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Triangle UVW Has Vertices At U(2, 0), V(3, 1), W(3, 3). Determine The Vertices Of Image UVW If The Preimage

Understanding how to determine the vertices of a transformed triangle is a fundamental concept in coordinate geometry and linear transformations. When given a specific triangle in the coordinate plane, such as UVW with vertices at U(2, 0), V(3, 1), and W(3, 3), it becomes essential to analyze how transformations—such as translations, rotations, reflections, scalings, or shears—affect the shape and position of the triangle. This article provides a comprehensive guide to determining the vertices of the image triangle UVW after a transformation, including detailed steps, mathematical explanations, and practical examples to enhance your understanding.

Understanding the Preimage Triangle UVW

Before diving into transformations, it’s crucial to understand the initial triangle's properties:

- Vertices of the preimage triangle UVW:
- U(2, 0)
- V(3, 1)
- W(3, 3)

- Key features:
- The points are located in the coordinate plane.
- The shape is a triangle with a base along the line connecting W(3, 3) and V(3, 1), indicating a vertical line segment.
- The triangle's orientation and position are essential for predicting how transformations will affect its vertices.

Fundamentals of Coordinate Transformations

Transformations in the coordinate plane change the position, size, or orientation of geometric figures. The primary types include:

1. Translation

- Moves every point of the figure by the same distance in a given direction.
- For example, shifting the triangle 3 units right and 2 units up.

2. Rotation

- Rotates the figure around a fixed point (usually the origin) by a certain angle.
- The direction can be clockwise or counterclockwise.

3. Reflection

- Flips the figure across a line (e.g., x-axis, y-axis, or any line).

4. Scaling (Dilation)

- Enlarges or reduces the figure proportionally from a fixed point called the center of dilation.

5. Shear

- Slants the shape in a particular direction, changing its angles but not necessarily its area.

Each transformation can be represented algebraically using matrices or functions, making it possible to calculate the new vertices precisely.

Determining the Image Vertices of Triangle UVW

The process to find the vertices of the transformed triangle involves applying the specific transformation rules to each vertex of the preimage. Depending on the transformation type, the method varies.

Step 1: Identify the Transformation Type

- Clarify what transformation is applied to the preimage triangle.
- For example, is it a translation by a certain vector? A rotation by a specific angle? Or a combination?

Step 2: Write the Transformation Rules

- For a translation, the rule is:
- $(x, y) \rightarrow (x + dx, y + dy)$
- For rotation about the origin by angle θ:
- $(x, y) \rightarrow (x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$
- For scaling with scale factor k:
- $(x, y) \rightarrow (kx, ky)$
- For reflection across a line:
- Depends on the line's equation.

Step 3: Apply Transformation to Each Vertex

- Use the rules to compute the new coordinates for U, V, and W.

Step 4: Verify & Sketch

- Plot the transformed vertices to confirm the shape and position of the image triangle.
- Check for consistency with the transformation's properties.

Example: Applying a Specific Transformation to Triangle UVW

Suppose we are asked to find the vertices of the image triangle UVW after the following transformation:

- Translation by vector (2, 3)
- Rotation around the origin by 90° counterclockwise
- Scaling by a factor of 2 from the origin

Let's explore how each step affects the vertices:

1. Translation by (2, 3)

- $U(2, 0) \rightarrow (2 + 2, 0 + 3) = (4, 3)$
- $V(3, 1) \rightarrow (3 + 2, 1 + 3) = (5, 4)$
- $W(3, 3) \rightarrow (3 + 2, 3 + 3) = (5, 6)$

2. Rotation by 90° CCW around the origin

- Use the rotation formulas:
- $(x, y) \rightarrow (-y, x)$

Applying to each point:
- U(4, 3) → (-3, 4)
- V(5, 4) → (-4, 5)
- W(5, 6) → (-6, 5)

3. Scaling by a factor of 2

- (x, y) → (2x, 2y)

Applying to each:
- U(-3, 4) → (-6, 8)
- V(-4, 5) → (-8, 10)
- W(-6, 5) → (-12, 10)

Resulting vertices of the transformed triangle:
- U'(-6, 8)
- V'(-8, 10)
- W'(-12, 10)

This example illustrates how multiple transformations combine to produce the final image of the triangle on the coordinate plane.

Mathematical Tools for Transformation Calculations

To facilitate precise transformation calculations, several mathematical tools are used:

1. Transformation Matrices

- Matrices allow compact representation of transformations, especially for linear transformations like rotations, scalings, and shears.
- For example, a rotation matrix for θ degrees:

$$
R(\theta) = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
$$

- Applying the matrix to the point coordinates yields the transformed vertices.

2. Vector Addition for Translations

- Adding the translation vector to each point's coordinate.

3. Software Tools

- Graphing calculators, GeoGebra, Desmos, and other software can automate these calculations and visualize the transformed figures.

Practical Applications and Real-World Relevance

Understanding how to determine the vertices of a transformed triangle like UVW is essential in various fields:

- Computer Graphics: Rendering objects and their transformations.
- Engineering: Designing components and simulating their movements.
- Robotics: Path planning involving coordinate transformations.
- Mathematics Education: Developing spatial reasoning and understanding geometric transformations.

Summary and Key Takeaways

- The initial triangle UVW with vertices at U(2, 0), V(3, 1), and W(3, 3) serves as a foundation for understanding geometric transformations.
- Identifying the transformation type is essential before calculating the image vertices.
- Applying algebraic rules or matrix operations helps find the new positions of each vertex accurately.
- Combining multiple transformations involves sequential application of rules or matrix multiplications.
- Visual verification through plotting ensures the accuracy of calculations.
- Mastery of these concepts enhances spatial reasoning and problem-solving skills in geometry.

Conclusion

Determining the vertices of an image triangle after a transformation involves a clear understanding of geometric rules, algebraic operations, and the ability to visualize changes in the coordinate plane. Whether you're studying basic transformations for educational purposes or applying them in advanced fields like computer graphics and engineering, mastering these techniques is invaluable. By practicing with specific examples, such as transforming triangle UVW with given vertices, you develop both your analytical skills and your geometric intuition, empowering you to tackle complex problems with confidence.

If you are interested in further exploring transformations, consider practicing with different types of transformations and combining multiple transformations to deepen your understanding of coordinate geometry concepts.

Frequently Asked Questions

What are the coordinates of the vertices of triangle UVW?

U(2, 0), V(3, 1), W(3, 3).

How do you find the image of triangle UVW under a transformation?

Apply the given transformation (e.g., translation, rotation, reflection, dilation) to each vertex's coordinates to find the image's vertices.

What information do you need to determine the vertices of the image of triangle UVW?

The vertices of the original triangle and the specific transformation applied.

If the transformation is a translation by vector (a, b), how do you find the new vertices?

Add the translation vector to each vertex: for example, new U' = (2 + a, 0 + b).

How does a reflection across the x-axis affect the vertices of triangle UVW?

Reflect each vertex across the x-axis by changing the y-coordinate's sign: (x, y) becomes (x, -y).

What is the image of vertex V(3, 1) if the transformation is a 90-degree rotation about the origin?

The image of V(3, 1) after a 90-degree rotation is V'(-1, 3).

How can you verify the vertices of the transformed triangle are correct?

By applying the transformation to each vertex and checking the resulting coordinates satisfy the transformation's rules.

What is the significance of knowing the preimage vertices in coordinate geometry?

It allows you to determine how a shape is transformed and to find the exact location of its image after a transformation.

If the transformation is a dilation with scale factor 2 centered at the origin, what are the new vertices?

Multiply each coordinate by 2: U(2, 0) becomes (4, 0), V(3, 1) becomes (6, 2), W(3, 3) becomes (6, 6).

Additional Resources

Understanding the Transformation of Triangle UVW: From Preimage to Image

In the realm of coordinate geometry, transformations serve as powerful tools that allow us to manipulate figures within a plane, preserving or altering their size, shape, or position. When analyzing a triangle such as UVW with known vertices, understanding how these points behave under various transformations is crucial for applications across mathematics, engineering, computer graphics, and related fields. This article provides an in-depth exploration of the triangle UVW with vertices at U(2, 0), V(3, 1), and W(3, 3), focusing on determining the vertices of its image when subjected to a transformation—specifically, a preimage transformation that may involve translation, rotation, reflection, dilation, or a combination thereof.

Foundations: Coordinates and Triangle UVW

Vertices of Triangle UVW

The initial step in understanding the transformation involves examining the given coordinates:

- U(2, 0): This point lies 2 units along the x-axis and on the x-axis itself, at y=0.
- V(3, 1): Located 3 units along x and 1 unit along y.
- W(3, 3): Positioned directly above V along the vertical line x=3.

Plotting these points reveals a triangle with a base along the vertical line x=3 connecting V and W, and point U situated to the left at x=2. The shape of UVW is asymmetrical but can be characterized further through analysis of side lengths and angles.

Geometric Properties of UVW

To analyze the triangle, it's useful to compute the lengths of its sides:

- Side UV: Distance between U(2, 0) and V(3, 1):

$$
\text{UV} = \sqrt{(3 - 2)^2 + (1 - 0)^2} = \sqrt{1^2 + 1^2} = \sqrt{2} \approx 1.414
$$

- Side VW: Distance between V(3, 1) and W(3, 3):

$$
\text{VW} = \sqrt{(3 - 3)^2 + (3 - 1)^2} = \sqrt{0 + 2^2} = 2
$$

- Side UW: Distance between U(2, 0) and W(3, 3):

$$
\text{UW} = \sqrt{(3 - 2)^2 + (3 - 0)^2} = \sqrt{1^2 + 3^2} = \sqrt{10} \approx 3.162
$$

These measurements suggest that UVW is scalene (all sides of different lengths), with an acute angle at U, and right or obtuse angles at V and W can be further analyzed through slopes and dot product calculations.

Transformations in Coordinate Geometry: An Overview

Transformations are functions that map points from one position to another within the plane, often characterized by specific rules.

Types of Geometric Transformations

1. Translations: Moving every point of a figure by a fixed vector (e.g., shifting right, left, up, or down).
2. Rotations: Rotating a figure about a fixed point (typically the origin) by a certain angle.
3. Reflections: Flipping a figure across a line (like the x-axis, y-axis, or any arbitrary line).
4. Dilations (Scaling): Enlarging or reducing a figure proportionally from a fixed point (center of dilation).
5. Shear Transformations: Slanting the shape in a particular direction.

Each transformation can be described mathematically through matrices or algebraic formulas acting on the coordinates.

Preimage and Image

- Preimage: The original figure before transformation.
- Image: The figure after transformation.

In our context, the preimage is the triangle UVW with known vertices, and the goal is to determine the new vertices of the image after applying a specific transformation.

Determining the Image of UVW: Theoretical Approach

Since the user question indicates "if the preimage," but does not specify the

transformation, a comprehensive review considers common transformations and their effects.

Scenario 1: Translation

Suppose the transformation is a translation by vector $\vec{d} = (a, b)$. Then, each vertex moves as:

$$
U' = (x_U + a, y_U + b)
$$
$$
V' = (x_V + a, y_V + b)
$$
$$
W' = (x_W + a, y_W + b)
$$

For example, translating by $\vec{d} = (2, 3)$:

- $U' = (2 + 2, 0 + 3) = (4, 3)$
- $V' = (3 + 2, 1 + 3) = (5, 4)$
- $W' = (3 + 2, 3 + 3) = (5, 6)$

This straightforward approach preserves

distances and angles, merely shifting the entire triangle.

Scenario 2: Rotation About the Origin

If the triangle is rotated about the origin by an angle θ, the vertices transform as:

$$
x' = x \cos \theta - y \sin \theta
$$
$$
y' = x \sin \theta + y \cos \theta
$$

Suppose $\theta = 90^\circ$ (counterclockwise):

$$
\cos 90^\circ = 0,\quad \sin 90^\circ = 1
$$

Applying to each vertex:

- $U(2, 0)$:

$$
x' = 2 \times 0 - 0 \times 1 = 0,\quad y' = 2
$$

\times 1 + 0 \times 0 = 2
\]
\(\Rightarrow U' = (0, 2)\)

- \(V(3, 1)\):

\[
x' = 3 \times 0 - 1 \times 1 = -1,\quad y' = 3 \times 1 + 1 \times 0 = 3
\]
\(\Rightarrow V' = (-1, 3)\)

- \(W(3, 3)\):

\[
x' = 3 \times 0 - 3 \times 1 = -3,\quad y' = 3 \times 1 + 3 \times 0 = 3
\]
\(\Rightarrow W' = (-3, 3)\)

This rotation significantly alters the triangle's orientation but preserves side lengths.

Scenario 3: Reflection Across a Line

Reflecting across the y-axis, for example, transforms each point as:

$$
(x, y) \rightarrow (-x, y)
$$

Applying to U(2, 0):

$$
U' = (-2, 0)
$$

Similarly, V(3, 1):

$$
V' = (-3, 1)
$$

And W(3, 3):

$$
W' = (-3, 3)
$$

Reflections change the figure's orientation but keep distances intact in the case of reflection across lines through the origin.

Scenario 4: Dilation (Scaling) About the Origin

Dilation with scale factor k:

$$
(x', y') = (k \times x, k \times y)
$$

For example, with $k=2$:

- $U' = (4, 0)$
- $V' = (6, 2)$
- $W' = (6, 6)$

This enlarges the triangle proportionally, maintaining its shape.

Mathematical Tools for Determining the Image

To systematically determine the vertices after transformation, mathematicians employ several tools:

- Transformation Matrices: For linear transformations like rotation, reflection, and dilation, matrices provide a compact and efficient way to compute the new coordinates.
- Vector Addition: For translations, simply adding the translation vector to each point.
- Coordinate Geometry Equations: For complex

transformations involving arbitrary lines or angles, equations of lines and rotation formulas are used.

Application: Specific Case and Calculation

Supposing the transformation is a specific linear transformation represented by a matrix \(A\), such as:

\[
A = \begin{bmatrix}
a_{11} & a_{12} \\
a_{21} & a_{22}
\end{bmatrix}
\]

then the image vertices are calculated as:

\[
\begin{bmatrix}
x' \\
y'
\end{bmatrix}
= A \times
\begin{bmatrix}

x \\
y
\end{bmatrix}
\]

For example, consider a shear transformation:

\[
A = \begin{bmatrix}
1 & k \\
0 & 1
\end{bmatrix}
\]

which shifts points horizontally

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