Through What Potential Difference Must An Electron Be Accelerated From Rest To Have A De Broglie Wavelength

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Understanding the relationship between electrons' wave-like behavior and their acceleration is fundamental in quantum physics and modern technology. When electrons are accelerated from rest through a potential difference, they gain kinetic energy that manifests as a specific de Broglie wavelength. This article explores the detailed process of determining the potential difference required for an electron to acquire a particular de Broglie wavelength, providing a comprehensive guide to the underlying physics principles and calculations involved.

Introduction to De Broglie Wavelength and Electron Acceleration

The concept of wave-particle duality, introduced by Louis de Broglie in 1924, revolutionized our understanding of matter at microscopic scales. De Broglie proposed that particles such as electrons exhibit wave-like properties characterized by a wavelength, known as the de Broglie wavelength. This wavelength (λ) is related to the particle's momentum (p) through the de Broglie relation:

- $\lambda = \frac{h}{p}$

where h is Planck's constant (6.626×10^{-34} Js).

When electrons are accelerated from rest through an electric potential difference (V), they gain kinetic energy, which in turn determines their momentum and thus their wavelength. To find the required potential difference for a specific de Broglie wavelength, we need to establish the connection between the electron's kinetic energy and the potential difference.

Fundamental Concepts and Equations

Before delving into the calculation, it is essential to understand some key physics principles and equations involved in the process.

Kinetic Energy of an Electron Accelerated Through a Potential Difference

When an electron is accelerated from rest through a potential difference V, it gains kinetic energy equal to the work done by the electric field:

- $KE = eV$

where:

- $e = 1.602 \times 10^{-19}$ C (elementary charge)
- V = potential difference in volts (V)

Since the electron starts from rest, all the energy gained manifests as kinetic energy:

- $KE = \frac{1}{2} m v^2$

where:

- $m = 9.109 \times 10^{-31}$ kg (electron mass)
- v = velocity of the electron

Equating the two expressions for energy:

$$
eV = \frac{1}{2} m v^2
$$

which allows us to find the electron's velocity after acceleration.

Momentum and de Broglie Wavelength

The momentum (p) of the electron is:

- $p = mv$

Using the de Broglie relation:

- $\lambda = \frac{h}{p} = \frac{h}{mv}$

Thus, to find the potential difference V for a given λ, we need to relate λ to V via the electron's velocity v.

Deriving the Formula for the Required Potential Difference

The process involves combining the above relationships to express V in terms of λ.

Step 1: Express Velocity in Terms of V

From the energy equation:

$$
eV = \frac{1}{2} m v^2
$$

Rearranged to solve for v:

$$
v = \sqrt{\frac{2 e V}{m}}
$$

Step 2: Express Momentum in Terms of V

Since $p = mv$:

$$
p = m \sqrt{\frac{2 e V}{m}} = \sqrt{2 m e V}
$$

Step 3: Express Wavelength in Terms of V

Using the de Broglie relation:

\[
\lambda = \frac{h}{p} = \frac{h}{\sqrt{2 m e V}}
\]

Rearranged to solve for \(V\):

\[
V = \frac{h^2}{2 m e \lambda^2}
\]

This is the key formula: the potential difference \(V\) needed to accelerate an electron from rest to have a de Broglie wavelength \(\lambda\).

Calculating the Potential Difference for a Given De Broglie Wavelength

Using the derived formula:

\[
V = \frac{h^2}{2 m e \lambda^2}
\]

we can plug in the known constants to find the required potential difference for any specified wavelength.

Constants:

- Planck's constant, \(h = 6.626 \times 10^{-34}\) Js
- Electron mass, \(m = 9.109 \times 10^{-31}\) kg
- Elementary charge, \(e = 1.602 \times 10^{-19}\) C

Example Calculation: Electron Wavelength of 1 nm

Suppose we want the electron to have a de Broglie wavelength of \(\lambda = 1 \text{ nm} = 1 \times 10^{-9} \text{ m}\).

Plug into the formula:

\[
V = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.109 \times 10^{-31} \times

1.602 \times 10^{-19} \times (1 \times 10^{-9})^2}
\]

Calculating numerator:

$$
(6.626 \times 10^{-34})^2 = 4.39 \times 10^{-67}
$$

Calculating denominator:

$$
2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 1 \times 10^{-18} \approx 2 \times 9.109 \times 1.602 \times 10^{-68}
$$

Simplifying:

$$
2 \times 9.109 \times 1.602 \approx 29.2
$$

So denominator:

$$
29.2 \times 10^{-68} = 2.92 \times 10^{-67}
$$

Therefore,

$$
V \approx \frac{4.39 \times 10^{-67}}{2.92 \times 10^{-67}} \approx 1.50 \text{ volts}
$$

Result: The electron must be accelerated through approximately 1.50 volts to have a de Broglie wavelength of 1 nm.

Implications and Applications

Knowing the potential difference required for specific de Broglie wavelengths is fundamental in various fields:

- **Electron Microscopy:** Achieving electrons with very short wavelengths improves resolution in electron microscopes.
- **Quantum Mechanics Experiments:** Understanding wave-particle duality and interference phenomena relies on precise wavelength control.

- **Particle Accelerators:** Designing accelerators to produce particles with specific wavelengths for research purposes.

Furthermore, the relationship underscores the importance of accelerating electrons to high voltages to probe phenomena at atomic and subatomic scales.

Limitations and Considerations

While the derived formula provides a straightforward way to calculate the required potential difference, some limitations and considerations include:

- **Relativistic Effects:** At very high energies, electrons approach the speed of light, and relativistic mechanics must be used instead of classical equations.
- **Quantum Limitations:** Extremely short wavelengths may require immense potentials, which are practically challenging to achieve.
- **Material Constraints:** High voltages may cause breakdown or damage in experimental setups.

For most practical purposes involving wavelengths in the nanometer range, the non-relativistic formula remains sufficiently accurate.

Summary

To determine the potential difference necessary to accelerate an electron from rest to attain a specific de Broglie wavelength, the key formula is:

$$
\boxed{
V = \frac{h^2}{2 m e \lambda^2}
}
$$

where:

- h is Planck's constant
- m is the electron mass

- e is the elementary charge
- λ is the desired de Broglie wavelength

This equation highlights the inverse-square relationship between the wavelength and the potential difference, emphasizing how shorter wavelengths require significantly higher voltages.

In conclusion, understanding the potential difference required to achieve a specific de Broglie wavelength equips scientists and engineers with critical insights for manipulating electron behavior in various technological and experimental applications. Whether in electron microscopy, quantum physics research, or particle acceleration, this fundamental relationship bridges classical electrostatics with quantum

Frequently Asked Questions

What is the relation between the de Broglie wavelength of an electron and the potential difference it is accelerated through?

The de Broglie wavelength λ of an electron accelerated through a potential difference V is given by $\lambda = h / \sqrt{(2meV)}$, where h is Planck's constant and me is the electron's mass.

How do you calculate the potential difference needed to give an electron a specific de Broglie wavelength?

Rearranging the de Broglie wavelength formula, $V = (h^2) / (2me\lambda^2)$. Substitute the desired λ to find the required potential difference V.

What physical constants are involved in determining the potential difference for a given de Broglie wavelength?

Planck's constant (h) and the mass of the electron (me) are involved in the calculation, along with the desired wavelength λ.

Why must an electron be accelerated from rest to observe wave-like behavior related to the de Broglie

wavelength?

Because an electron's wave properties are observed when it has a finite momentum, which is obtained by accelerating it through a potential difference from rest.

What is the approximate potential difference needed to give an electron a de Broglie wavelength of 0.1 nm?

Using $V \approx 12.4 / \lambda^2$ (with λ in nm), $V \approx 12.4 / (0.1)^2 = 1240$ volts. So, approximately 1240 V is needed.

How does increasing the potential difference affect the de Broglie wavelength of an electron?

Increasing the potential difference increases the electron's kinetic energy and momentum, which decreases its de Broglie wavelength.

Is the potential difference required to achieve a certain de Broglie wavelength dependent on the electron's rest mass?

Yes, because the calculation involves the electron's mass; a different mass would alter the relation between potential difference and wavelength.

Can the de Broglie wavelength be observed directly, and how does the potential difference facilitate this observation?

Direct observation is challenging, but the wavelength influences phenomena like electron diffraction. Accelerating electrons through a potential difference ensures they have the appropriate wavelength to produce observable interference patterns.

What is the significance of understanding the potential difference in experiments involving electron wave properties?

It allows precise control of the electron's momentum and wavelength, enabling the study of wave-particle duality and quantum phenomena such as diffraction and interference.

Additional Resources

Potential Difference Required for Electron Acceleration to a Specific De Broglie Wavelength: An In-Depth Analysis

Understanding the relationship between potential difference and the resulting de Broglie wavelength of an electron is fundamental in quantum mechanics and electron optics. This comprehensive review delves into the physics principles, mathematical derivations, and practical implications of determining the potential difference necessary to accelerate an electron from rest to a state where its wave-like nature, characterized by the de Broglie wavelength, becomes significant.

Introduction to Electron Wave Nature and De Broglie Wavelength

The dual wave-particle nature of electrons is a cornerstone of quantum physics. Louis de Broglie hypothesized that particles such as electrons exhibit wave-like properties, leading to the concept of the de Broglie wavelength:

$$
\lambda = \frac{h}{p}
$$

where:
- λ is the de Broglie wavelength,
- h is Planck's constant ($6.626 \times 10^{-34}\, \mathrm{Js}$),
- p is the momentum of the particle.

This relationship implies that the wavelength is inversely proportional to the particle's momentum. For electrons accelerated from rest, their kinetic energy and momentum increase with the potential difference applied, directly affecting their wavelength.

The Significance of the Potential Difference in Electron Acceleration

Applying a potential difference V to an electron initially at rest imparts kinetic energy to the particle. This process is fundamental in electron microscopes, particle accelerators, and wave-particle experiments, where controlling the electron's wavelength is crucial.

Key points:
- The potential difference determines the electron's kinetic energy.
- The kinetic energy influences the de Broglie wavelength.
- Precise control of V allows for desired wave properties.

Physical Principles Governing Electron Acceleration

When an electron is accelerated through a potential difference V, it gains kinetic energy equal to the work done by the electric field:

$$
K.E. = eV
$$

where:
- e is the elementary charge ($1.602 \times 10^{-19}\, \mathrm{C}$),
- V is the potential difference in volts.

Assuming non-relativistic speeds (which is valid for relatively small V), the kinetic energy relates to the electron's momentum as:

$$
K.E. = \frac{p^2}{2m}
$$

with:
- $m = 9.109 \times 10^{-31}\, \mathrm{kg}$ (electron mass),
- p is the momentum.

By combining these equations, we can derive the relationship between V and the de Broglie wavelength.

Derivation of the Potential Difference for a Given de Broglie Wavelength

Step 1: Equate kinetic energy from electric work to the classical kinetic energy:

$$
eV = \frac{p^2}{2m}
$$

Step 2: Express momentum in terms of de Broglie wavelength:

$$
p = \frac{h}{\lambda}
$$

Step 3: Substitute p into the kinetic energy equation:

$$
eV = \frac{(h/\lambda)^2}{2m}
$$

Step 4: Solve for V:

$$
V = \frac{h^2}{2me \lambda^2}
$$

This formula gives the potential difference required to accelerate an electron from rest to a state where its de Broglie wavelength is λ.

Numerical Calculation and Practical Examples

To solidify understanding, let's evaluate the potential difference needed for specific wavelengths.

Constants:
- $h = 6.626 \times 10^{-34}\, \mathrm{Js}$,
- $m = 9.109 \times 10^{-31}\, \mathrm{kg}$,
- $e = 1.602 \times 10^{-19}\, \mathrm{C}$.

Example 1: Electron wavelength of 0.1 nm (typical in electron microscopy).

$$
\lambda = 0.1\, \mathrm{nm} = 1.0 \times 10^{-10}\, \mathrm{m}
$$

Plug into the formula:

$$
V = \frac{(6.626 \times 10^{-34})^2}{2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times (1.0 \times 10^{-10})^2}
$$

Calculating numerator:

$$
(6.626 \times 10^{-34})^2 = 4.39 \times 10^{-67}
$$

Calculating denominator:

$$
2 \times 9.109 \times 10^{-31} \times 1.602 \times 10^{-19} \times 1.0 \times 10^{-20} \approx 2.92 \times 10^{-69}
$$

Result:

$$
V \approx \frac{4.39 \times 10^{-67}}{2.92 \times 10^{-69}} \approx 150.3\, \mathrm{V}
$$

Interpretation: To achieve a de Broglie wavelength of 0.1 nm, the electron must be accelerated through approximately 150 volts.

Example 2: Electron wavelength of 1 nm:

$$
\lambda = 1 \times 10^{-9}\, \mathrm{m}
$$

Following similar calculations, the potential difference reduces to approximately 1.5 volts.

Relativistic Corrections and Their Impact

At higher energies (typically above a few tens of keV), the non-relativistic approximation becomes invalid because electrons approach speeds comparable to the speed of light ($c = 3 \times 10^8\, \mathrm{m/s}$).

Relativistic energy considerations:

$$
K.E. = (\gamma - 1) m c^2
$$

where:
- $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$.

The relativistic momentum:

$$
p = \gamma m v
$$

and the de Broglie wavelength:

$$
\lambda = \frac{h}{p}
$$

Relativistic relationship between kinetic energy and momentum:

$$
K.E. = \sqrt{(pc)^2 + (m c^2)^2} - m c^2
$$

Adjusted formula for potential difference:

$$
V = \frac{(\gamma - 1) m c^2}{e}
$$

Given the complexity, for high energies, use relativistic formulas to accurately determine the potential difference required.

Practical Implications and Applications

Understanding the potential difference in relation to the de Broglie wavelength has several technological and scientific applications:

- Electron Microscopy: Achieving high resolution requires electrons with very short wavelengths (~0.1 nm), demanding accelerations of around 100–200 V, depending on relativistic effects.

- Particle Accelerators: In high-energy physics, electrons are accelerated to MeV energies, where relativistic corrections are essential, and the de Broglie wavelength becomes exceedingly small.

- Quantum Experiments: Precise control over the electron's wavelength enables interference experiments, diffraction studies, and tests of quantum mechanics principles.

Limitations and Assumptions in Derivations

While the derivation provides valuable insights, it is based on certain assumptions:

- Non-Relativistic Approximation: Valid only for low to moderate energies; at high energies, relativistic effects dominate.
- Electrostatic Acceleration: Assumes electrons are accelerated solely by a potential difference without other forces or fields.
- Neglect of Electron-Photon Interactions: Assumes ideal conditions without energy losses or scattering.
- Point Particle Model: Assumes electrons behave as point particles, ignoring quantum field effects.

Recognizing these limitations helps refine experimental designs and theoretical calculations.

Summary and Final Remarks

- The potential difference V required to accelerate an electron from rest to achieve a particular de Broglie wavelength λ is given by:

$$
V = \frac{h^2}{2 m e \lambda^2}
$$

- For typical electron wavelengths used in microscopy (~0.1 nm), the necessary accelerating voltage is approximately 150 V.
- As the wavelength decreases (implying higher resolution or energy), the required potential difference increases quadratically.
- Incorporating relativistic effects becomes essential at higher energies, modifying the simple formula and demanding more sophisticated calculations.
- Practical applications leverage this relationship to control electron wave properties in scientific research, imaging technology, and particle physics.

In conclusion, understanding the potential difference needed to produce a certain de Broglie wavelength is fundamental for manipulating electron wave properties. The derived formula provides a straightforward method to estimate this potential, enabling scientists and engineers to tailor electron beams for various advanced technological applications.

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