Question 3 Find Whether The Vectorrs Are Parallel. (−2,1,−1) And (0,3,1) A. Parallel B. Collinearly Parallel

Question 3 Find Whether The Vectorrs Are Parallel. (−2,1,−1) And (0,3,1) A. Parallel B. Collinearly Parallel

Understanding whether two vectors are parallel or collinearly parallel is a fundamental aspect of vector algebra and is crucial in various fields such as physics, engineering, and computer graphics. When analyzing vectors, the primary goal is to determine their directional relationship—whether they point in the same or exactly opposite directions, or if they are unrelated. In this article, we will explore how to assess whether the vectors $\mathbf{v}_1 = (-2, 1, -1)$ and $\mathbf{v}_2 = (0, 3, 1)$ are parallel or collinearly parallel, including detailed steps, concepts, and relevant mathematical principles.

Understanding the Concepts: Parallel and Collinearly Parallel Vectors

What Does It Mean for Vectors to Be Parallel?

Vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ are said to be parallel if they have the same or exactly opposite direction. Mathematically, this occurs if one vector is a scalar multiple of the other:

$$
\mathbf{v}_1 = k \mathbf{v}_2 \quad \text{or} \quad \mathbf{v}_2 = k \mathbf{v}_1
$$

for some scalar k.

This scalar k can be positive or negative:
- If $k > 0$, the vectors point in the same direction.
- If $k < 0$, they point in opposite directions.

Example:

$$
(2, 4, 6) \text{ and } (1, 2, 3) \quad \text{are parallel because} \quad (2, 4, 6) = 2 \times (1, 2, 3)
$$

What Are Collinearly Parallel Vectors?

Collinearly parallel vectors are vectors that lie along the same line, regardless of their magnitude. In essence, these are vectors that are scalar multiples of each other, but they may have different lengths or directions (positive or negative scalar).

In the context of vectors, "collinearly parallel" is often used interchangeably with "parallel," emphasizing their alignment along a single line in space.

Mathematical Approach to Determine Parallelism

To determine whether two vectors are parallel or collinearly parallel, mathematicians typically employ the following methods:

Method 1: Scalar Multiple Check

Check if one vector can be expressed as a scalar multiple of the other:

$$
\mathbf{v}_2 = k \mathbf{v}_1
$$

This involves comparing each component:

$$
\frac{v_{2x}}{v_{1x}} = \frac{v_{2y}}{v_{1y}} = \frac{v_{2z}}{v_{1z}}
$$

where the denominators are non-zero.

Method 2: Cross Product Test

Calculate the cross product:

$$
\mathbf{v}_1 \times \mathbf{v}_2
$$

If the cross product results in the zero vector:

$$
\mathbf{v}_1 \times \mathbf{v}_2 = \mathbf{0}
$$

then the vectors are parallel or collinearly parallel.

Applying the Methods to the Given Vectors

Let's analyze the vectors:

$$
\mathbf{v}_1 = (-2, 1, -1)
$$

$$
\mathbf{v}_2 = (0, 3, 1)
$$

Step 1: Scalar Multiple Check

Attempt to find a scalar \(k\) such that:
\[
(0, 3, 1) = k \times (-2, 1, -1)
\]

This leads to the following component-wise equations:
1. \(0 = k \times (-2) \implies 0 = -2k \implies k = 0\)
2. \(3 = k \times 1 \implies 3 = k\)
3. \(1 = k \times (-1) \implies 1 = -k \implies k = -1\)

The scalar \(k\) derived from the first component is 0, from the second is 3, and from the third is -1. Since these values do not match, the vectors are not scalar multiples of each other.

Step 2: Cross Product Calculation

Calculate the cross product:
\[
\mathbf{v}_1 \times \mathbf{v}_2 =
\begin{vmatrix}
\mathbf{i} & \mathbf{j} & \mathbf{k} \\
-2 & 1 & -1 \\
0 & 3 & 1
\end{vmatrix}
\]

Expanding the determinant:
\[
\mathbf{i} (1 \times 1 - (-1) \times 3) - \mathbf{j} (-2 \times 1 - (-1) \times 0) + \mathbf{k} (-2 \times 3 - 1 \times 0)
\]

Calculate each component:
- \(\mathbf{i}\) component:
\[
1 \times 1 - (-1) \times 3 = 1 + 3 = 4
\]
- \(\mathbf{j}\) component:
\[
-2 \times 1 - (-1) \times 0 = -2 - 0 = -2
\]
- \(\mathbf{k}\) component:
\[
-2 \times 3 - 1 \times 0 = -6 - 0 = -6
\]

So, the cross product:
\[
\mathbf{v}_1 \times \mathbf{v}_2 = (4, 2, -6)
\]

Since this result is not the zero vector, the vectors are not parallel.

Conclusion: Are the Vectors Parallel or Collinearly Parallel?

Based on the calculations:
- The vectors $\mathbf{v}_1 = (-2, 1, -1)$ and $\mathbf{v}_2 = (0, 3, 1)$ are not scalar multiples of each other.
- The cross product is non-zero, indicating they are not parallel.

Therefore, the correct conclusion is that the vectors are neither parallel nor collinearly parallel.

Additional Insights and Applications

Understanding the relationship between vectors has practical applications:
- Physics: Determining forces acting along the same line.
- Computer Graphics: Calculating angles and directions for rendering.
- Engineering: Structural analysis involving parallel components.
- Mathematics: Solving systems of vector equations and analyzing geometric configurations.

Summary

- To determine if vectors are parallel, check if one is a scalar multiple of the other.
- The scalar multiple condition requires component-wise ratios to be equal.
- The cross product provides a quick test: zero vector indicates parallelism.
- For the vectors $(-2, 1, -1)$ and $(0, 3, 1)$, neither condition is satisfied, confirming they are not parallel.

Final Remarks

Determining whether vectors are parallel is a fundamental skill in vector algebra that underpins many advanced mathematical and physical concepts. Always start with the scalar multiple check, and complement it with the cross product test for verification. In the case of the vectors in question, the analysis clearly shows they are not parallel, emphasizing the importance of systematic approaches in vector analysis.

Keywords: vectors, parallel vectors, collinearly parallel, scalar multiple, cross product, vector algebra, mathematical analysis, vector relationships, physics, engineering

Frequently Asked Questions

Are the vectors (−2, 1, −1) and (0, 3, 1) parallel?

No, the vectors are not parallel because their components are not scalar multiples of each other.

How can you determine if two vectors are parallel?

Two vectors are parallel if one is a scalar multiple of the other, meaning their components are proportional.

Are the vectors (−2, 1, −1) and (0, 3, 1) collinearly parallel?

No, they are not collinearly parallel since they are not scalar multiples of each other.

What is the scalar multiple test for vector parallelism?

The scalar multiple test involves checking if all components of one vector are proportional to the corresponding components of the other vector by the same scalar.

Calculate if vectors (−2, 1, −1) and (0, 3, 1) are parallel.

Since the ratios (−2/0), (1/3), and (−1/1) are not all equal or defined consistently, the vectors are not parallel.

Can vectors be considered parallel if one has a zero component?

Yes, as long as the non-zero components are scalar multiples; zero components can still satisfy the proportionality condition if the other vector's corresponding component is also zero.

Is the vector (0, 3, 1) a scalar multiple of (−2, 1, −1)?

No, because the ratios between corresponding components are not consistent, so they are not scalar multiples.

What is the conclusion about the vectors (−2, 1, −1) and (0, 3, 1)?

They are not parallel or collinearly parallel.

How do you classify vectors that are not scalar multiples?

They are classified as neither parallel nor collinearly parallel.

What is the importance of checking vector components to determine parallelism?

Checking components helps verify if one vector is a scalar multiple of the other, which is essential for establishing parallelism.

Additional Resources

Question 3: Find whether the vectors (-2, 1, -1) and (0, 3, 1) are parallel or collinearly parallel

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Introduction

Understanding the relationship between vectors is fundamental in vector algebra, a branch of mathematics with widespread applications in physics, engineering, computer science, and more. Among the core concepts are the notions of parallelism and collinearity, which describe how vectors relate to each other in space. The question at hand involves determining whether the two vectors (-2, 1, -1) and (0, 3, 1) are parallel or collinearly parallel. This inquiry not only deepens our grasp of vector properties but also exemplifies essential analytical techniques used to analyze spatial relationships.

Definitions and Basic Concepts

Before delving into the problem, it is vital to establish clear definitions of the key terms involved:

1. Parallel Vectors

Two vectors are considered parallel if they have the same or exactly opposite direction, regardless of their magnitude. Formally, vectors A and B are parallel if there exists a scalar k such that:

$$\mathbf{A} = k \mathbf{B} \quad \text{or} \quad \mathbf{A} = -k \mathbf{B}$$

for some real number k. This scalar multiplication indicates that one vector is just a scaled version of the other, pointing along the same or directly opposite line.

2. Collinear Vectors

Collinearity extends the idea of parallelism. Two vectors are collinear if they lie along the same line, which means they are scalar multiples of each other, regardless of direction. In the context of vectors originating from the same point or considering their directions, collinearity implies that the vectors are parallel or anti-parallel.

3. Vector Components and Notation

Given vectors:

$$
\mathbf{A} = (a_1, a_2, a_3) \quad \text{and} \quad \mathbf{B} = (b_1, b_2, b_3)
$$

they can be analyzed for parallelism or collinearity through their components.

Analyzing the Given Vectors

The vectors in question are:

$$
\mathbf{A} = (-2, 1, -1)
$$

$$
\mathbf{B} = (0, 3, 1)
$$

To ascertain whether these vectors are parallel or collinear, we must examine their components for a scalar multiple relationship.

Step 1: Check for Scalar Multiplication

The primary test for parallelism is to determine if:

$$
\mathbf{A} = k \mathbf{B} \quad \text{or} \quad \mathbf{A} = -k \mathbf{B}
$$

for some scalar k.

This requires solving the following system of equations:

$$
-2 = k \times 0
$$

\[

$$
1 = k \times 3
$$

$$
-1 = k \times 1
$$

Let's analyze each component:

- From the first component:

$$
-2 = 0 \times k \implies -2 = 0
$$

which is not true, indicating that A cannot be a scalar multiple of B based on the first component alone.

- From the second component:

$$
1 = 3k \implies k = \frac{1}{3}
$$

- From the third component:

$$
-1 = 1 \times k \implies k = -1
$$

Since the scalar k derived from the second and third components do not match (i.e., $k = 1/3$ vs. $k = -1$), the vectors cannot be scalar multiples of each other.

Step 2: Conclusion from Scalar Multiplication

Because the scalar k is inconsistent across components, the vectors (-2, 1, -1) and (0, 3, 1) are not parallel. They do not point along the same or opposite line in space.

Alternative Approach: Vector Cross Product

Another robust method to verify whether two vectors are parallel or not involves calculating their cross product.

1. Cross Product Definition

The cross product of vectors A and B:

$$
\mathbf{A} \times \mathbf{B} = (a_2b_3 - a_3b_2, \; a_3b_1 - a_1b_3, \; a_1b_2 - a_2b_1)
$$

If the cross product yields the zero vector, then A and B are parallel or collinear; otherwise, they are not.

2. Calculation of Cross Product

Using the given vectors:

$$
\mathbf{A} = (-2, 1, -1)
$$

$$
\mathbf{B} = (0, 3, 1)
$$

Compute:

$$
\mathbf{A} \times \mathbf{B} = (1 \times 1 - (-1) \times 3, \; (-1) \times 0 - (-2) \times 1, \; -2 \times 3 - 1 \times 0)
$$

Simplify each component:

- First component:

$$
1 \times 1 - (-1) \times 3 = 1 + 3 = 4
$$

- Second component:

$$
(-1) \times 0 - (-2) \times 1 = 0 + 2 = 2
$$

- Third component:

$$
-2 \times 3 - 1 \times 0 = -6 - 0 = -6
$$

Thus,

$$
\mathbf{A} \times \mathbf{B} = (4, 2, -6)
$$

Since the cross product is not the zero vector, the vectors are not parallel.

Implications and Final Conclusions

The combined analytical approaches – scalar multiple verification and cross product calculation – conclusively demonstrate that the vectors (-2, 1, -1) and (0, 3, 1) are neither parallel nor collinearly parallel.

Summary of Findings:

- Scalar Multiple Test: No consistent scalar k exists to relate these vectors component-wise. Therefore, they are not scalar multiples of each other.

- Cross Product Test: The non-zero cross product indicates the vectors are not parallel.

Geometric Interpretation:

In three-dimensional space, the vectors in question point in different directions and do not lie along the same line. This means they are 'skew' relative to each other, not parallel or collinear.

Broader Context and Significance

Understanding vector relationships is crucial in multiple scientific and engineering contexts. For example:

- Physics: Determining whether forces are aligned or act along the same line affects equilibrium analysis.
- Computer Graphics: Calculating object orientations and lighting involves vector comparisons.
- Navigation and Robotics: Path planning relies on understanding directional vectors.
- Mathematical Modelling: Recognizing when vectors are parallel simplifies calculations, reduces computational complexity, and aids in solving systems of equations.

In educational settings, problems like these reinforce essential concepts such as vector scalar multiples and the geometric significance of the cross product.

Conclusion

In summary, the vectors (-2, 1, -1) and (0, 3, 1) are definitively not parallel nor collinearly parallel. This conclusion is grounded in the fundamental principles of vector algebra and verified through multiple analytical methods. Recognizing the relationships between vectors enhances spatial reasoning and mathematical problem-solving skills, vital for advanced studies and practical applications across various scientific disciplines.

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3 1 A Parallel B Collinearly Parallel

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