinant of the coefficient matrix.
4. Use Cramer's Rule to find the determinants for P and Q.
5. Derive explicit formulas for equilibrium price and quantity.
6. Interpret the results in an economic context, considering parameter effects and conditions for validity.

Understanding these steps equips you with a systematic approach to solving similar problems involving linear demand and supply functions, reinforcing fundamental concepts in microeconomics and algebra.

Frequently Asked Questions

What is the general form of the demand function in Problem Set C (1)?

The demand function is given as La = A - bP, where La is the quantity demanded, A is the intercept, b is the slope, and P is the price.

What is the general form of the supply function in Problem Set C (1)?

The supply function is Qs = DP - C, where Qs is the quantity supplied, D is the slope, P is the price, and C is the intercept.

How do you set up the simultaneous equations for demand and supply in this problem?

You set La = Qs, which leads to the equations A - bP = DP - C, representing the equilibrium condition.

What is the first step in applying Cramer's Rule to solve for the equilibrium price P?

Rearrange the equations into standard linear form and write the coefficient matrix and constant matrix for the system.

How do you construct the coefficient matrix for the demand and supply equations?

The matrix consists of the coefficients of P in both equations: [[b + D, 0], [0, -1]], depending on the specific rearranged equations.

What determinants do you calculate when using Cramer's Rule to find the equilibrium price?

Calculate the main determinant of the coefficient matrix and the determinants replacing the relevant column with the constants to find P.

How is the equilibrium price P expressed using Cramer's Rule in this context?

P = (determinant of the matrix with the P-column replaced by constants) divided by (determinant of the coefficient matrix).

What are the conditions for a unique solution when applying Cramer's Rule to these functions?

The determinant of the coefficient matrix must be non-zero; otherwise, the system has no unique solution.

Why is Cramer's Rule useful in solving for equilibrium in demand and supply functions?

It provides a straightforward algebraic method to solve for variables explicitly when dealing with linear systems, such as demand and supply equations.

Additional Resources

Problem Set C (1) For Demand Function La = A-bP And Supply Function Qs = DP-C, Using Cramer's Rule Determine

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Introduction

In the realm of microeconomics, understanding how markets reach equilibrium is crucial for analyzing the interaction between demand and supply. When demand and supply functions are expressed algebraically, economists often employ mathematical tools to find the equilibrium price and quantity—that is, the point where the quantity demanded equals the quantity supplied. One such powerful tool is Cramer's Rule, a method rooted in linear algebra that provides explicit solutions to systems of linear equations.

This article delves into a specific problem set involving demand and supply functions:
- Demand function: La = A - bP

- Supply function: Qs = DP - C

Our objective is to determine the equilibrium price and quantity by applying Cramer's Rule. We will explore the derivation process step-by-step, providing clarity and insight into both the mathematical methodology and its economic significance.

Understanding the Functions: Demand and Supply

Before jumping into the solution, it’s essential to understand what these functions represent:

- Demand Function (La): This expresses the quantity demanded (La) as a function of the price (P). The parameters A and b are constants, where:
- A reflects the maximum demand when the price is zero (intercept).
- b indicates the rate at which demand decreases as the price increases (slope).

- Supply Function (Qs): This describes the quantity supplied (Qs) as a function of price (P). The parameters D and C are constants, where:
- D measures the responsiveness of supply to price changes.
- C is the fixed cost or baseline supply level.

For equilibrium, the quantity demanded equals the quantity supplied at the same price point, leading to the fundamental condition:

$$ La = Qs $$

Formulating the Equilibrium Conditions

In algebraic terms, the equilibrium condition becomes:

$$
A - bP = DP - C
$$

Rearranging this equation brings all variables to one side:

$$
A + C = DP + bP
$$

Or equivalently:

\[
A + C = P(D + b)
\]

This is a linear equation in terms of P (price). To incorporate the demand and supply functions into a system suitable for applying Cramer's Rule, we need to consider two equations—one for the demand and one for the supply—if additional conditions are involved. However, since demand and supply functions are expressed directly as functions of P, the standard approach involves setting them equal and solving for P.

Setting Up the System of Equations

In the typical application of Cramer's Rule, you work with a system of linear equations in multiple variables. Here, however, we have a single equation involving P. To illustrate the use of Cramer's Rule, let's consider a scenario where the problem involves two equations with two unknowns—say, price (P) and quantity (Q).

Suppose we are asked to find the equilibrium price (P) and equilibrium quantity (Q) given:

- Demand function: \(Q_d = A - bP \)
- Supply function: \(Q_s = D P - C \)

At equilibrium:

\[
Q_d = Q_s = Q^
\]

which yields two equations:

\[
Q^ = A - bP
\]
\[
Q^ = D P - C
\]

Now, the system becomes:

\[

$$
A - bP = D P - C
$$

Rearranged as:

$$
A + C = P(D + b)
$$

We can now express these as a system of two equations in two unknowns (Q and P). For clarity, let’s explicitly write the system:

1. $Q = A - bP$
2. $Q = D P - C$

At equilibrium, both equations equal Q, so:

$$
A - bP = D P - C
$$

which simplifies to:

$$
A + C = P(D + b)
$$

To apply Cramer's Rule, consider if we had two equations with two unknowns, perhaps derived from additional constraints or parameters. For example, suppose we have:

- Demand: $Q = A - bP$
- Supply: $Q = D P - C$

and an additional condition, such as a fixed quantity $Q = Q_0$, or perhaps a different parameterization involving two equations with P and Q as unknowns.

Note: For illustration purposes, assume the following system:

$$
\begin{cases}
Q = A - bP \\
Q = D P - C
\end{cases}

\]

which can be rewritten as:

\[
\begin{cases}
Q + bP = A \\
Q - D P = -C
\end{cases}
\]

This forms a system of two linear equations in two variables \(Q\) and \(P\):

\[
\boxed{
\begin{cases}
Q + bP = A \\
Q - D P = -C
\end{cases}
}
\]

Applying Cramer's Rule

Cramer's Rule provides a straightforward method to solve systems of linear equations:

\[
ax + by = e
\]
\[
cx + dy = f
\]

The solutions are:

\[
x = \frac{\det \mathbf{A}_x}{\det \mathbf{A}}
\]
\[
y = \frac{\det \mathbf{A}_y}{\det \mathbf{A}}
\]

where:

$$
\mathbf{A} = \begin{bmatrix}
a & b \\
c & d
\end{bmatrix}
$$

and matrices $\mathbf{A}_x$ and $\mathbf{A}_y$ are formed by replacing the respective columns with the constants vector.

In our case, the system:

$$
\begin{cases}
Q + bP = A \\
Q - D P = -C
\end{cases}
$$

can be written as:

$$
\begin{bmatrix}
1 & b \\
1 & -D
\end{bmatrix}
\begin{bmatrix}
Q \\
P
\end{bmatrix}
=
\begin{bmatrix}
A \\
-C
\end{bmatrix}
$$

The coefficient matrix:

$$
\mathbf{A} = \begin{bmatrix}

1 & b \\
1 & -D
\end{bmatrix}
\]

and the constants vector:

\[
\mathbf{B} = \begin{bmatrix}
A \\
-C
\end{bmatrix}
\]

Calculating the Determinants

First, compute the determinant of \(\mathbf{A}\):

\[
\det(\mathbf{A}) = (1)(-D) - (b)(1) = -D - b
\]

Provided that \(\det(\mathbf{A}) \neq 0\), solutions exist.

Next, find \(\det(\mathbf{A}_Q)\), where the first column is replaced by \(\mathbf{B}\):

\[
\mathbf{A}_Q = \begin{bmatrix}
A & b \\
-C & -D
\end{bmatrix}
\]

The determinant:

\[
\det(\mathbf{A}_Q) = A \times (-D) - b \times (-C) = -A D + b C
\]

Similarly, for \(P\):

\[
\mathbf{A}_P = \begin{bmatrix}
1 & A \\
1 & -C
\end{bmatrix}
\]

with determinant:

\[
\det(\mathbf{A}_P) = 1 \times (-C) - A \times 1 = -C - A
\]

Thus, the solutions are:

\[
Q^ = \frac{\det(\mathbf{A}_Q)}{\det(\mathbf{A})} = \frac{-A D + b C}{-D - b}
\]

\[
P^ = \frac{\det(\mathbf{A}_P)}{\det(\mathbf{A})} = \frac{-C - A}{-D - b}
\]

Note that the negative signs can be simplified:

\[
Q^ = \frac{A D - b C}{D + b}
\]

\[
P^ = \frac{C + A}{D + b}
\]

This provides explicit formulas for the equilibrium quantity and price based on the parameters of the demand and supply functions.

Interpretation of the Results

The derived formulas have significant economic interpretations:

- Equilibrium Price (\(P^\)):

$$
P^{\wedge} = \frac{A + C}{D + b}
$$

This indicates that the equilibrium price depends on the maximum demand at zero price (A), the fixed supply component (C), and the responsiveness parameters (b and D). If the demand is highly sensitive to price (b large), the equilibrium price tends to decrease, assuming other parameters are constant.

- Equilibrium Quantity ($Q^{\wedge}$):

$$
Q^{\wedge} = \frac{A D - b C}{D + b}
$$

Similarly, the equilibrium quantity hinges on the interplay between demand intercept A, supply responsiveness D, and the fixed component C.

[Problem Set C 1 For Demand Function La A Bp And Supply Function Qs Dp C Using Cramer S Rule Determine]()

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